RELATED RATES HW #1 NAME:____________________ PER:____

Find the missing value

1) Given $x^2 + y^2 = 3\cancel{6}4$ find $\frac{dx}{dt}$,

when $x = 5$, $y = 3$, and $\frac{dy}{dt} = 2$.

$\rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2x\frac{dx}{dt} = -2y\frac{dy}{dt}$

$\frac{dx}{dt} = -\frac{y}{x} \cdot \frac{dy}{dt}$

$\left.\frac{dx}{dt}\right|_{\substack{x=5 \\ y=3 \\ dy/dt=2}} = -\frac{6}{5}$

2) Given $x^2y = \overset{98}{\cancel{18}}$ find $\frac{dy}{dt}$,

when $x = -7$, $y = 2$, and $\frac{dx}{dt} = 5$.

$\rightarrow 2xy\frac{dx}{dt} + x^2\frac{dy}{dt} = 0$

$x^2\frac{dy}{dt} = -2xy\frac{dx}{dt}$

$\frac{dy}{dt} = -\frac{2y}{x}\frac{dx}{dt}$

$\left.\frac{dy}{dt}\right|_{\substack{x=-7 \\ y=2 \\ dx/dt=5}} = \frac{20}{7}$

3) The radius of a sphere is increasing at a rate of 8 ft/min. $\left(V = \frac{4}{3}\pi r^3\right)$ $\left(A = 4\pi r^2\right)$

a) Find the rate of change of the volume when $r = 4$.

a) $\frac{dr}{dt} = 8$

$\left.\frac{dV}{dt}\right|_{r=4}$?

$\frac{dV}{dt} = \frac{4\pi}{3} \times 3r^2\frac{dr}{dt}$

$= 4\pi r^2\frac{dr}{dt}$

$\left.\frac{dV}{dt}\right|_{r=4} = 64\pi \times 8$

$= \underline{512\pi \text{ ft}^3/\text{min}}$

b) How fast is the surface area changing when $r = 4$?

$\frac{dA}{dt} = 8\pi r\frac{dr}{dt}$

$\left.\frac{dA}{dt}\right|_{r=4} = 32\pi \times 8$

$= \underline{256\pi \text{ ft}^2/\text{min}}$

4) All edges of a cube are expanding at a rate of 6 cm/s. How fast is the......

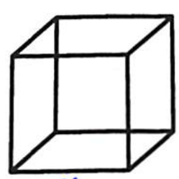
a) volume changing when each edge is 2cm?

$V = x^3$

$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$

$\left.\frac{dV}{dt}\right|_{x=2} = 12 \times 6$

$= \underline{72 \text{ cm}^3/\text{s}}$



$\frac{dx}{dt} = 6$

b) surface area changing when each edge is 2cm?

$A = 6x^2$

$\frac{dA}{dt} = 12x\frac{dx}{dt}$

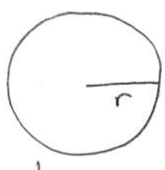
$\left.\frac{dA}{dt}\right|_{x=2} = 12(2) \cdot 6$

$= 144 \text{ cm}^2/\text{s}$

5) Water is spilling onto the ground and forming a circular shape. The radius of the puddle is changing at the rate of 2 inches per second. How fast is the....

a) area of the circle changing when the area is 121π?

$$A = \pi r^2$$



$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$\frac{dr}{dt} = 2$

• when $A = 121\pi$, then $r = 11$

$$\Rightarrow \left.\frac{dA}{dt}\right|_{A=121\pi} = 22\pi \times 2$$

$$= \underline{44\pi \text{ in}^2/\text{s}}$$

b) circumference changing when the radius is 7 in.?

$$C = 2\pi r$$

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

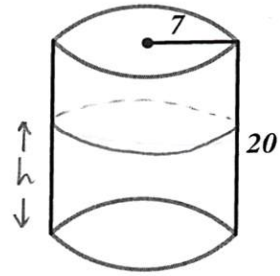
$$\left.\frac{dC}{dt}\right|_{r=7} = \underline{4\pi \text{ in/s}}$$

6) A cylinder with radius 7 ft and height 20 feet is losing water at a rate of 6 ft^3/min. How fast is the height changing when $h = 8$? How fast is the height changing when $h = 16$?

$$V = 49\pi h$$

$$\frac{dV}{dt} = 49\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{49\pi} \cdot \frac{dV}{dt}$$



$$\left.\frac{dh}{dt}\right|_{h=8} = \left.\frac{dh}{dt}\right|_{h=16} = \underline{-\frac{6}{49\pi} \text{ ft/min}}$$

doesn't depend on h...

