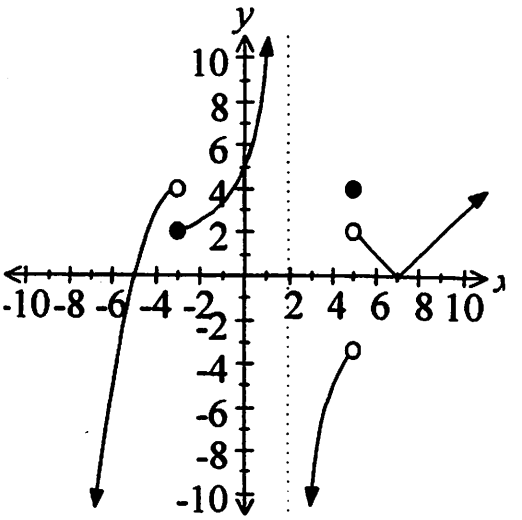
MIDTERM REVIEW

Chapter 2 LIMITS

1. For the function f shown, evaluate the following: **[3]**



a) $\lim_{x\to 5+} f(x)$ = _____

b) $\lim_{x\to 5} f(x)$ = _____

c) $f(5)$ = _____

d) $\lim_{x\to -3-} f(x)$ = _____

e) $\lim_{x\to -\infty} f(x)$ = _____

f) $\lim_{x\to \infty} f(x)$ = _____

g) $\lim_{x\to 2-} f(x)$ = _____

h) $\lim_{x\to 2+} f(x)$ = _____

2. For what value(s) of x is the function $f(x)=\begin{cases}\dfrac{x^2-1}{x-1} & x\neq 1\\ 3 & x=1\end{cases}$ **continuous?** Justify your answer.

3. Evaluate each of the following limits. **Show your work for all questions, including (f).**

a) $\lim_{x\to 1} \dfrac{3x+7}{x-2}$

b) $\lim_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$

c) $\lim_{h\to 0} \dfrac{\dfrac{1}{1+h}-1}{h}$

d) $\lim_{x\to 2+} \dfrac{|2-x|}{x^2-4}$

[4]

e) $\lim_{x\to 3-} \dfrac{1-4x^2}{x+3}$

f) $\lim_{x\to -\infty} \dfrac{1-4x^2}{x+3}$

[4]

g) $\lim_{x\to \infty} \dfrac{4-x^3}{2x^3+1}$

h $\lim_{x\to 3} \dfrac{9-x^2}{x-3}$

i) $\lim_{x\to 2+} \frac{|2-x|}{x^2-4}$

j) $\lim_{x\to 0} \frac{\sin x}{x^2-x}$

k) $\lim_{x\to\infty} \frac{4x^4-2x^2+1}{x^2}$

l) $\lim_{x\to 4-} f(x)$ where $f(x)=\begin{cases} 3x-2 & x<4 \\ x^2 & x\geq 4 \end{cases}$

m) $\lim_{t\to -1.5} \frac{2t+3}{\sqrt{6+t}-\sqrt{3-t}}$

4. Let the function f be defined by $f(x)=\begin{cases} x+c & x<2 \\ cx^2+1 & x\geq 2 \end{cases}$

For what value(s) of c does $\lim_{x\to 3} f(x)$ exist?

Justify your answer.

5. For the function $f(x) = \dfrac{1-4x^2}{x^2-2x+1}$

a) identify any vertical asymptote(s) and determine the behaviour of the function around the asymptote(s) – i.e. evaluate the one sided limits on each side of each asymptote.

b) Identify any horizontal asymptotes.

