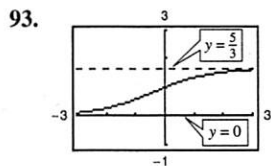
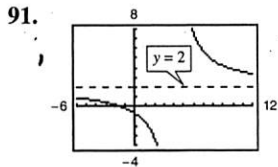
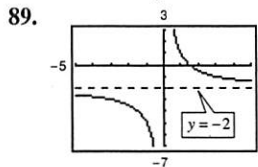


61. (a) -4 (b) 4 (c) Limit does not exist.  
 63.  $x = \pm 3$  65.  $x = \pm 8$  67.  $x = \pm 5$  69.  $-\infty$   
 71.  $\frac{1}{3}$  73.  $-\infty$  75.  $\frac{4}{5}$  77.  $-\infty$   
 79. (a) \$14,117.65; \$80,000.00; \$720,000.00  
 (b)  $\infty$ ; No matter how much the company spends, the company will never be able to remove 100% of the pollutants.  
 81. 8 83.  $\frac{2}{3}$  85.  $-\infty$  87. 6



**AP® Exam Practice Questions for Chapter 1 (page 120)**

1. B 2. D 3. A 4. B 5. D 6. B  
 7. C 8. B 9. C 10. D  
 11. (a)  $\lim_{x \rightarrow 0} f(x) = 8$  (b)  $\lim_{x \rightarrow 0} (f(x) + 4) = 12$   
 (c)  $y = 0, y = 10$ ;

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{10}{1 + \frac{1}{4}e^{-x}} = \lim_{x \rightarrow \infty} \frac{10}{1 + \frac{1}{4e^x}} = \frac{10}{1 + \frac{1}{4e^\infty}} = \frac{10}{1 + 0} = 10$$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{10}{1 + \frac{1}{4}e^{-x}} = \frac{10}{1 + \frac{1}{4}e^\infty} = \frac{10}{\infty} = 0$$

12. (a)  $f(x)$  has discontinuities at  $x = -3$  and  $x = -\frac{1}{2}$ .  
 (b)  $\lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow -3} \frac{x^2 + 5x + 6}{2x^2 + 7x + 3} = \lim_{x \rightarrow -3} \frac{(x+2)(x+3)}{(2x+1)(x+3)} = \lim_{x \rightarrow -3} \frac{x+2}{2x+1} = \frac{-3+2}{2(-3)+1} = \frac{1}{-5} = -\frac{1}{5}$

(c)  $x = -\frac{1}{2}$

(d)  $y = \frac{1}{2}$ ;

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x^2 + 5x + 6}{2x^2 + 7x + 3} = \lim_{x \rightarrow \infty} \frac{1 + \frac{5}{x} + \frac{6}{x^2}}{2 + \frac{7}{x} + \frac{3}{x^2}} = \frac{1 + 0 + 0}{2 + 0 + 0} = \frac{1}{2}$$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x^2 + 5x + 6}{2x^2 + 7x + 3} = \lim_{x \rightarrow -\infty} \frac{1 + \frac{5}{x} + \frac{6}{x^2}}{2 + \frac{7}{x} + \frac{3}{x^2}} = \frac{1 + 0 + 0}{2 + 0 + 0} = \frac{1}{2}$$

13. (a) 6;  $\lim_{x \rightarrow 1} (f(x) + 4) = \lim_{x \rightarrow 1} f(x) + \lim_{x \rightarrow 1} 4 = 2 + 4 = 6$   
 (b) 5;  $\lim_{x \rightarrow 3} \frac{5}{g(x)} = \frac{5}{1} = 5$   
 (c) 0;  $\lim_{x \rightarrow 2} (f(x) \cdot g(x)) = 2 \cdot 0 = 0$   
 (d) -2;

$$\lim_{x \rightarrow 3} \frac{f(x)}{g(x) - 1} = \lim_{x \rightarrow 3} \frac{-2x + 6}{(x - 2) - 1} = \lim_{x \rightarrow 3} \frac{-2x + 6}{x - 3} = \lim_{x \rightarrow 3} \frac{-2(x-3)}{x-3} = -2$$

