The EXISTENCE Theorems

These theorems guarantee the existence of a point satisfying a certain condition…

IVT

The Intermediate Value Theorem (IVT)

If f is continuous on the closed interval [a, b] then f takes on every value between $f(a)$ and $f(b)$.

Suppose k is any number between $f(a)$ and $f(b)$, then there is at least one number c in [a, b] such that $f(c) = k$.

The IVT tells you that if a function is continuous on a closed interval, it will take all the y-values between the two end points.

EVT

The Extreme Value Theorem

If f is continuous on a closed interval [a, b], then f has both a minimum and a maximum on the interval

The EVT tells you that if a function is continuous on a closed interval, there will be a point in that interval that is the max and a point that is the min.

MVT

The Mean Value Theorem

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then *there exists* a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

The MVT tells you that if a function is continuous on a closed interval and differentiable on the open one, there is a point where the instantaneous rate of change (derivative) equals the average rate of change of the function over the interval (the slope of a tangent line will be the same than the slope of the secant line joining the end points).

Applications:

The **IVT** is usually used to show that an equation has at least one solution.

The **EVT** is usually used to be able to give a name to a point that will be an extremum without really knowing where it is. It is useful to prove many theorems, but is extremely rare on an AP exam…

The **MVT** is usually used to show that a function has a local min or max in a certain interval.

2014 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters / minute)	0	100	40	−120	−150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

(a) Find the average acceleration of train A over the interval $2 \le t \le 8$.

(b) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

Solutions :

(a) $$\frac{V(8) - V(2)}{8-2} = \frac{-120-100}{6} = -\frac{220}{6} = -\frac{110}{3} \ m/min^2$$

(b) The velocity is a continuous function of time.

$\therefore V_A(t)$ is continuous on $[5,8]$

$V_A(5) = 40 > -100$

$V_A(8) = -120 < -100$

Therefore, the IVT applies and guarantees the existence of a time $t \in (5,8)$ such that

$$V_A(t) = -100.$$

Examples from the AP Exam…

Example for **MVT:**

2013 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

   (a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

   (b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

Solutions :

(a) $C'(3.5) \simeq \dfrac{C(4) - C(3)}{4-3} = \dfrac{12.8 - 11.2}{1} = 1.6$ ounces/min

(b) We have: $\dfrac{C(4) - C(2)}{4-2} = \dfrac{12.8 - 8.8}{2} = 2$.

Moreover,
C is differentiable on $[0,6]$. Therefore it is continuous on $[2,4]$ and differentiable on $(2,4)$
The MVT applies and guarantees the existence of a time $t \in (2,4)$ such that $C'(t) = 2$.