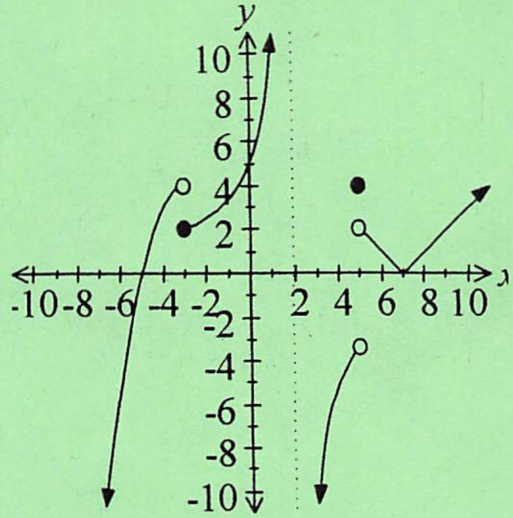
Sol'ns

MIDTERM REVIEW

Chapter 2 LIMITS



1. For the function f shown, evaluate the following: **[3]**

a) $\lim_{x \to 5+} f(x)$ = 2

b) $\lim_{x \to 5} f(x)$ = DNE

c) $f(5)$ = 4

d) $\lim_{x \to -3-} f(x)$ = 4

e) $\lim_{x \to -\infty} f(x)$ = $-\infty$

f) $\lim_{x \to \infty} f(x)$ = $+\infty$

g) $\lim_{x \to 2-} f(x)$ = $+\infty$

h) $\lim_{x \to 2+} f(x)$ = $-\infty$

2. For what value(s) of x is the function $f(x) = \begin{cases} \dfrac{x^2-1}{x-1} & x \neq 1 \\ 3 & x = 1 \end{cases}$ **continuous**? Justify your answer.

$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{(x-1)(x+1)}{(x-1)} = 2$

Since $\lim_{x \to 1} f(x) \neq 3$, $f(x)$ is continuous for all x except $x = 1$

3. Evaluate each of the following limits. **Show your work for all questions, including (f).**

a) $\lim_{x \to 1} \frac{3x+7}{x-2}$

$= \frac{3(1)+7}{(1)-2}$

$= -10$

b) $\lim_{x \to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} \cdot \frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}$

$\lim_{x \to 0} \frac{(2+x)-2}{x(\sqrt{2+x}+\sqrt{2})} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

c) $\lim_{h\to 0}\dfrac{\frac{1}{1+h}-1}{h}$

$= \lim_{h\to 0}\dfrac{\frac{1}{1+h}-\frac{(1+h)}{1+h}}{h}$

$= \lim_{h\to 0}\dfrac{-\cancel{h}}{1+h}\cdot\dfrac{1}{\cancel{h}}$

$= -1$

d) $\lim_{x\to 2+}\dfrac{|2-x|}{x^2-4}$

when $x > 2$,
$|2-x| = -(2-x)$
$= x-2$

$= \lim_{x\to 2^+}\dfrac{x-\cancel{2}}{(x-\cancel{2})(x+2)}$

$= \dfrac{1}{4}$

[4]

e) $\lim_{x\to -3-}\dfrac{1-4x^2}{x+3}$

$= \dfrac{1-4(-3)^2}{0^-}$

$= \dfrac{-35}{0^-}$

$= \infty$

f) $\lim_{x\to -\infty}\dfrac{1-4x^2}{x+3}$

$= \lim_{x\to -\infty}\dfrac{-4x^{\cancel{2}}}{\cancel{x}}$ EBM

$= \infty$

[4]

g) $\lim_{x\to\infty}\dfrac{4-x^3}{2x^3+1} =$

$= \lim_{x\to\infty} -\dfrac{\cancel{x^3}}{2\cancel{x^3}}$

$= -\dfrac{1}{2}$

h $\lim_{x\to 3}\dfrac{9-x^2}{x-3}$

$= \lim_{x\to 3}\dfrac{(3-\cancel{x})(3+x)}{-(3-\cancel{x})}$

$= \dfrac{6}{-1}$

$= -6$

i) $\lim\limits_{x\to 2+}\dfrac{|2-x|}{x^2-4}$ when $x > 2$

$|2-x| = -(2-x)$

$= x-2$

$= \lim\limits_{x\to 2^+}\dfrac{\cancel{x-2}}{(\cancel{x-2})(x+2)}$

$= \dfrac{1}{4}$

j) $\lim\limits_{x\to 0}\dfrac{\sin x}{x^2-x}$

$= \lim\limits_{x\to 0}\dfrac{\sin x}{x}\cdot\dfrac{1}{x-1}$

$= 1 \cdot -1$

$= -1$

k) $\lim\limits_{x\to\infty}\dfrac{4x^4-2x^2+1}{x^2}$

$= \lim\limits_{x\to\infty}\dfrac{4x^{\cancel{4}\,2}}{\cancel{x^2}}$

$= \infty$

l) $\lim\limits_{x\to 4-} f(x)$ where $f(x)=\begin{cases}3x-2 & x<4\\ x^2 & x\geq 4\end{cases}$