$\frac{dV}{dt} = -6$

$\left.\frac{dh}{dt}\right|_{h=8}$? $\left.\frac{dh}{dt}\right|_{h=16}$?

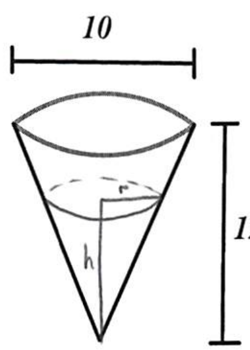
7) A conical tank (with vertex down) is 10 feet across the top and 12 feet deep. If water is flowing into the tank at a rate of 10 ft^3 / min., find the rate of change of the depth of the water when the water is 8 feet deep?

$$V = \frac{1}{3} \times \pi r^2 \times h$$

• $\frac{r}{5} = \frac{h}{12}$

$r = \frac{5}{12} h$

$\frac{dr}{dt} = \frac{5}{12}\frac{dh}{dt}$



$$\frac{dV}{dt} = \frac{\pi}{3}\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right)$$

• when $h = 8$:

$$10 = \frac{\pi}{3}\left(2 \times \frac{10}{3} \times 8 \times \frac{5}{12}\frac{dh}{dt} + \frac{100}{9} \times \frac{dh}{dt}\right)$$

• when $h = 8$: $r = \frac{10}{3}$

$$\frac{30}{\pi} = \frac{200}{9}\frac{dh}{dt} + \frac{100}{9}\frac{dh}{dt}$$

$$\frac{30}{\pi} = \frac{100}{3}\frac{dh}{dt}$$

$$\left.\frac{dh}{dt}\right|_{h=8} = \underline{\frac{9}{10\pi} \text{ ft/min}}$$

$\frac{dV}{dt} = 10$

$\left.\frac{dh}{dt}\right|_{h=8}$?

RELATED RATES HW #2 NAME:________________________________PER:____

1) 25 foot ladder is leaning against the wall of a house. The base of the ladder is pulled away at 2 ft. per second.

a) How fast is the ladder sliding down the wall when the base of the ladder is 7 ft. from the wall?

$x^2 + h^2 = 25^2$

$2x\frac{dx}{dt} + 2h\frac{dh}{dt} = 0$

$\frac{dh}{dt} = -\frac{x}{h}\frac{dx}{dt}$

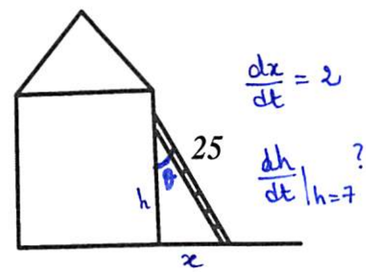
• when $h = 7$, $x^2 + 49 = 625$

$x^2 = 576$

$x = 24$

$\Rightarrow \left.\frac{dh}{dt}\right|_{h=7} = -\frac{24}{7} \times 2$

$= -\frac{48}{7}$ ft/s



b) How fast is the area of the triangle formed changing at this time?

$A = \frac{1}{2}xh$

$\frac{dA}{dt} = \frac{1}{2}\left(\frac{dx}{dt}h + x\frac{dh}{dt}\right)$

$\left.\frac{dA}{dt}\right|_{h=7} = \frac{1}{2}\left(2 \times 7 + 24 \times \left(-\frac{48}{7}\right)\right)$

$= -\frac{527}{7}$ ft²/s

c) How fast is the angle between the ladder and the wall of the house changing at this time?

$\cos\theta = \frac{h}{25}$

$-\sin\theta \cdot \frac{d\theta}{dt} = \frac{1}{25}\frac{dh}{dt}$

$\frac{d\theta}{dt} = -\frac{\csc\theta}{25} \cdot \frac{dh}{dt}$

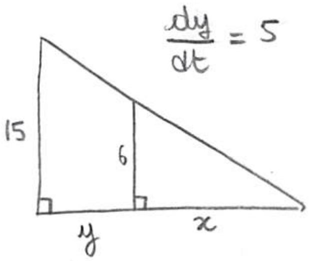
when $h = 7$, $\sin\theta = \frac{24}{25}$

$\Rightarrow \left.\frac{d\theta}{dt}\right|_{h=7} = -\frac{1}{24} \times \left(-\frac{48}{7}\right)$

$= \frac{2}{7}$ rad/s

2) A person 6 ft. tall walks away from a streetlight that is 15 feet above the ground. The person is walking away from the light at a constant rate of 5 feet per second. At what rate, in feet per second,......