14. (a)  $\frac{1}{e^2}$   
 (b)  $f(0)$  is defined as  $f(0) = e^{2(0)} = 1$ .  $\lim_{x \rightarrow 0^-} f(x) = 1$  and  $\lim_{x \rightarrow 0^+} f(x) = 1$ , so  $\lim_{x \rightarrow 0} f(x) = 1$ . Also,  $\lim_{x \rightarrow 0} f(x) = f(0) = 1$ . So,  $f$  is continuous at  $x = 0$ .  
 (c) 0  
 15. (a) 172; Because  $T(x)$  is continuous on  $[0, 10)$ ,  $\lim_{x \rightarrow 4} T(x) = T(4) = 172$ .  
 (b) -2 degrees per minute  
 (c) (6, 8);  $T(x)$  is continuous and when  $x = 6, T(x) > 166.5^\circ$  and when  $x = 8, T(x) < 166.5^\circ$ .  
 (d) Slope = -2

16. (a) Because  $s(t)$  is a continuous function and  $s(1) = 392$  and  $s(2) = 377.3$ , there exists a time  $t, 1 < t < 2$  where  $s(t)$  is a value between 377.3 and 392.  
 (b) 9 seconds  
 (c) -83.3 m/sec; This is a good measure of velocity because it is the object's average rate of change right before it hits the ground; It can be improved by finding the instantaneous rate of change using calculus.

(d)  $\lim_{t \rightarrow 3} \frac{s(t) - s(3)}{t - 3} = \lim_{t \rightarrow 3} \frac{(-4.9t^2 + 396.9) - (-4.9(3)^2 + 396.9)}{t - 3} = \lim_{t \rightarrow 3} \frac{-4.9t^2 + 4.9(9)}{t - 3} = \lim_{t \rightarrow 3} \frac{-4.9(t^2 - 9)}{t - 3} = \lim_{t \rightarrow 3} \frac{-4.9(t-3)(t+3)}{t-3} = \lim_{t \rightarrow 3} -4.9(t+3) = -4.9(3+3) = -29.4 \text{ m/sec}$

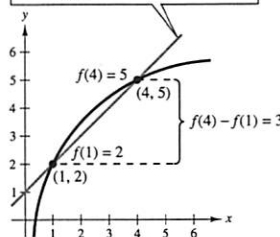
17. (a)  $a = 2, b = 3$  (b) 9 (c) 5

**Chapter 2**

**Section 2.1 (page 131)**

1.  $m_1 = 0, m_2 = \frac{5}{2}$

3. (a)-(c)  $y = \frac{f(4) - f(1)}{4 - 1} (x - 1) + f(1) = x + 1$



5.  $m = -5$  7.  $m = 8$  9.  $m = 3$  11.  $f'(x) = 0$   
 13.  $f'(x) = -5$  15.  $h'(s) = \frac{2}{3}$  17.  $f'(x) = 2x + 1$

3.

$n$	$x_n$	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1	1.6	-0.0292	-0.9996	0.0292	1.5708
2	1.5708	0	-1	0	1.5708

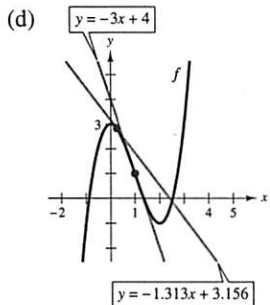
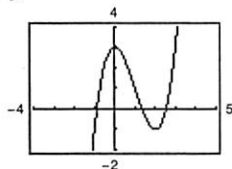
5. -1.587    7. 0.682    9. 1.250, 5.000    11. 0.567

13. 0.900, 1.100, 1.900    15. 1.935    17. 0.567

19. 4.493    21. (a) Proof    (b)  $\sqrt{5} \approx 2.236$ ;  $\sqrt{7} \approx 2.646$

23.  $f'(x_1) = 0$     25. 0.74    27. 1.12    29. Proof

31. (a) (b) 1.347    (c) 2.532



x-intercept of  $y = -3x + 4$  is  $\frac{4}{3}$ .

x-intercept of  $y = -1.313x + 3.156$  is approximately 2.404.