$= \lim\limits_{x\to 4^-} 3x-2$

$= 10$

m) $\lim\limits_{t\to -1.5}\dfrac{2t+3}{\sqrt{6+t}-\sqrt{3-t}}$ $= 3\sqrt{2}$

$= \lim\limits_{t\to -1.5}\dfrac{2}{\frac{1}{2}(6+t)^{-1/2}+\frac{1}{2}(3-t)^{-1/2}}$

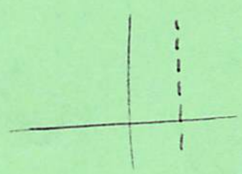
$= \dfrac{4}{\frac{1}{\sqrt{4.5}}+\frac{1}{\sqrt{4.5}}}$

$= \dfrac{4}{\frac{2}{\sqrt{4.5}}} = 4\cdot\dfrac{\sqrt{4.5}}{2} = 2\sqrt{\dfrac{9}{2}} = \dfrac{6}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = 3\sqrt{2}$

4. Let the function f be defined by $f(x)=\begin{cases}x+c & x<2\\ cx^2+1 & x\geq 2\end{cases}$

For what value(s) of c does $\lim\limits_{x\to 3}f(x)$ 2 exist?

Justify your answer.



$x+c = cx^2+1$ when $x=2$

$2+c = c(4)+1$

$1 = 3c$

$\boxed{c = 1/3}$ when $c = 1/2$ LH Lim = RH Lim

5. For the function $f(x) = \dfrac{1-4x^2}{x^2-2x+1}$

a) identify any vertical asymptote(s) and determine the behaviour of the function around the asymptote(s) – i.e. evaluate the one sided limits on each side of each asymptote.

b) Identify any horizontal asymptotes.

[3]

a) vert as. when $x^2-2x+1=0$

or $(x-1)(x-1)=0$

$x=1$

$$\lim_{x\to1^+}\frac{(1-2x)(1+2x)}{(x-1)(x-1)} = -\infty$$

$$\lim_{x\to1^-}\frac{(1-2x)(1+2x)}{(x-1)(x-1)} = -\infty$$

b) hor as. when $x\to\pm\infty$

$$\lim_{x\to\pm\infty}\frac{-4x^2}{x^2} = -4$$

6. Let f be the function defined by $f(x) = \begin{cases} \sin x & x<0 \\ 2 & x=0 \\ x^2 & 0<x<2 \\ x+1 & x\geq 2 \end{cases}$

a) Evaluate $\lim_{x\to0} f(x)$. Show your work. [1]

$$\left.\begin{aligned}\lim_{x\to0^-}\sin x &= 0\\ \lim_{x\to0^+}x^2 &= 0\end{aligned}\right\} \therefore \lim_{x\to0} f(x) = 0$$

b) At what value(s) of x is f continuous? Give reasons for your answer

$$\left.\begin{aligned}\lim_{x\to2^-}x^2 &= 4\\ \lim_{x\to2^+}x+1 &= 3\end{aligned}\right\} \lim_{x\to2} \text{ DNE}$$

$\therefore$ cont. on $(-\infty,0)\cup(0,2)\cup[2,\infty)$

c) At what value(s) of x is f differentiable? Give reasons for your answer

all x except $x=0,2$

7. Let $f(x) = \dfrac{x^2+2x+1}{x^2-1}$. Evaluate the following limits. [

a) $\lim_{x\to 1} f(x)$

$= \lim_{x\to 1} \dfrac{\cancel{(x+1)}(x+1)}{\cancel{(x+1)}(x-1)}$

$= \lim_{x\to 1^-} \dfrac{x+1}{x-1} = -\infty$

$\lim_{x\to 1^+} \dfrac{x+1}{x-1} = \infty$

lim DNE

b) $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} \dfrac{x+1}{x-1}$

$= \dfrac{0}{-2}$

$= 0$

c) $\lim_{x\to -1} f(x)$

$= 0$

d) $\lim_{x\to \infty} f(x) = \dfrac{x^2}{x^2} = 1$

8. Let $g(x) = \dfrac{x^2+1}{ax}$. For what value(s) of x is $\lim_{x\to\infty} g(x) = -\infty$? Give reasons for your answer.