~~a)~~ is the length of the shadow changing? ~~b) is the tip of the shadow changing?~~



$\frac{x}{x+y} = \frac{6}{15}$

$5x = 2(x+y)$

$5x = 2x + 2y$

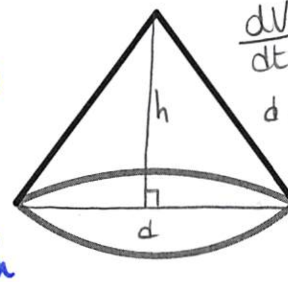
$3x - 2y = 0$

$3\frac{dx}{dt} - 2\frac{dy}{dt} = 0$

$\frac{dx}{dt} = \frac{2}{3}\frac{dy}{dt}$

$\frac{dx}{dt} = \frac{10}{3}$ ft/s

3) At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at a rate of 10 ft^3/min. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 15 feet high?



$\frac{dV}{dt} = 10$

$d = 3h$

$\left.\frac{dh}{dt}\right|_{h=15}$

- $V = \frac{1}{3}\pi\left(\frac{d}{2}\right)^2 h$

$V = \frac{\pi}{12} d^2 h$

$V = \frac{\pi}{12}(3h)^2 h$

$V = \frac{3\pi}{4} h^3$

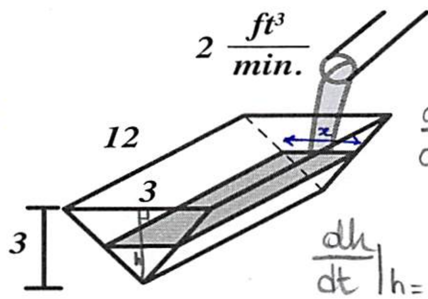
- $\frac{dV}{dt} = \frac{3\pi}{4} \times 3h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{4}{9\pi h^2}\frac{dV}{dt}$

- $\left.\frac{dh}{dt}\right|_{h=15} = \frac{4}{2025\pi} \times 10$

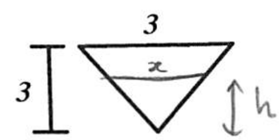
$= \frac{8}{405\pi}$ ft/min

4) A trough is 12 feet long and 3 feet across the top (see the figure). Its ends are isosceles triangles with altitudes of 3 feet. If the water is being pumped into the trough at 2 cubic feet per minute, how fast is the water level rising when the depth is 1 foot?



$\frac{dV}{dt} = 2$

$\left.\frac{dh}{dt}\right|_{h=1}$?



side view (cross section)

- $V = \frac{1}{2}xh \times 12$

$V = 6xh$

$\frac{dV}{dt} = 6\left(\frac{dx}{dt}h + x\frac{dh}{dt}\right)$

$2 = 6\left(2h\frac{dh}{dt}\right)$

$\frac{1}{6} = \left.\frac{dh}{dt}\right|_{h=1}$

(ft/min)

- $\frac{x}{3} = \frac{h}{3} \Rightarrow x = h$

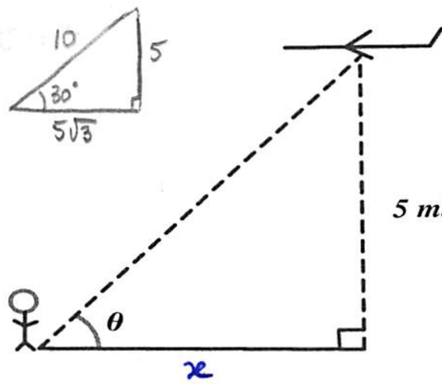
EC: How fast is the area of the surface of the water (shaded in figure) changing at this time?