(e) If the initial estimate  $x = x_1$  is not sufficiently close to the desired zero of a function, then the x-intercept of the corresponding tangent line to the function may approximate a second zero of the function.

33. Answers will vary. Sample answer:

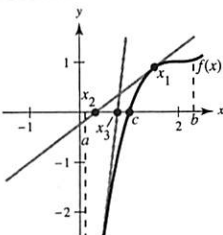
If  $f$  is a function continuous on  $[a, b]$  and differentiable on  $(a, b)$ , where  $c \in [a, b]$  and  $f(c) = 0$ , then Newton's Method uses tangent lines to approximate  $c$ . First, estimate an initial  $x_1$  close to  $c$ . (See graph.)

Then determine  $x_2$  using

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ . Calculate a third estimate  $x_3$  using

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$ . Continue this process until  $|x_n - x_{n+1}|$  is

within the desired accuracy, and let  $x_{n+1}$  be the final approximation of  $c$ .



35. 4.486 hours    37. True    39. 0.217    41. A

### Review Exercises for Chapter 2 (page 204)

1.  $f'(x) = 0$     3.  $f'(x) = 2x - 4$     5. 5

7.  $f$  is differentiable for all  $x \neq 3$ .

9.  $f$  is differentiable on the interval  $(1, \infty)$ .

11. 0    13.  $3x^2 - 22x$     15.  $\frac{3}{\sqrt{x}} + \frac{1}{3\sqrt{x^2}}$     17.  $-\frac{4}{3t^3}$

19.  $4 - 5 \cos \theta$     21.  $-3 \sin t - 4e^t$     23. -1    25. 0

27. (a) 50 (vibrations/sec)/lb    (b) 33.33 (vibrations/sec)/lb

29. (a)  $s(t) = -16t^2 - 30t + 600$ ;  $v(t) = -32t - 30$

(b) -94 ft/sec

(c)  $v'(1) = -62$  ft/sec;  $v'(3) = -126$  ft/sec

(d) About 5.258 sec

(e) About -198.256 ft/sec

31.  $4(5x^3 - 15x^2 - 11x - 8)$     33.  $\frac{1}{2\sqrt{x}} \sin x + \sqrt{x} \cos x$

35.  $-\frac{x^2 + 1}{(x^2 - 1)^2}$     37.  $\frac{4x^3 \cos x + x^4 \sin x}{\cos^2 x}$

39.  $3x^2 \sec x \tan x + 6x \sec x$     41.  $4xe^x + 4e^x + \csc^2 x$

43.  $y = 4x + 10$     45.  $y = -8x + 1$     47.  $y = \frac{1}{2}x + 3$

49.  $-48t$     51.  $\frac{225}{4}\sqrt{x}$     53.  $6 \sec^2 \theta \tan \theta$

55. (a) Proof    (b)  $-7920\sqrt{3}$  mi/degree

57.  $28(7x + 3)^3$     59.  $-\frac{2x}{(x^2 + 4)^2}$     61.  $-45 \sin(9x + 1)$

63.  $\sin^2 x$     65.  $(6x + 1)^4(36x + 1)$     67.  $\frac{3}{(x^2 + 1)^{3/2}}$

69.  $\frac{1}{4}te^{t/4}(t + 8)$     71.  $\frac{e^{2x} - e^{-2x}}{\sqrt{e^{2x} + e^{-2x}}}$     73.  $\frac{x(2 - x)}{e^x}$

75.  $\frac{1}{2x}$     77.  $\frac{1 + 2 \ln x}{2\sqrt{\ln x}}$     79.  $\frac{x}{(a + bx)^2}$     81.  $\frac{1}{x(a + bx)}$

83.  $-\frac{3x^2}{2\sqrt{1 - x^3}}$ ; -2    85.  $-\frac{8x}{(x^2 + 1)^2}$ ; 2

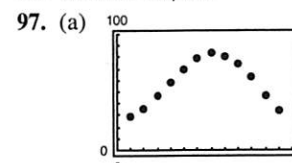
87.  $-(\csc 2x)\cot 2x$ ; 0    89.  $384(8x + 5)$     91.  $2 \csc^2 x \cot x$

93. (a)  $-18.667^\circ\text{F/h}$     (b)  $-7.284^\circ\text{F/h}$

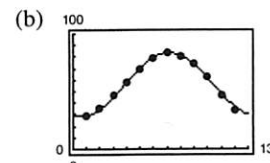
(c)  $-3.240^\circ\text{F/h}$     (d)  $-0.747^\circ\text{F/h}$

As the time increases, the rate of change of the temperature decreases.

95.  $0.04224 \text{ cm/sec}^2$

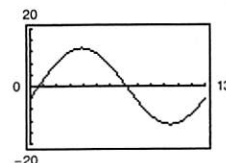


$T(t) = 56.1 + 27.6 \sin(0.48t - 1.86)$



The model is a good fit.

(c)  $T'(t) = 13.248 \cos(0.48t - 1.86)$



(d) The temperature changes most rapidly around spring (March–May) and fall (Oct.–Nov.). The temperature changes most slowly around winter (Dec.–Feb.) and summer (June–Aug.). Yes. Explanations will vary.

99.  $-\frac{x}{y}$     101.  $\frac{y(y^2 - 3x^2)}{x(x^2 - 3y^2)}$     103.  $\frac{y \sin x + \sin y}{\cos x - x \cos y}$

105.  $y = -6x + 28$     107.  $y = -\frac{1}{2}x + 5$

109. Tangent line:    111. Tangent line:

$y = -3x + 10$

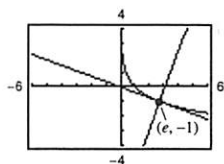
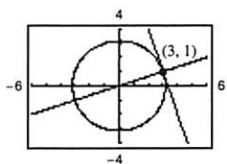
$y = -\frac{1}{e}x$

Normal line:

$y = \frac{1}{3}x$

Normal line:

$y = ex - e^2 - 1$



113.  $\frac{x^3 + 8x^2 + 4}{(x + 4)^2 \sqrt{x^2 + 1}}$     115.  $\frac{1}{3(\sqrt[3]{-3})^2} \approx 0.160$     117.  $\frac{3}{4}$

119.  $(1 - x^2)^{-3/2}$     121.  $\frac{x}{|x|\sqrt{x^2 - 1}} + \operatorname{arcsec} x$

123. (a)  $2\sqrt{2}$  units/sec    (b) 4 units/sec    (c) 8 units/sec

125.  $450\pi$  km/h    127.  $\frac{d\theta}{dt} = \frac{1}{25} \cos^2 \theta, -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$

129.  $-0.347, -1.532, 1.879$     131. 1.202

133. 0.264, 1, 1.737

AP® Exam Practice Questions for Chapter 2 (page 208)

1. D    2. A    3. D    4. A    5. C    6. A    7. B  
8. C    9. B    10. D    11. B    12. D

13. (a)  $12xe^{2x^2}$     (b) 0.158    (c)  $\frac{1 \pm \sqrt{5}}{4}$

14. (a)  $\cos x$     (b)  $\frac{1}{3x^{2/3}}$     (c)  $\frac{1}{8}$     (d)  $-\frac{1}{25}$

15. (a)  $-\frac{1}{25}$     (b)  $-14$     (c)  $\frac{7}{4}$

16. (a) (i) (0, 1), (4.4, 5)  
(ii) (2, 4.4)  
(iii) (1, 2)

- (b) (i)  $-2$  ft/sec<sup>2</sup>  
(ii) 5 ft/sec<sup>2</sup>

17. (a)  $x = \frac{\pi}{2}, \frac{3\pi}{2}$ ;  $\tan x$  and  $\sec^2 x$  are undefined at these values.

(b)  $k = 0$

Chapter 3

Section 3.1 (page 217)

1.  $f'(0) = 0$     3.  $f'(2) = 0$     5.  $f'(-2)$  is undefined.

7. 2, absolute maximum (and relative maximum)

9. 1, absolute maximum (and relative maximum)

2, absolute minimum (and relative minimum)

3, absolute maximum (and relative maximum)

11.  $x = \frac{3}{4}$     13.  $t = \frac{8}{3}$

15.  $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$     17.  $t = \frac{1}{2}$     19.  $x = 0$

21. Minimum: (2, 1); Maximum: (-1, 4)

23. Minimum: (-3, -13); Maximum: (0, 5)

25. Minimum:  $(-1, -\frac{5}{2})$ ; Maximum: (2, 2)

27. Minimum: (0, 0); Maximum: (-1, 5)

29. Minimum: (1, -1); Maximum:  $(0, -\frac{1}{2})$

31. Minimum: (-1, -1); Maximum: (3, 3)

33. Minimum value is -2 for  $-2 \leq x < -1$ ; Maximum: (2, 2)

35. Minimum:  $(\frac{3\pi}{2}, -1)$ ; Maximum:  $(\frac{5\pi}{6}, \frac{1}{2})$

37. Minimum:  $(\pi, -3)$ ; Maxima: (0, 3) and  $(2\pi, 3)$

39. Minimum: (0, 0); Maximum:  $(-2, \arctan 4)$

41. Minimum:  $(2, 5e^2 - e^4)$ ; Maximum:  $(\ln \frac{5}{2}, \frac{25}{4})$

43. Minima: (0, 0) and  $(\pi, 0)$ ; Maximum:  $(\frac{3\pi}{4}, \frac{\sqrt{2}}{2} e^{3\pi/4})$

45.  $x = 0$  is a critical number but not a relative extremum;  $x = -\frac{3}{2}$  and  $x = 3$

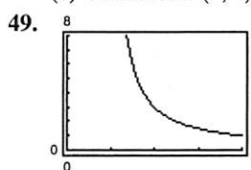
47. (a) Minimum: (0, -3)

(b) Minimum: (0, -3)

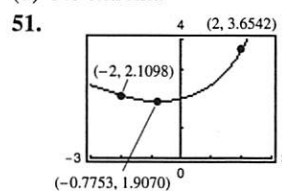
Maximum: (2, 1)

(c) Maximum: (2, 1)

(d) No extrema

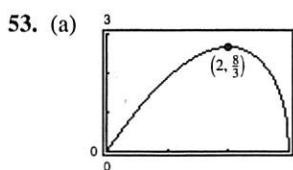


Minimum: (4, 1)



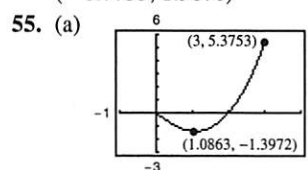
Minimum:

$(-0.7753, 1.9070)$



(b) Minima: (0, 0) and (3, 0)

Maximum:  $(2, \frac{8}{3})$



(b) Minimum

$(1.0863, -1.3972)$

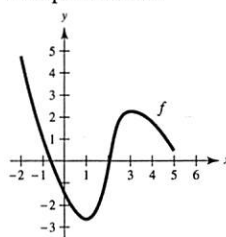
57. Maximum:  $|f''(\sqrt[3]{-10 + \sqrt{108}})| = f''(\sqrt{3} - 1) \approx 1.47$

59. Maximum:  $|f''(0)| = 1$

61.  $f$  is continuous on  $[0, \frac{\pi}{4}]$  but not on  $[0, \pi]$ .

63. Answers will vary.

Sample answer:



65. (a) Yes    (b) No

67. Maximum:  $P(12) = 72$ ; No,  $P$  is decreasing for  $I < 12$ .

69.  $\theta = \operatorname{arccsc} \sqrt{3} \approx 0.9553$  rad    71. True

73. Proof    75. D

77. (a)  $y = -\frac{1}{2}x + (\frac{\pi}{6} + \frac{\sqrt{3}}{4})$

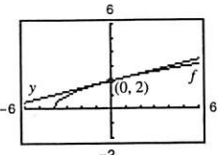
(b)  $x = \frac{\pi}{4}$ , relative maximum;  $x = \frac{3\pi}{4}$ , relative minimum;

$x = \frac{5\pi}{4}$ , relative maximum;  $x = \frac{7\pi}{4}$ , relative minimum

39.  $f(x) = \sqrt{x}$ ,  $dy = \frac{1}{2\sqrt{x}} dx$   
 $f(99.4) \approx \sqrt{100} + \frac{1}{2\sqrt{100}}(-0.6) = 9.97$   
 Calculator: 9.97

41.  $f(x) = \sqrt[4]{x}$ ,  $dy = \frac{1}{4x^{3/4}} dx$   
 $f(624) \approx \sqrt[4]{625} + \frac{1}{4(625)^{3/4}}(-1) = 4.998$   
 Calculator: 4.998

43.  $y - f(0) = f'(0)(x - 0)$   
 $y - 2 = \frac{1}{4}x$   
 $y = 2 + \frac{x}{4}$



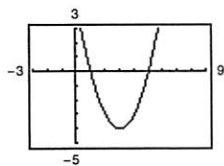
45. (a)  $f(x) = \sqrt{x}$ ;  $dy = \frac{1}{2\sqrt{x}} dx$   
 $f(4.02) \approx \sqrt{4} + \frac{1}{2\sqrt{4}}(0.02) = 2 + \frac{1}{4}(0.02)$   
 (b)  $f(x) = \tan x$ ;  $dy = \sec^2 x dx$   
 $f(0.05) \approx \tan 0 + (\sec^2 0)(0.05) = 0 + 1(0.05)$

47. True  
 49. False; Let  $f(x) = \sqrt{x}$ ,  $x = 1$ , and  $\Delta x = dx = 3$ . 51. A  
 53. (a)  $P' = \frac{1}{4}e^{-x/400}(400 - x)$  (b) 400 units  
 (c) \$503; 5.8%

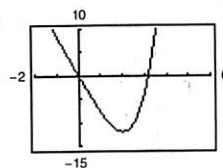
Review Exercises for Chapter 3 (page 274)

1. Maximum: (0, 0)      3. Maximum: (4, 0)  
 Minimum:  $(-\frac{5}{2}, -\frac{25}{4})$       Minimum: (0, -2)  
 5. Maximum:  $(3, \frac{2}{3})$       7. Maximum:  $(2\pi, 17.57)$   
 Minimum:  $(-3, -\frac{2}{3})$       Minimum:  $(2.73, 0.88)$   
 9.  $f(0) \neq f(4)$       11. Not continuous on  $[-2, 2]$   
 13.  $f'(\frac{2744}{729}) = \frac{3}{7}$       15.  $f$  is not differentiable at  $x = 5$ .  
 17.  $f'(0) = 1$   
 19. No; The function has a discontinuity at  $x = 0$ , which is in the interval  $[-2, 1]$ .  
 21. Increasing on  $(-\frac{3}{2}, \infty)$ ; Decreasing on  $(-\infty, -\frac{3}{2})$   
 23. Increasing on  $(-\infty, 1)$  and  $(\frac{7}{3}, \infty)$ ; Decreasing on  $(1, \frac{7}{3})$   
 25. Increasing on  $(1, \infty)$ ; Decreasing on  $(0, 1)$   
 27. Increasing on  $(-\infty, 2 - \frac{1}{\ln 2})$ ; Decreasing on  $(2 - \frac{1}{\ln 2}, \infty)$

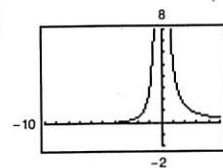
29. (a) Critical number:  $x = 3$   
 (b) Increasing on  $(3, \infty)$ ; Decreasing on  $(-\infty, 3)$   
 (c) Relative minimum:  $(3, -4)$   
 (d)



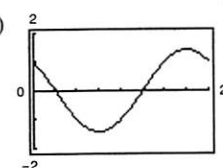
31. (a) Critical number:  $t = 2$   
 (b) Increasing on  $(2, \infty)$ ; Decreasing on  $(-\infty, 2)$   
 (c) Relative minimum:  $(2, -12)$   
 (d)