Since numerator is always >0, denominator must be $ax < 0$; for positive a, $x<0$ and for negative a, $x > 0$

9 Let $h(x) = \dfrac{(x+1)(x-4)}{(x+a)^2}$. **No written solutions are required for this question.**

a) For what value(s) of a does h have a vertical asymptote? **[1]**

$a \in \mathbb{R}$

b) For what value(s) of a does h have a removable discontinuity (i.e., a "hole")? **[1]**

$a = 1, -4$

c) State the equation of the horizontal asymptote of h. **[1]**

$y = 1$

10. Use first principles (i.e. the limit definition) to determine the derivative of $y = \dfrac{2}{x+1}$.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2}{x+h+1} - \frac{2}{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2(x+1) - 2(x+h+1)}{(x+h+1)(x+1)}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x} + \cancel{2} - \cancel{2x} - 2h - \cancel{2}}{(x+h+1)(x+1)\cancel{h}}$$

$$= \lim$$

$$= \lim_{h \to 0} \frac{-2}{(x+h+1)(x+1)}$$

$$= \frac{-2}{(x+1)^2}$$

11. Use first principles to determine the slope of the tangent line to $f(x) = \sqrt{x+4}$ at the point where $x = 0$.

$$\lim_{h \to 0} \frac{\sqrt{x+h+4} - \sqrt{x+4}}{h} \cdot \frac{\sqrt{x+h+4} + \sqrt{x+4}}{\sqrt{x+h+4} + \sqrt{x+4}}$$

$$= \lim_{h \to 0} \frac{(\cancel{x} + \cancel{h} + \cancel{4}) - (\cancel{x} + \cancel{4})}{\cancel{h}(\sqrt{x+h+4} + \sqrt{x+4})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+4} + \sqrt{x+4}}$$

$$f'(x) = \frac{1}{2\sqrt{x+4}}$$

$$f'(0) = \frac{1}{4}$$

@ $x = 0,\ y = 2$

$$y = mx + b$$

$$2 = \frac{1}{4}(0) + b$$

$$b = 2$$

$$\therefore \boxed{y = \frac{1}{4}x + 2} \text{ tangent}$$

2. Find the derivative of $y = \dfrac{2x}{1-3x^2}$

A) $-\dfrac{1}{3x}$

B) $-\dfrac{12x}{(1-3x^2)^2}$

C) $\dfrac{6x^2+2}{(1-3x^2)^2}$

D) $\dfrac{9x^2-2}{(1-3x^2)^2}$

E) $\dfrac{2x}{3(1-3x^2)^2}$

3. For $y = \sqrt{3-2x}$, $\dfrac{dy}{dx}$ equals

A) $\dfrac{1}{2\sqrt{3-2x}}$

B) $-\dfrac{1}{\sqrt{3-2x}}$

C) $-\dfrac{(3-2x)^{\frac{3}{2}}}{3}$

D) $-\dfrac{1}{3-2x}$

E) $\dfrac{2}{3}(3-2x)^{\frac{3}{2}}$

4. If $y = (3x^2+5)^5(x+2)^4$ then $\dfrac{dy}{dx} =$

A) $2(x+2)^3(3x^2+5)^4$
B) $2(21x^2+30x+10)(x+2)^3(3x^2+5)^4$
B) $(x+2)^3(3x^2+5)(21x^2+30x+10)$
D) $24(x+2)^3(3x^2+5)^4(21x^2+30x+10)$
E) $12(x+2)^3(3x^2+5)^4(21x+30)$

$5(3x^2+5)^4 \cdot 6x(x+2)^4 + (3x^2+5)^5 4(x+2)$
$= 2(3x^2+5)^4(x+2)^3[15x(x+2)+2(3x^2+5$
$= 2(3x^2+5)^4(x+2)^3(15x^2+30x+6x^2+10$
$= 2(3x^2+5)^4(x+2)^3(21x^2+30x+10)$

5. If $y = \cos^2 3x$, then $\frac{dy}{dx} =$

A) $-6\sin 3x \cos 3x$
B) $-2\cos 3x$
C) $2\cos 3x$
D) $6\cos 3x$
E) $2\sin 3x \cos 3x$