[3]

6. Let f be the function defined by $f(x) = \begin{cases} \sin x & x < 0 \\ 2 & x = 0 \\ x^2 & 0 < x < 2 \\ x+1 & x \geq 2 \end{cases}$

a) Evaluate $\lim_{x \to 0} f(x)$. Show your work. **[1]**

b) At what value(s) of x is f continuous? Give reasons for your answer

c) At what value(s) of x is f differentiable? Give reasons for your answer

7. Let $f(x) = \dfrac{x^2+2x+1}{x^2-1}$. Evaluate the following limits. [

a) $\lim_{x\to 1} f(x)$

b) $\lim_{x\to -1^+} f(x)$

c) $\lim_{x\to -1} f(x)$

d) $\lim_{x\to \infty} f(x)$

8. Let $g(x) = \dfrac{x^2+1}{ax}$. For what value(s) of x is $\lim_{x\to\infty} g(x) = -\infty$? Give reasons for your answer.

9 Let $h(x) = \dfrac{(x+1)(x-4)}{(x+a)^2}$. **No written solutions are required for this question.**

a) For what value(s) of a does h have a vertical asymptote? **[1]**

b) For what value(s) of a does h have a removable discontinuity (i.e., a "hole")? **[1]**

c) State the equation of the horizontal asymptote of h. **[1]**

10. Use first principles (i.e. the limit definition) to determine the derivative of $y = \frac{2}{x+1}$.

11. Use first principles to determine the slope of the tangent line to $f(x) = \sqrt{x+4}$ at the point where $x = 0$.

2. Find the derivative of $y = \dfrac{2x}{1-3x^2}$

A) $-\dfrac{1}{3x}$

B) $-\dfrac{12x}{(1-3x^2)^2}$

C) $\dfrac{6x^2+2}{(1-3x^2)^2}$

D) $\dfrac{9x^2-2}{(1-3x^2)^2}$

E) $\dfrac{2x}{3(1-3x^2)^2}$

3. For $y = \sqrt{3-2x}$, $\dfrac{dy}{dx}$ equals

A) $\dfrac{1}{2\sqrt{3-2x}}$

B) $-\dfrac{1}{\sqrt{3-2x}}$

C) $-\dfrac{(3-2x)^{\frac{3}{2}}}{3}$

D) $-\dfrac{1}{3-2x}$

E) $\dfrac{2}{3}(3-2x)^{\frac{3}{2}}$

4. If $y = (3x^2+5)^5\,(x+2)^4$ then $\dfrac{dy}{dx} =$

A) $2(x+2)^3\,(3x^2+5)^4$

B) $2(21x^2+30x+10)\,(x+2)^3\,(3x^2+5)^4$

B) $(x+2)^3\,(3x^2+5)\,(21x^2+30x+10)$

D) $24(x+2)^3\,(3x^2+5)^4\,(21x^2+30x+10)$

E) $12(x+2)^3\,(3x^2+5)^4\,(21x+30)$

5. If $y = \cos^2 3x$, then $\frac{dy}{dx} =$

A) $-6\sin 3x \cos 3x$
B) $-2\cos 3x$
C) $2\cos 3x$
D) $6\cos 3x$
E) $2\sin 3x \cos 3x$

6 $\frac{d}{dx} 2^{\cos x} =$

A) $-2^{\cos x} \ln(\sin x)$
B) $-2^{\cos x} \sin x \ln 2$
C) $2^{-\cos x} \sin x \ln 2$
D) $2^{-\sin x} \ln 2$
E) $-2^{\sin x} \ln 2$

7 If $y = \arctan\left(e^{2x}\right)$ then $\frac{dy}{dx} =$

A) $\frac{2e^{2x}}{\sqrt{1-e^{4x}}}$
B) $\frac{2e^{2x}}{1+e^{4x}}$
C) $\frac{e^{2x}}{1+e^{4x}}$
D) $\frac{1}{\sqrt{1-e^{4x}}}$
E) $\frac{1}{1+e^{4x}}$

8. If $y = (\sin x)^x$ then $\frac{dy}{dx} =$

A) $x \ln(\sin x)$
B) $(\sin x)^x \cot x$
C) $x (\sin x)^{x-1} \cos x$
C) $(\sin x)^x (x \cos x + \sin x)$
E) $(\sin x)^x (x \cot x + \ln(\sin x))$