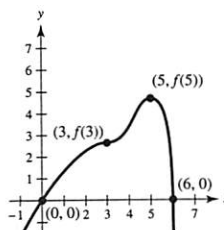
33. (a) Critical number:  $x = -8$ ; Discontinuity:  $x = 0$   
 (b) Increasing on  $(-8, 0)$   
 Decreasing on  $(-\infty, -8)$  and  $(0, \infty)$   
 (c) Relative minimum:  $(-8, -\frac{1}{16})$   
 (d)



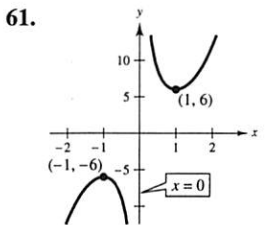
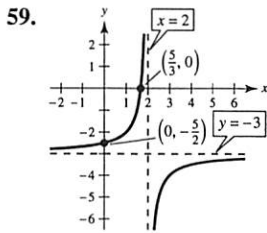
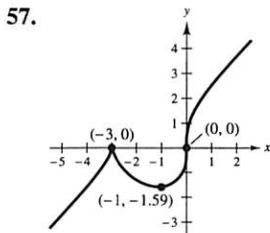
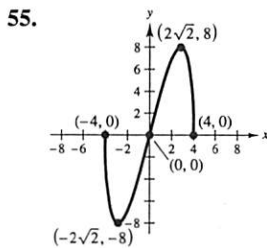
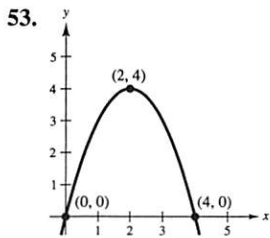
35. (a) Critical numbers:  $x = \frac{3\pi}{4}, \frac{7\pi}{4}$   
 (b) Increasing on  $(\frac{3\pi}{4}, \frac{7\pi}{4})$   
 Decreasing on  $(0, \frac{3\pi}{4})$  and  $(\frac{7\pi}{4}, 2\pi)$   
 (c) Relative minimum:  $(\frac{3\pi}{4}, -\sqrt{2})$   
 Relative maximum:  $(\frac{7\pi}{4}, \sqrt{2})$   
 (d)



37.  $(3, -54)$ ; Concave upward:  $(3, \infty)$ ;  
 Concave downward:  $(-\infty, 3)$   
 39. No points of inflection; Concave upward:  $(-5, \infty)$   
 41.  $(\frac{\pi}{2}, \frac{\pi}{2}), (\frac{3\pi}{2}, \frac{3\pi}{2})$ ; Concave upward:  $(\frac{\pi}{2}, \frac{3\pi}{2})$ ;  
 Concave downward:  $(0, \frac{\pi}{2}), (\frac{3\pi}{2}, 2\pi)$   
 43. Relative minimum:  $(-9, 0)$   
 45. Relative maxima:  $(\pm \frac{\sqrt{2}}{2}, \frac{1}{2})$ ; Relative minimum:  $(0, 0)$   
 47. Relative maximum:  $(-3, -12)$ ; Relative minimum:  $(3, 12)$   
 49.



51.  $x = \sqrt{\frac{2Qs}{r}}$



63.  $x = 50$  ft,  $y = \frac{200}{3}$  ft    65.  $(0, 0), (5, 0), (0, 10)$

67. 14.05 ft    69.  $\frac{32\pi r^3}{81}$  units<sup>3</sup>

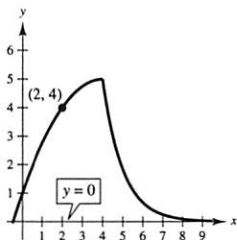
71.  $\Delta y = 0.03005$ ;  $dy = 0.03$

73.  $(1 + x \sin x - \cos x) dx$     75.  $\Delta p = -\frac{1}{4}$ ;  $dp = -\frac{1}{4}$

AP® Exam Practice Questions for Chapter 3 (page 276)

1. B    2. B    3. B    4. C    5. D    6. A    7. A

8. (a)  $(-\infty, 4)$   
 (b) Yes; It goes from increasing to decreasing and  $f(x)$  is continuous.  
 (c)  $x = 4$ ; At this value,  $f''(x)$  changes sign.  
 (d) No;  $f(x)$  is not differentiable on  $(3, 5)$ .  
 (e) Answers will vary. Sample answer:



9. (a)  $x \approx \pm 0.7108$     (b)  $y = \frac{3\pi^2}{8}x - \frac{\pi^3}{8}$   
 (c)  $f(1.5) \approx 1.6759$ ; This is an underestimate because  $f'(\frac{\pi}{2}) > f'(1.5)$ .