$2\cos 3x \cdot -\sin 3x \cdot 3$

$= 6\cos 3x \cdot \sin 3x$

6 $\frac{d}{dx} 2^{\cos x} =$

A) $-2^{\cos x}\ln(\sin x)$
B) $-2^{\cos x}\sin x \ln 2$
C) $2^{-\cos x}\sin x \ln 2$
D) $2^{-\sin x}\ln 2$
E) $-2^{\sin x}\ln 2$

$y = 2^{\cos x}$

$\ln y = \cos x \ln 2$

$\frac{y'}{y} = -\ln 2 \sin x$

$y' = -2^{\cos x}\ln 2 \sin x$

7 If $y = \arctan(e^{2x})$ then $\frac{dy}{dx} =$

A) $\frac{2e^{2x}}{\sqrt{1-e^{4x}}}$
B) $\frac{2e^{2x}}{1+e^{4x}}$
C) $\frac{e^{2x}}{1+e^{4x}}$
D) $\frac{1}{\sqrt{1-e^{4x}}}$
E) $\frac{1}{1+e^{4x}}$

$\frac{1}{1+(e^{2x})^2} \cdot 2e^{2x}$

$= \frac{2e^{2x}}{1+e^{4x}}$

8. If $y = (\sin x)^x$ then $\frac{dy}{dx} =$

A) $x\ln(\sin x)$
B) $(\sin x)^x \cot x$
C) $x(\sin x)^{x-1}\cos x$
C) $(\sin x)^x (x\cos x + \sin x)$
E) $(\sin x)^x (x\cot x + \ln(\sin x))$

$\ln y = x \ln \sin x$

$\frac{y'}{y} = \ln \sin x + \frac{x}{\sin x} \cdot \cos x$

$y' = (\sin x)^x [\ln \sin x + x\cot x]$

9. If $y^2 - 2xy = 16$ then $\frac{dy}{dx} =$

A) $\frac{x}{y-x}$

B) $\frac{y}{x-y}$

C) $\frac{y}{y-x}$

D) $\frac{y}{2y-x}$

E) $\frac{2y}{x-y}$

$2yy' - (2y + 2xy') = 0$

$y'(2y - 2x) = 2y$

$y' = \frac{2y}{2y-2x}$

$y' = \frac{y}{y-x}$

10. An equation of the line **normal** to the graph of $y = x^3 + 3x^2 + 7x - 1$ at the point where $x = -1$ is

A) $4x + y = -10$
B) $x - 4y = 23$
C) $4x - y = 2$
C) $x + 4y = 25$
E) $x + 4y = -25$

$y' = 3x^2 + 6x + 7$

$y'(-1) = 3 - 6 + 7 = 4$

$m \perp = -\frac{1}{4}$ @ $x = -1$, $y = -1 + 3 - 7 - 1$
$= -6$

$y = mx + b$

$-6 = -\frac{1}{4}(-1) + b$

$-6 - \frac{1}{4} = b$

$b = -\frac{25}{4}$

$y = -\frac{1}{4}x - \frac{25}{4}$

$4y = -x - 25$

$x + 4y = -25$

11 If $f(x) = \ln(\ln x)$, thne $f'(x) =$

A) $\frac{1}{x}$

B) $\frac{1}{\ln x}$

C) $\frac{\ln x}{x}$

D) x

E) $\frac{1}{x \ln x}$

$y' = \frac{1}{\ln x} \cdot \frac{1}{x}$

12 $\frac{d}{dx}\left(\frac{1}{x^3}-\frac{1}{x}+x^2\right)$ at $x=-1$ is

(A) -6
(B) -4
(C) 0
(D) 2
(E) 6

$y = x^{-3} - x^{-1} + x^2$

$y' = -3x^{-4} + x^{-2} + 2x$

$y'(-1) = -3 + 1 - 2$

$= -4$

9. For $x+\cos(x+y)=\pi$, $\frac{dy}{dx}$ equals

A) $\csc(x+y)-1$
B) $\csc(x+y)$
C) $\frac{x}{\sin(x+y)}$
D) $\frac{1}{\sqrt{1-x^2}}$
E) $\frac{1-\sin x}{\sin y}$