RELATED RATES

1) An airplane flies at an altitude of 5 miles toward a point directly over an observer. The speed of the plane is 600 miles per hour. Find the rates at which the angle of elevation θ is changing when $\theta = 30°$.

$$\tan\theta = \frac{5}{x}$$

• when $\theta = 30°$,



$$\frac{dx}{dt} = -600$$

$$\left.\frac{d\theta}{dt}\right|_{\theta=30°}$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = -\frac{5}{x^2}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = -\frac{5\cos^2\theta}{x^2} \cdot \frac{dx}{dt}$$

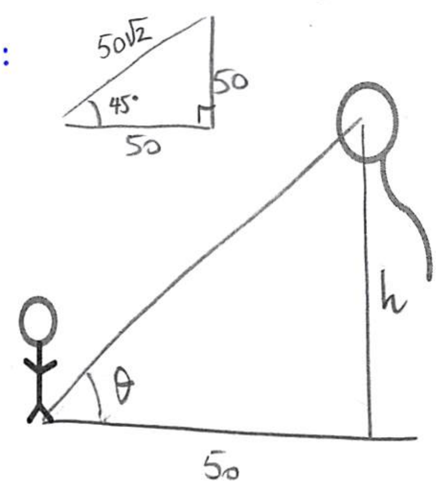
$$\left.\frac{d\theta}{dt}\right|_{\theta=30°} = -\frac{5 \times \frac{3}{4}}{75} \cdot (-600)$$

$$= 30 \text{ rad}/h$$

2) A balloon rises at a rate of 4 m/s from a point on the ground 50 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 50 meters above the ground.

$$\tan\theta = \frac{h}{50}$$

• when $h = 50$:



$$\frac{dh}{dt} = 4$$

$$\left.\frac{d\theta}{dt}\right|_{h=50} ?$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{50}\frac{dh}{dt}$$

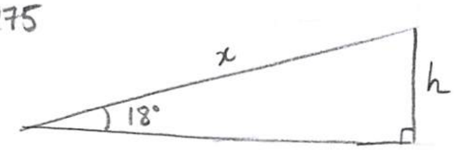
$$\frac{d\theta}{dt} = \frac{\cos^2\theta}{50} \cdot \frac{dh}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{100} \times 4$$

$$\frac{d\theta}{dt} = \frac{1}{25} \text{ rad}/s$$

3) An airplane is flying in still air with an airspeed of 275 miles per hour. If it is climbing at an angle of 18°, find the rate at which it is gaining altitude.

$\frac{dx}{dt} = 275$



$\frac{dh}{dt} = ?$

$$\sin 18 = \frac{h}{x}$$

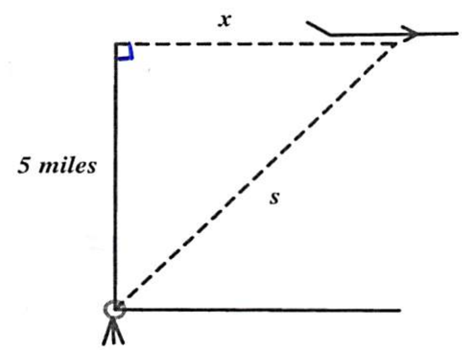
$$h = x \sin 18$$

$$\frac{dh}{dt} = \sin 18 \frac{dx}{dt}$$

$$\frac{dh}{dt} = \sin 18 \times 275$$

$$\frac{dh}{dt} \simeq 85 \text{ mi/h}$$

4) An airplane is flying at an altitude of 5 miles and passes over a radar antenna. When the plane is 10 miles away ($s = 10$), the radar detects that the distance s is changing at a rate of 240 miles per hour. What is the speed of the plane?



$\left.\frac{ds}{dt}\right|_{s=10} = 240$

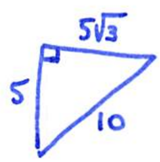
$\frac{dx}{dt}$?

$$s^2 = x^2 + 25$$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{s}{x} \cdot \frac{ds}{dt}$$

when $s = 10$:



$$\frac{dx}{dt} = \frac{2}{\sqrt{3}} \cdot 240$$

$$= \frac{480}{\sqrt{3}} \text{ mi/h}$$

CH.2 Related Rates WS Name: ____________________ Per.____

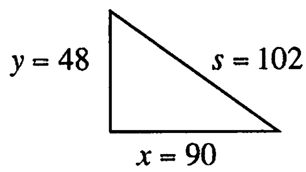
1) Find $\frac{dx}{dt}$ given $x = 5$ and $\frac{dy}{dt} = 7$ for the equation $3x^2 - 5y^3 = 35$.

$3(5)^2 - 5y^3 = 35$ $\quad y = 2$ $\quad 6x\frac{dx}{dt} - 15y^2\frac{dy}{dt} = 0$ $\quad 6(5)\frac{dx}{dt} - 15(2)^2(7) = 0$ $\quad \underline{\frac{dx}{dt} = 14}$

2) Two trains leave the station at the same time with one train traveling north at 16 mph and the other train traveling east at 30 mph. How fast is the distance between the two trains changing after 3 hours?

$x^2 + y^2 = s^2$ $\quad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2s\frac{ds}{dt}$

$2(90)(30) + 2(48)(16) = 2(102)\frac{ds}{dt}$ $\quad \underline{\frac{ds}{dt} = 34 \text{ mph}}$



3) The radius of a circle is increasing at the rate of 4 feet per minute.

a) Find the rate at which the area $(A = \pi r^2)$ is increasing when the radius is 12 feet. $\underline{\frac{dA}{dt} = 96\pi \ \text{ft}^2/\text{min.}}$

b) Find the rate at which the circumference $(C = 2\pi r)$ is increasing at the same time. $\underline{\frac{dC}{dt} = 8\pi \ \text{ft}/\text{min.}}$

a) $A = \pi r^2$ $\quad \frac{dA}{dt} = 2\pi r\frac{dr}{dt}$ $\quad \frac{dA}{dt} = 2\pi(12)(4)$ $\quad \frac{dA}{dt} = 96\pi \ \text{ft}^2/\text{min.}$

b) $C = 2\pi r$ $\quad \frac{dC}{dt} = 2\pi\frac{dr}{dt}$ $\quad \frac{dC}{dt} = 2\pi(4)$ $\quad \frac{dC}{dt} = 8\pi \ \text{ft}/\text{min.}$

4) A spherical balloon $\left(V = \frac{4}{3}\pi r^3\right)$ is inflated at the rate of 11 cubic feet per minute.

a) How fast is the radius of the balloon changing at the instant the radius is 5 feet? $\underline{\frac{dr}{dt} = \frac{11}{100\pi} \ \text{ft}/\text{min.}}$

b) How fast is the surface area $(A = 4\pi r^2)$ of the balloon changing at the same time? $\underline{\frac{dA}{dt} = \frac{22}{5} \ \text{ft}^2/\text{min.}}$

a) $V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$ $\quad 11 = 4\pi(5)^2\frac{dr}{dt}$ $\quad \frac{dr}{dt} = \frac{11}{100\pi} \ \text{ft}/\text{min.}$

b) $A = 4\pi r^2$ $\quad \frac{dA}{dt} = 8\pi r\frac{dr}{dt}$ $\quad \frac{dA}{dt} = 8\pi(5)\frac{11}{100\pi}$ $\quad \frac{dA}{dt} = \frac{22}{5} \ \text{ft}^2/\text{min.}$

5) The height of a cylinder with a radius of 4 ft. is increasing at a rate of 2 feet per minute. Find the rate of change of the volume of the cylinder when the height is 6 feet. $(V = \pi r^2 h)$

$V = \pi r^2 h$ $\quad V = 16\pi h$ $\quad \frac{dV}{dt} = 16\pi\frac{dh}{dt}$ $\quad \frac{dV}{dt} = 16\pi(2)$ $\quad \underline{\frac{dV}{dt} = 32\pi \ \text{ft}^3/\text{min.}}$

6) A conical tank is 20 feet across the top and 15 feet deep. If water is flowing into the tank at the rate of 9 cubic feet per minute, $\left(V = \frac{1}{3}\pi r^2 h\right)$

a) find the rate of change of the depth of the water the instant that it is 2 feet deep. $\frac{dh}{dt} = \frac{81}{16\pi}$ ft/min.

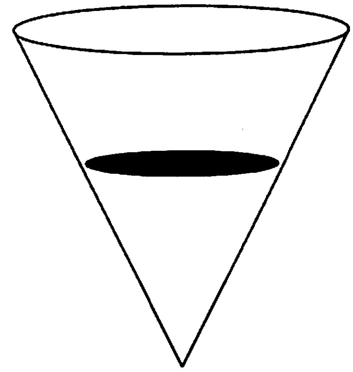
b) find the rate of change of the surface of the water at the same time. $\frac{dA}{dt} = 9$ ft^2/min.

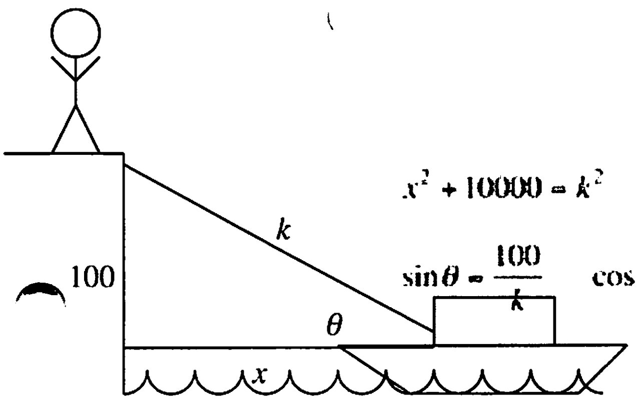
$a)\quad \frac{r}{h} = \frac{10}{15} \qquad r = \frac{2}{3}h \qquad V = \frac{1}{3}\pi\left(\frac{2}{3}h\right)^2 h \qquad V = \frac{4}{27}\pi h^3$

$\frac{dV}{dt} = \frac{4}{9}\pi h^2 \frac{dh}{dt} \qquad 9 = \frac{4}{9}\pi(2)^2\frac{dh}{dt} \qquad \frac{dh}{dt} = \frac{81}{16\pi}$ ft/min.

$b)\quad A = \pi r^2 \qquad A = \pi\left(\frac{2}{3}h\right)^2 \qquad A = \frac{4}{9}\pi h^2$

$\frac{dA}{dt} = \frac{8}{9}\pi h \frac{dh}{dt} \qquad \frac{dA}{dt} = \frac{8}{9}\pi(2)\frac{81}{16\pi} \qquad \frac{dA}{dt} = 9$ ft^2/min.





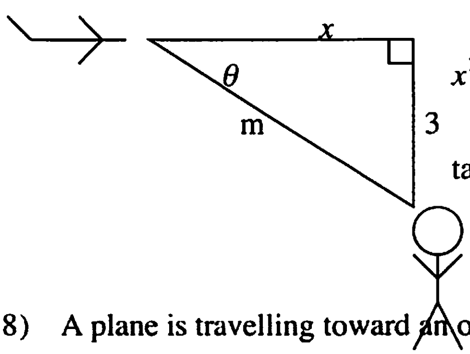
$x^2 + 10000 = k^2 \qquad 2x\frac{dx}{dt} = 2k\frac{dk}{dt} \qquad 2(70)(88) = 2\left(\sqrt{14900}\right)\frac{dk}{dt}$

$\sin\theta = \frac{100}{k} \qquad \cos\theta\frac{d\theta}{dt} = \frac{-100}{k^2}\frac{dk}{dt} \qquad \frac{70}{\sqrt{14900}}\frac{d\theta}{dt} = \frac{-100}{14900}\left(\frac{6160}{\sqrt{14900}}\right)$

7) A man standing on a 100 ft. cliff watches a boat heading away from the cliff. The boat is travelling at a rate of 88 ft/s.

a) How fast is the distance k between the boat and the man changing when the boat is 70 ft. from the cliff? 50.465 ft/s.

b) How fast is the angle θ changing at this time? -0.591 radians/s



$x^2 + 9 = m^2 \qquad 2x\frac{dx}{dt} = 2m\frac{dm}{dt} \qquad 2(4)(-300) = 2(5)\frac{dm}{dt}$

$\tan\theta = \frac{3}{x} \qquad \sec^2\theta\frac{d\theta}{dt} = \frac{-3}{x^2}\frac{dx}{dt} \qquad \left(\frac{5}{4}\right)^2\frac{d\theta}{dt} = \frac{-3}{4^2}(-300)$

8) A plane is travelling toward an observer at 300 mph. The plane is flying 3 miles above the ground.

a) How fast is the distance m between the plane and the man changing when the plane is 5 miles from the man ($m = 5$)? $\frac{dx}{dt} = -240$ mph

b) How fast is the angle of depression θ changing at this time? $\frac{d\theta}{dt} = 36$ radians/hr