10. (a)  $2\pi + 1$ ; Critical number:  $x = \frac{\pi}{4}$

$$f(\pi) = 2\pi + 1 > f\left(\frac{\pi}{4}\right) > f(0)$$

- (b)  $f(x)$  is continuous on  $[0, \pi]$  and differentiable on  $(0, \pi)$ ;

$$x = \frac{\pi}{2}$$

11. (a)  $(0, 2)$   
 (b)  $(-2, -0.8), (1.3, \infty)$ ;  $f(x)$  is concave downward when  $f'(x)$  is decreasing.

- (c) Negative;  $f'(x)$  is negative so the slope of  $f(x)$  is negative there.

12. (a)  $(-\infty, 0), (0, \infty)$   
 (b)  $f(x)$  is never concave downward because  $f''(x) > 0$  for all real  $x$ .  
 (c)  $f(x)$  has no points of inflection because it does not change concavity.

13. (a)  $f'(-1) = -\frac{2}{3}$ ;  $f''(-1) = -\frac{1}{3}$

- (b)  $x = -3$ ;  $f'(x)$  changes from positive to negative, so  $f$  has a relative maximum at  $x = -3$ .

- (c)  $(-4, 0)$  and  $(1, 4)$ ;  $f$  is concave downward when  $f'$  is decreasing.

- (d)  $x = -4, x = 0$ , and  $x = 1$ ;  $f''(0)$  and  $f''(1)$  are undefined, and  $f''(-4) = 0$ .

- (e) Decreasing;

$$\begin{aligned} g(x) &= f(x) + \sin^2 x \\ g'(x) &= f'(x) + 2 \sin x \cos x \\ g'\left(-\frac{\pi}{4}\right) &= f'\left(-\frac{\pi}{4}\right) + 2 \sin\left(-\frac{\pi}{4}\right) \cos\left(-\frac{\pi}{4}\right) \\ &= f'\left(-\frac{\pi}{4}\right) + 2\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= f'\left(-\frac{\pi}{4}\right) - 1 \end{aligned}$$

From the graph,  $f'\left(-\frac{\pi}{4}\right)$  is negative. So,  $g'\left(-\frac{\pi}{4}\right)$  is negative, which means that  $g$  is decreasing at  $x = -\frac{\pi}{4}$ .

Chapter 4

Section 4.1 (page 287)

1. Proof    3.  $y = 3t^3 + C$     5.  $y = \frac{2}{5}x^{5/2} + C$

Original Integral	Rewrite	Integrate	Simplify
7. $\int \sqrt[3]{x} dx$	$\int x^{1/3} dx$	$\frac{x^{4/3}}{4/3} + C$	$\frac{3}{4}x^{4/3} + C$

9. $\int \frac{1}{x\sqrt{x}} dx$	$\int x^{-3/2} dx$	$\frac{x^{-1/2}}{-1/2} + C$	$-\frac{2}{\sqrt{x}} + C$
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11.  $\frac{3}{4}x^4 - 2x^3 + 2x + C$     13.  $\frac{2}{5}x^{5/2} + x^2 + x + C$

15.  $\frac{3}{5}x^{5/3} + C$     17.  $-\frac{1}{4x^4} + C$     19.  $\frac{2}{3}x^{3/2} + 12x^{1/2} + C$

21.  $x^3 + \frac{1}{2}x^2 - 2x + C$     23.  $5 \sin x - 4 \cos x + C$

25.  $-2 \cos x - 5e^x + C$     27.  $\tan y + C$

29.  $x^2 - \frac{4^x}{\ln 4} + C$     31.  $\frac{1}{2}x^2 - 5 \ln|x| + C$