$1 - \sin(x+y)(1+y') = 0$

$1 - \sin(x+y) - y'\sin(x+y) = 0$

$\frac{1-\sin(x+y)}{\sin(x+y)} = y'$

$y' = \csc(x+y) - 1$

10. $\frac{d}{dx}(\arcsin 2x) =$

$\frac{1}{\sqrt{1-u^2}}$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$

$y' = \frac{2}{\sqrt{1-4x^2}}$

(B) $\frac{-2}{\sqrt{4x^2-1}}$

(C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$

(E) $\frac{2}{\sqrt{4x^2-1}}$

11. If $y = \text{Arctan}(\cos x)$, then $\frac{dy}{dx} =$

$\frac{1}{1+u^2}$

(A) $\frac{-\sin x}{1+\cos^2 x}$

$y' = \frac{1}{1+\cos^2 x} \cdot -\sin x$

$= \frac{-\sin x}{1+\cos^2 x}$

$=$

(B) $-(\text{Arcsec}(\cos x))^2 \sin x$

(C) $(\text{Arcsec}(\cos x))^2$

(D) $\frac{1}{(\text{Arccos}\, x)^2+1}$

(E) $\frac{1}{1+\cos^2 x}$

12. If $f(x) = \ln(\ln x)$, thne $f'(x) =$

(A) $\frac{1}{x}$

(B) $\frac{1}{\ln x}$

(C) $\frac{\ln x}{x}$

(D) x

(E) $\frac{1}{x \ln x}$

13. If $y = (\sin x)^x$ then $\dfrac{dy}{dx} =$

A) $x \ln(\sin x)$

B) $(\sin x)^x \cot x$

C) $x (\sin x)^{x-1} \cos x$

D) $(\sin x)^x (x \cos x + \sin x)$

E) $(\sin x)^x (x \cot x + \ln(\sin x))$

14. $\dfrac{d}{dx} 3^{\csc x} =$

$3^{\csc x} \ln 3 \cdot -\csc x \cot x$

A) $\csc x (3)^{\csc x - 1}$
B) $-3^{\csc x} \csc x \cot x$
C) $-3^{\csc x} \ln 3 \csc x \cot x$
D) $-3^{\csc x} \cot^2 x$
E) $-3^{\csc x} \ln 3 \cot^2 x$

15. For $y = 2\sqrt{x} - \dfrac{1}{2\sqrt{x}}$, $\dfrac{dy}{dx}$ equals

A) $x + \dfrac{1}{x\sqrt{x}}$ B) $x^{-1/2} + x^{-3/2}$ C) $\dfrac{4x-1}{4x\sqrt{x}}$

D) $\dfrac{1}{\sqrt{x}} + \dfrac{1}{4x\sqrt{x}}$ E) $\dfrac{4}{\sqrt{x}} + \dfrac{1}{x\sqrt{x}}$

$y = 2x^{1/2} - \frac{1}{2} x^{-1/2}$
$y' = x^{-1/2} + \frac{1}{4} x^{-3/2}$
$= \frac{1}{\sqrt{x}} + \frac{1}{4x\sqrt{x}}$

16. For $y = \dfrac{1+x^2}{1-x^2}$, $\dfrac{dy}{dx}$ equals

A) $-\dfrac{4x}{(1-x^2)^2}$ B) $\dfrac{4x}{(1-x^2)^2}$ C) $\dfrac{-4x^3}{(1-x^2)^2}$

D) $\dfrac{2x}{1-x^2}$ E) $\dfrac{4}{1-x^2}$

$y' = \dfrac{2x(1-x^2) + (1+x^2)(2x)}{(1-x^2)^2}$
$= (2x - \cancel{2x^3} + 2x + \cancel{2x^3}) / (1-x^2)^2$
$= \dfrac{4x}{(1-x^2)^2}$

17 For $y = \arctan\dfrac{x}{2}$, $\dfrac{dy}{dx}$ equals

A) $\dfrac{4}{4+x^2}$ B) $\dfrac{1}{2\sqrt{4-x^2}}$ C) $\dfrac{2}{\sqrt{4-x^2}}$ D) $\dfrac{1}{2+x^2}$

E) $\dfrac{2}{x^2+4}$

$y' = \dfrac{1}{1+\frac{x^2}{4}} \cdot \frac{1}{2} = \dfrac{1/2}{\frac{4}{4}+\frac{x^2}{4}} = \dfrac{4}{4+x^2} \cdot \frac{1}{2} = \dfrac{2}{4+x^2}$