9. If $y^2 - 2xy = 16$ then $\frac{dy}{dx} =$

A) $\frac{x}{y-x}$

B) $\frac{y}{x-y}$

C) $\frac{y}{y-x}$

D) $\frac{y}{2y-x}$

E) $\frac{2y}{x-y}$

10. An equation of the line **normal** to the graph of $y = x^3 + 3x^2 + 7x - 1$ at the point where $x = -1$ is

A) $4x + y = -10$

B) $x - 4y = 23$

C) $4x - y = 2$

C) $x + 4y = 25$

E) $x + 4y = -25$

11 If $f(x) = \ln(\ln x)$, thne $f'(x) =$

A) $\frac{1}{x}$

B) $\frac{1}{\ln x}$

C) $\frac{\ln x}{x}$

D) x

E) $\frac{1}{x \ln x}$

12 $\frac{d}{dx}\left(\frac{1}{x^3}-\frac{1}{x}+x^2\right)$ at $x=-1$ is

(A) -6
(B) -4
(C) 0
(D) 2
(E) 6

9. For $x+\cos(x+y)=\pi$, $\frac{dy}{dx}$ equals

A) $\csc(x+y)-1$
B) $\csc(x+y)$
C) $\frac{x}{\sin(x+y)}$
D) $\frac{1}{\sqrt{1-x^2}}$
E) $\frac{1-\sin x}{\sin y}$

10. $\frac{d}{dx}(\arcsin 2x) =$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$

(B) $\frac{-2}{\sqrt{4x^2-1}}$

(C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$

(E) $\frac{2}{\sqrt{4x^2-1}}$

11. If $y = \text{Arctan}(\cos x)$, then $\frac{dy}{dx} =$

(A) $\frac{-\sin x}{1+\cos^2 x}$

(B) $-(\text{Arcsec}(\cos x))^2 \sin x$

(C) $(\text{Arcsec}(\cos x))^2$

(D) $\frac{1}{(\text{Arccos}\, x)^2 + 1}$

(E) $\frac{1}{1+\cos^2 x}$

12. If $f(x) = \ln(\ln x)$, thne $f'(x) =$

(A) $\frac{1}{x}$

(B) $\frac{1}{\ln x}$

(C) $\frac{\ln x}{x}$

(D) x

(E) $\frac{1}{x \ln x}$

13. If $y = (\sin x)^x$ then $\frac{dy}{dx} =$

A) $x \ln(\sin x)$

B) $(\sin x)^x \cot x$

C) $x (\sin x)^{x-1} \cos x$

D) $(\sin x)^x (x \cos x + \sin x)$

E) $(\sin x)^x (x \cot x + \ln(\sin x))$

14. $\frac{d}{dx} 3^{\csc x} =$

A) $\csc x(3)^{\csc x - 1}$

B) $-3^{\csc x} \csc x \cot x$

C) $-3^{\csc x} \ln 3 \csc x \cot x$

D) $-3^{\csc x} \cot^2 x$

E) $-3^{\csc x} \ln 3 \cot^2 x$

15. For $y = 2\sqrt{x} - \frac{1}{2\sqrt{x}}$, $\frac{dy}{dx}$ equals

A) $x + \frac{1}{x\sqrt{x}}$ B) $x^{-1/2} + x^{-3/2}$ C) $\frac{4x-1}{4x\sqrt{x}}$

D) $\frac{1}{\sqrt{x}} + \frac{1}{4x\sqrt{x}}$ E) $\frac{4}{\sqrt{x}} + \frac{1}{x\sqrt{x}}$

16. For $y = \frac{1+x^2}{1-x^2}$, $\frac{dy}{dx}$ equals

A) $-\frac{4x}{(1-x^2)^2}$ B) $\frac{4x}{(1-x^2)^2}$ C) $\frac{-4x^3}{(1-x^2)^2}$

D) $\frac{2x}{1-x^2}$ E) $\frac{4}{1-x^2}$