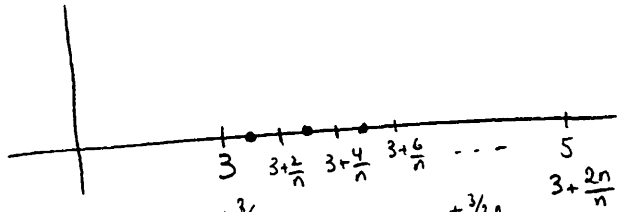
7. $\lim\limits_{x\to\infty} \sum_{k=1}^{n} \frac{2}{n}\log\left(3+\frac{2k-1}{n}\right)$

$$\frac{2}{n}\log\left(3+\frac{1}{n}\right) + \frac{2}{n}\log\left(3+\frac{3}{n}\right) + \dots + \frac{2}{n}\log\left(3+\frac{2n-1}{n}\right)$$

$$= \int_3^5 \log x\,dx$$

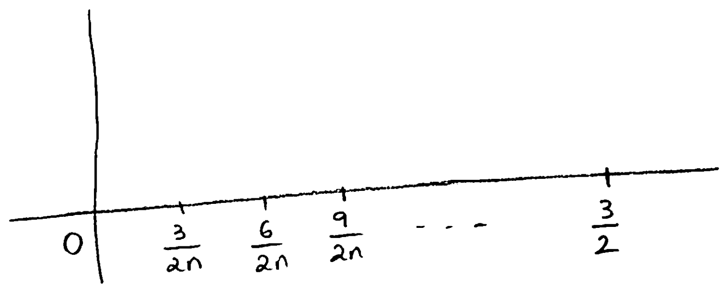


8. $\lim\limits_{x\to\infty} \sum_{k=1}^{n} \frac{5}{n}\ln\left(\frac{3k}{2n}\right)$

+3/2n   +3/2n

$$\frac{5}{n}\ln\left(\frac{3}{2n}\right) + \frac{5}{n}\ln\left(\frac{6}{2n}\right) + \frac{5}{n}\ln\left(\frac{9}{2n}\right) + \dots + \frac{5}{n}\ln\left(\frac{3n}{2n}\right)$$

$$= \frac{10}{3}\int_0^{3/2} \ln x\,dx$$

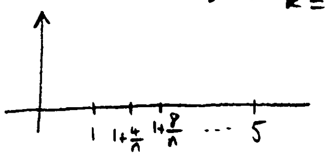


$$\frac{10}{3}\cdot\frac{3}{2n}\left(\ln\frac{3}{2n} + \ln\frac{6}{2n} + \ln\frac{9}{2n} + \dots + \ln\left(\frac{3n}{2n}\right)\right)$$

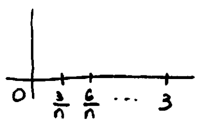
Limits of Riemann Sums – Extra Practice

Rewrite the following integrals as a limit of a Riemann Sum :

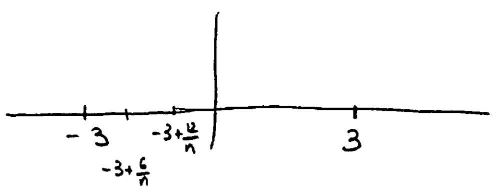
1. $\int_1^5 \frac{1}{x}dx = \lim\limits_{n\to+\infty} \sum\limits_{k=1}^{n} \frac{1}{1+\frac{4k}{n}} \cdot \frac{4}{n}$



2. $\int_0^3 \sqrt{2+x}dx = \lim\limits_{n\to+\infty} \sum\limits_{k=1}^{n} \sqrt{2+\frac{3k}{n}} \cdot \frac{3}{n}$



3. $\int_{-3}^{3} \cos(2x)\, dx = \lim\limits_{n\to+\infty} \sum\limits_{k=1}^{n} \cos\left(2\left(-3+\frac{6k}{n}\right)\right) \cdot \frac{6}{n}$



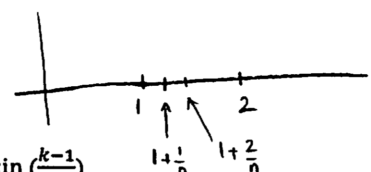
Rewrite these limits as Integrals :

4. $\lim\limits_{x\to\infty} \sum_{k=1}^{n} \frac{1}{n}(1+\frac{k}{n})^5 = \int_1^2 x^5 dx$

$k=1: \ \frac{1}{n}\left(1+\frac{1}{n}\right)^5$

$k=2: \ \frac{1}{n}\left(1+\frac{2}{n}\right)^5$

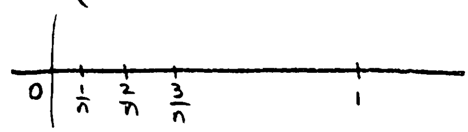
$k=n: \ \frac{1}{n}\left(1+\frac{n}{n}\right)^5$



5. $\lim\limits_{x\to\infty} \sum_{k=1}^{n} \frac{3\sin\left(\frac{k-1}{n}\right)}{n}$

$= 3\int_0^1 \sin x\, dx$

$\frac{3}{n}\left(\sin 0 + \sin\frac{1}{n} + \sin\frac{2}{n} + \dots + \sin\frac{n-1}{n}\right)$



6. $\lim\limits_{x\to\infty} \sum_{k=1}^{n} \frac{2}{n}\sqrt{4+\frac{2k}{n}}$

$= \int_4^6 \sqrt{x}\, dx$

$\frac{2}{n}\sqrt{4+\frac{2}{n}} + \frac{2}{n}\sqrt{4+\frac{4}{n}} + \frac{2}{n}\sqrt{4+\frac{6}{n}} + \dots + \frac{2}{n}\sqrt{4+\frac{2n}{n}}$