18 For $y = \sec^2\sqrt{x}$, $\dfrac{dy}{dx}$ equals

A) $\dfrac{\sec\sqrt{x}\tan\sqrt{x}}{\sqrt{x}}$ B) $\dfrac{\tan\sqrt{x}}{\sqrt{x}}$ C) $2\sec\sqrt{x}\tan^2\sqrt{x}$

D) $\dfrac{\sec^2\sqrt{x}\tan\sqrt{x}}{\sqrt{x}}$ E) $2\sec^2\sqrt{x}\tan\sqrt{x}$

$y' = 2\sec\sqrt{x} \cdot \sec\sqrt{x}\tan\sqrt{x} \cdot \frac{1}{2}x^{-1/2}$
$= \dfrac{\sec^2\sqrt{x}\tan\sqrt{x}}{\sqrt{x}}$

19 For $y = \sin(2^x)$, $\dfrac{dy}{dx}$ equals

A) $2^x\cos(2^x)$ B) $2^x\ln 2\cos(2^x)$ C) $x2^{x-1}\cos(2^x)$

D) $\dfrac{2^x\cos(2^x)}{\ln 2}$ E) $\cos(2^x)+\sin(2^x\ln 2)$

$y' = \cos 2^x \cdot 2^x \ln 2$
$= 2^x \ln 2 \cos 2^x$

20 For $y = x^{\ln x}$, $\dfrac{dy}{dx}$ equals

A) $\dfrac{2}{x}$ B) $2\dfrac{\ln x}{x}$ C) $\dfrac{2x^{\ln x}\ln x}{x}$ D) $\dfrac{2x^{\ln x}}{x}$

E) $x^{\ln x-1}\ln x$

$\ln y = \ln x \cdot \ln x$

$\dfrac{y'}{y} = \dfrac{\ln x}{x} + \dfrac{\ln x}{x}$

$y' = x^{\ln x}\left(\dfrac{2\ln x}{x}\right)$

$= \dfrac{2x^{\ln x}\ln x}{x}$

1. Find the equation of the tangent line to the curve $e^{xy} + x^2 - y^2 = 0$ at the point (0, 1).

[4]

$$e^{xy}[y + xy'] + 2x - 2yy' = 0$$

$$y'[xe^{xy} - 2y] = -(2x + ye^{xy})$$

$$y' = \frac{2x + ye^{xy}}{2y - xe^{xy}}$$

@ (0,1) $y' = \frac{1}{2}$

$$y - y_1 = m(x - x_1)$$

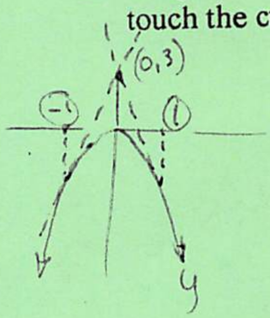
$$y - 1 = \frac{1}{2}(x - 0)$$

$$\boxed{y = \frac{1}{2}x + 1} \quad \text{or} \quad 2y = x + 2$$

$$-2 = x - 2y$$

2. Tangents are drawn from (0, 3) to the curve $y = -3x^2$. Find the point(s) at which these tangents touch the curve.

[4]



$$m = -6x$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-6x = \frac{(-3x^2) - 3}{x - 0}$$

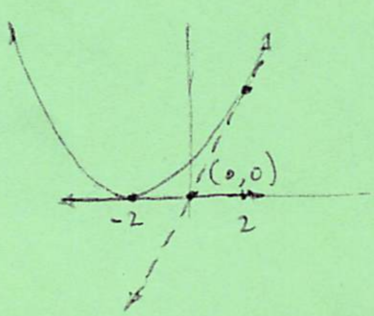
$$-6x^2 = -3x^2 - 3$$

$$3 = 3x^2$$

$$\pm 1 = x$$

3. Find all values of x for which the tangent line to $y = (x + 2)^2$ passes through the origin.

[4]



$$y' = 2(x+2)$$

$$m = 2x + 4$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$2x + 4 = \frac{(x+2)^2 - 0}{x - 0}$$

$$2x + 4 = \frac{x^2 + 4x + 4}{x}$$

$$2x^2 + 4x = x^2 + 4x + 4$$

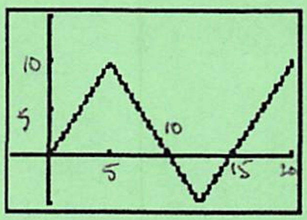
$$x^2 = 4$$

$$x = \pm 2$$

$\therefore$ The tangent to $f(x)$ @ $x = \pm 2$ passes through $(0,0)$

Part B (18 marks): