AP® Exam Practice Questions for Chapter 1

What You Need to Know . . .

- The AP® Exam, especially the free-response section, stresses the major applications of calculus, rather than the foundational limit concept.
- You are not required to use the formal epsilon-delta definition of a limit on the AP® Exam.
- Algebraic limit evaluation methods (Section 1.3) are not explicitly tested on the free-response section of the AP® Exam. They are, however, helpful on some multiple-choice questions.
- You should be able to apply the Intermediate Value Theorem, whether the function is presented as an equation or by a table.
- You should be able to find both vertical and horizontal asymptotes given a function or a graph.
- The AP® Exam frequently uses limits at infinity as a way of describing horizontal asymptotes.

Practice Questions

Section 1, Part A, Multiple Choice, No Technology

1. What is the limit of $h(x) = 2$ as x approaches π?

(A) 0 (B) 2
(C) π (D) nonexistent

2. What is the limit of

$$g(x) = \frac{|x+4|}{x+4}$$

as x approaches -4?

(A) -4 (B) 0
(C) 1 (D) nonexistent

3. $\lim_{x\to\pi} \frac{\sin x}{x}$ is

(A) 0. (B) 1.
(C) π. (D) nonexistent.

4. $\lim_{x\to-2} \frac{3x^2+5x+7}{x-4}$ is

(A) $-\frac{5}{2}$. (B) $-\frac{3}{2}$.
(C) -1. (D) 3.

5. If $\lim_{x\to5} f(x) = 10$ and $\lim_{x\to5} g(x) = 1$, what is the limit of $\lim_{x\to5} [5f(x) - g(x)]$?

(A) 9 (B) 15
(C) 45 (D) 49

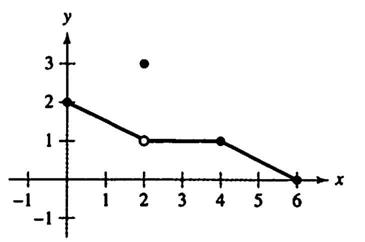
6. $\lim_{x\to\infty} \frac{4x^3+3x^2-x+5}{6x^4+x^3-x^2+4x-1}$ is

(A) -5. (B) 0.
(C) $\frac{2}{3}$. (D) nonexistent.

7. What are the horizontal asymptotes of the graph of $y = \frac{3+4^x}{1-4^x}$?

(A) $y = -1$ only (B) $y = 3$ only
(C) $y = -1$ and $y = 3$ (D) $y = 0$ and $y = 3$

8.



The graph of function g is shown above. Which of the following is true?

I. $\lim_{x\to2} g(x) = 1$
II. $\lim_{x\to2} g(x) = g(2)$
III. g is continuous at $x = 3$.

(A) I only (B) I and III only
(C) III only (D) I, II, and III

Section 1, Part B, Multiple Choice, Technology Permitted

9. $\lim_{x\to1} \frac{x-1}{\sqrt{x}-1}$ is

(A) 0. (B) 1.
(C) 2. (D) nonexistent.

10. $\lim_{x\to\infty} \frac{x^3}{\sqrt{x^2+4}}$ is

(A) $-\infty$. (B) 0.
(C) 4. (D) ∞.

Section 2, Part A, Free Response, Technology Permitted

11. The function f is defined as $f(x) = \dfrac{10}{1 + \frac{1}{4}e^{-x}}$.

(a) Find $\lim\limits_{x \to 0} f(x)$.

(b) Find $\lim\limits_{x \to 0} (f(x) + 4)$.

(c) State the equation(s) for the horizontal asymptote(s) of the graph of $y = f(x)$. Show the work that leads to your answer.

Section 2, Part B, Free Response, No Technology

12. The function f is defined as $f(x) = \dfrac{x^2 + 5x + 6}{2x^2 + 7x + 3}$.

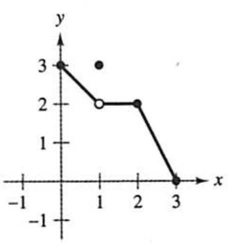
(a) State the value(s) of x for which f is not continuous.

(b) Evaluate $\lim\limits_{x \to -3} f(x)$. Justify your answer.

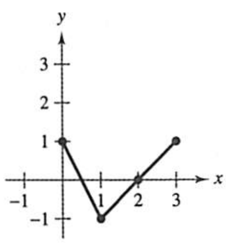
(c) State the equation(s) for the vertical asymptote(s) of the graph of $y = f(x)$.

(d) State the equation(s) for the horizontal asymptote(s) of the graph of $y = f(x)$. Show the work that leads to your answer.

13.



Graph of f



Graph of g

The graphs of functions f and g are shown above. Evaluate each limit using the graphs provided. Show the computations that lead to your answer.

(a) $\lim\limits_{x \to 1} [f(x) + 4]$

(b) $\lim\limits_{x \to 3^-} \dfrac{5}{g(x)}$

(c) $\lim\limits_{x \to 2} [f(x) \cdot g(x)]$

(d) $\lim\limits_{x \to 3^-} \dfrac{f(x)}{g(x) - 1}$ (Assume that f and g are linear on the interval $[2, 3]$.)

14. Let f be a function defined by

$$f(x) = \begin{cases} e^{2x}, & x \le 0 \\ 4 - 3\cos x, & x > 0 \end{cases}.$$

(a) Find $\lim\limits_{x \to -1} f(x)$.

(b) Show that f is continuous at $x = 0$.

(c) Find $\lim\limits_{x \to -\infty} f(x)$.

15. A hot cup of tea is placed on a counter and left to cool. The temperature (in degrees Fahrenheit) of the tea x minutes after the cup is placed on the counter is modeled by a continuous function $T(x)$ for $0 \le x < 10$. Values of $T(x)$ at various times x are shown in the table.

x	0	3	4	6	8	9
$T(x)$	180°	174°	172°	168°	164°	162°

(a) Find $\lim\limits_{x \to 4} T(x)$. Justify your answer.

(b) Use the data to find the average rate of change in the temperature of the tea for $3 \le x \le 8$. Include units in your final answer.

(c) Use the data to identify the shortest interval during which there must exist a time x for which the temperature of the tea is 166.5°. Justify your answer.

(d) Use the data to find the best estimate of the slope of the line tangent to the graph of T at $x = 8$.

16. The position function $s(t) = -4.9t^2 + 396.9$ gives the height (in meters) of an object that has fallen from a height of 396.9 meters after t seconds.

(a) Explain why there must exist at a time t, $1 < t < 2$, at which the height of the object must be 382 meters above the ground.

(b) After how many seconds does the object hit the ground?

(c) Find the average rate of change in s over the interval $[8, 9]$. Include units of measure. Explain why this is a good estimate of the velocity at which the object hits the ground. How can this estimate be improved?

(d) Find

$$\lim_{t \to 3} \frac{s(t) - s(3)}{t - 3}.$$

Show the work that leads to your answer. Include units.

17. Let a and b represent real numbers. Define

$$f(x) = \begin{cases} ax^2 + x - b, & \text{if } x \le 2 \\ ax + b, & \text{if } 2 < x < 5. \\ 2ax - 7, & \text{if } x \ge 5 \end{cases}$$

(a) Find the values of a and b such that f is continuous on the entire real number line.

(b) Evaluate $\lim\limits_{x \to 3} f(x)$.

(c) Let $g(x) = \dfrac{f(x)}{x - 1}$. Evaluate $\lim\limits_{x \to 1} g(x)$.

AP® Exam Practice Questions for Chapter 2

What You Need to Know . . .

- The definition of the derivative is primarily tested on the multiple-choice section of the AP® Exam.
- The AP® Exam requires that you have proficiency with using a function's equation, table of values, or graph when finding the average velocity or average rate of change.
- Extremely complex examples of the Product Rule, Quotient Rule, and Chain Rule do not typically appear on the AP® Exam. The more difficult problems in this chapter will help you master and remember the concepts.
- Although you should know the derivatives of the six trigonometric functions, the derivatives of the sine, cosine, and tangent functions are the most commonly tested.
- Related rate problems make frequent appearances on the AP® Exam because they represent a powerful application of implicit derivatives.

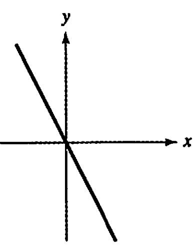
Practice Questions

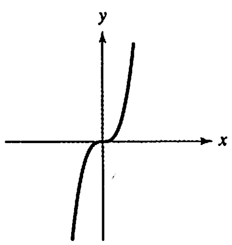
Section 1, Part A, Multiple Choice, No Technology

1. What is an equation of the tangent line to the graph of $f(x) = 4e^x - x + 6$ at $(0, 10)$?

(A) $y = 4x + 10$
(B) $y = 4x - 10$
(C) $y = 10x - 4$
(D) $y = 3x + 10$

2. Which graph shows a function whose derivative is always negative?

(A)


(B)


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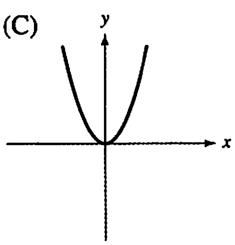
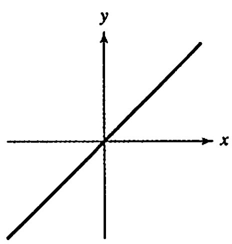


(D)


3. If $y = \dfrac{6x^4 - 3x^5 + 5x^3}{x^3}$, then $\dfrac{d^2y}{dx^2} =$

(A) $6 - 6x$.
(B) 6.
(C) $6x$.
(D) -6.

4. If $h(x) = |2x - 5|$, which of the following is true?

(A) h is continuous but is not differentiable at $x = \frac{5}{2}$.
(B) h is not continuous but is differentiable at $x = \frac{5}{2}$.
(C) h is continuous and differentiable at $x = \frac{5}{2}$.
(D) h is neither continuous nor differentiable at $x = \frac{5}{2}$.

5. If $f(x) = \dfrac{\sin x}{x^2}$, then $f'(x) =$

(A) $\dfrac{\cos x}{2x}$.
(B) $\dfrac{x \cos x - 2 \sin x}{x^2}$.
(C) $\dfrac{x \cos x - 2 \sin x}{x^3}$.
(D) $\dfrac{\cos x - 2 \sin x}{x^2}$.

6. If $y = \sqrt[4]{8x + 3}$, then $y' =$

(A) $\dfrac{2}{(8x + 3)^{3/4}}$.
(B) $\dfrac{1}{4(8x + 3)^{3/4}}$.
(C) $\dfrac{1}{4}(8x + 3)^{3/4}$.
(D) $\dfrac{8}{(8x + 3)^{3/4}}$.

7. If $y = 6 \cos 2x$, then $y^{(6)} =$

(A) $384 \cos 2x$.
(B) $-384 \cos 2x$.
(C) $384 \sin 2x$.
(D) $-384 \sin 2x$.

8. The table shows the position $s(t)$ of a particle that moves along a straight line at several times t, where t is measured in seconds and s is measured in meters.

t	2.0	2.7	3.2	3.8
$s(t)$	5.2	7.8	10.6	12.2

Which of the following best estimates the velocity of the particle at $t = 3$?

(A) 3.7 m/sec (B) 3.9 m/sec
(C) 5.6 m/sec (D) 7.8 m/sec

9. If $2y^3 - 3xy + x^2 = 4$, then $\frac{dy}{dx} =$

(A) $-\frac{2x}{6y^2 - 3}$. (B) $\frac{2x - 3y}{3x - 6y^2}$.
(C) $\frac{2x - 3}{6y^2}$. (D) $-\frac{2x}{6y^2 - 3x}$.

10. The volume of a cylinder with radius r and height h is given by $V = \pi r^2 h$. The radius of the cylinder is increasing at a rate of $1/3$ centimeter per second and the height of the cylinder is increasing at a rate of $1/2$ centimeter per second. At what rate, in cubic centimeters per second, is the volume of the cylinder increasing when its height is 9 centimeters and the radius is 4 centimeters?

(A) $\frac{4\pi}{3}$ (B) $\frac{8\pi}{3}$ (C) 6π (D) 32π

Section 1, Part B, Multiple Choice, Technology Permitted

11. Two roads intersect at right angles. You are standing 25 meters north of the intersection on one of the roads. You are watching a car traveling west at 30 meters per second. At how many meters per second is the car traveling away from you 3 seconds after it passes through the intersection?

(A) 23.047 (B) 28.906
(C) 29.032 (D) 30

12. The position $s(t)$ of a particle moving along the x-axis at time t is given by $s(t) = -t^3 + 2t^2 + \frac{3}{2}$, where s is measured in meters and t is measured in seconds. At what time is the particle's instantaneous velocity equal to its average velocity on the interval $[0, 4]$?

(A) 1.097 seconds (B) 2 seconds
(C) 2.333 seconds (D) 2.431 seconds

Section 2, Part A, Free Response, Technology Permitted

13. The function f is defined as $f(x) = 3e^{2x^2}$.

(a) Find $f'(x)$.

(b) For what value of x is the slope of the tangent line to the graph of f equal to 2?

(c) For what value(s) of x does the tangent line to the graph of f intersect the x-axis at the point $\left(\frac{1}{2}, 0\right)$?

Section 2, Part B, Free Response, No Technology

14. Evaluate each limit analytically.

(a) $\lim_{h\to 0} \frac{\sin(x + h) - \sin x}{h}$ (b) $\lim_{h\to 0} \frac{\sqrt[3]{x + h} - \sqrt[3]{x}}{h}$

(c) $\lim_{h\to 0} \frac{\sqrt{16 + h} - 4}{h}$ (d) $\lim_{h\to 0} \frac{\frac{1}{5 + h} - \frac{1}{5}}{h}$

15. Given:

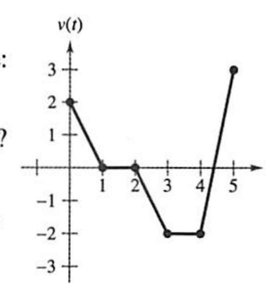
x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	-3	1	5	-2
5	4	7	-1	2

(a) If $h(x) = \frac{f(x)}{g(x)}$, find $h'(2)$.

(b) If $j(x) = f(g(x))$, find $j'(2)$.

(c) If $k(x) = \sqrt{f(x)}$, find $k'(5)$.

16. The figure below shows the graph of the velocity, in feet per second, for a particle moving along the line $x = 4$.



(a) During which time interval(s) is the particle:

(i) moving upward?

(ii) moving downward?

(iii) at rest?

(b) What is the acceleration of the particle at

(i) $t = 0.75$ and

(ii) $t = 4.2$? Be sure to include units.

17. Given: $g(x) = f(x) \cdot \tan x + kx$, where k is a real number. f is differentiable for all x; $f(\pi/4) = 4$; $f'(\pi/4) = -2$.

(a) For what values of x, if any, in the interval $0 < x < 2\pi$ will the derivative of g fail to exist? Justify your answer.

(b) If $g'\left(\frac{\pi}{4}\right) = 6$, find the value of k.

AP® Exam Practice Questions for Chapter 3

What You Need to Know . . .

- On some free-response questions, there may be more than one way of applying derivatives and theorems to justify your answer.
- Be prepared to apply the Mean Value Theorem. It may be referred to directly, or it may be necessary to use the theorem to justify your answer.
- Questions that involve position, velocity, and acceleration functions are very common on the AP® Exam.
- Be prepared to apply the Second Derivative Test to justify whether a point is a local maximum, a local minimum, or a point of inflection. For a point of inflection, make sure to also check for a sign change.
- Tangent line approximations, and whether such an approximation overestimates or underestimates a function value, is commonly tested on the free-response section.

Practice Questions

Section 1, Part A, Multiple Choice, No Technology

1. What are the critical numbers of

$f(x) = 4x^3 + 6x^2 - 72x - 9$?

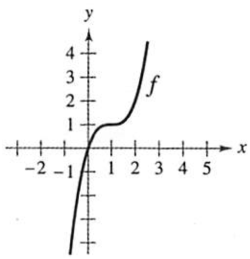
(A) $x = -2$ and $x = 3$

(B) $x = -3$ and $x = 2$

(C) $x = -2$

(D) $x = -3$

2. The graph of the function f is shown. Which of the following is true?



I. $f'(x) > 0$ on the entire real number line.

II. $f''(x) < 0$ on the interval $(-\infty, 1)$.

III. $f''(x) > 0$ on the interval $(1, \infty)$.

(A) I only (B) II and III only

(C) I and III only (D) I, II and III

3. The position of an object along a vertical line is given by $s(t) = -t^3 + 3t^2 + 9t + 5$, where s is measured in feet and t is measured in seconds. The maximum velocity of the object in the time interval $[0, 4]$ is

(A) 9 feet per second.

(B) 12 feet per second.

(C) 16 feet per second.

(D) 32 feet per second.

4. The function g is continuous and differentiable on the interval $[2, 6]$. The table shows selected values of g on $[2, 6]$. Which of the following statements must be true?

x	2	3	4	5	6
$g(x)$	7	4	1	4	7

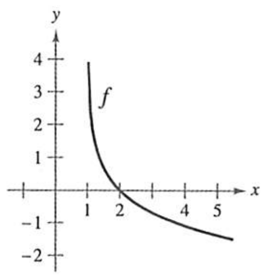
(A) The minimum value of g on $[2, 6]$ is 1.

(B) The maximum value of g on $[2, 6]$ is 7.

(C) There exists a number c, with $2 < c < 6$, for which $g'(c) = 0$.

(D) $g'(x) < 0$ for $2 < x < 4$

5. Consider the graph of $y = f(x)$ shown below. If f is a function such that f' and f'' are defined in a region around $x = 2$, then which of the following must be true?



(A) $f''(2) < f(2)$ (B) $f''(2) < f'(2)$

(C) $f(2) = f'(2)$ (D) $f''(2) > f(2)$

6. If $y = \arctan 4x$, then $dy =$

(A) $\dfrac{4}{1 + 16x^2}\,dx.$ (B) $\dfrac{4x}{1 + 16x^2}\,dx.$

(C) $-\dfrac{4x}{1 + 16x^2}\,dx.$ (D) $-\dfrac{4}{1 + 16x^2}\,dx.$

Section 1, Part B, Multiple Choice, Technology Permitted

7. If the Mean Value Theorem is applied to the function $f(x) = \ln(x - 3)$ on the interval $[4, 8]$, then the number c that must exist in $(4, 8)$ is

(A) 5.485.

(B) 5.885.

(C) 6.

(D) 6.368.

Section 2, Part A, Free Response, Technology Permitted

8. The table below shows the behavior of a function f that is continuous on the entire real number line. For the function, $f(2) = 4$, and $\lim_{x\to\infty} f(x) = 0$.

	$x < 4$	$x = 4$	$x > 4$
$f'(x)$	positive	does not exist	negative
$f''(x)$	negative	does not exist	positive

(a) For what values of x is f increasing?

(b) Does f have a relative maximum at $x = 4$? Explain.

(c) If possible, name the x-coordinate of the point of inflection on the graph of f. Justify your answer.

(d) Does the Mean Value Theorem apply over the interval $[3, 5]$? Justify your answer.

(e) Sketch a possible graph of f.

9. Consider the function $f(x) = \dfrac{x^3}{2} - \sin x + 1$.

(a) Approximate the relative extrema of f.

(b) Find the tangent line approximation of f at $\dfrac{\pi}{2}$.

(c) Use your tangent line approximation to approximate the value of $f(1.5)$. Is your approximation an underestimate or an overestimate of the actual value of $f(1.5)$? Justify your answer.

Section 2, Part B, Free Response, No Technology

10. Consider the function

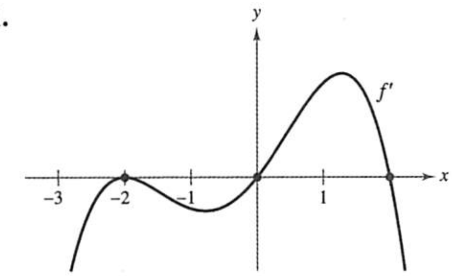
$$f(x) = 2x + \cos 2x$$

on the interval $[0, \pi]$.

(a) Find the maximum value of f. Justify your answer.

(b) Explain how the conditions of the Mean Value Theorem are satisfied by f for $0 \le x \le \pi$. Find the value of x whose existence is guaranteed by the Mean Value Theorem.

11.



The figure above shows the graph of f', the derivative of f. The function f is a twice differentiable function on $x \in (-\infty, \infty)$, $f''(-0.8) = 0$, and $f''(1.3) = 0$.

(a) For what values of x is f increasing?

(b) For what values of x is the graph of f concave downward? Justify your answer.

(c) Is

$$\frac{f(-0.5) - f(0)}{-0.5 - 0}$$

positive or negative? Justify your answer.

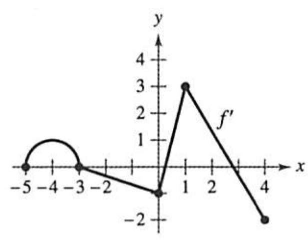
12. Consider the function $f(x) = \dfrac{1 - 4x^2}{x}$.

(a) For what values of x is f decreasing?

(b) For what values of x is the graph of f concave downward? Justify your answer.

(c) Does the graph of f have any points of inflection? Justify your answer.

13.



The figure above shows the graph of f', the derivative of f, on the interval $[-5, 4]$. The function f is differentiable on the interval and $f''(-4) = 0$.

(a) Find $f'(-1)$ and $f''(-1)$.

(b) At which x-values does f have a relative extrema on the interval $(-5, 0)$? Justify your answer.

(c) Find all intervals on which the graph of f is concave downward. Explain your reasoning.

(d) Find the x-coordinate of each of the points of inflection of the graph of f. Justify your answer.

(e) If $g(x) = f(x) + \sin^2 x$, is g increasing or decreasing at $x = -\pi/4$? Justify your answer.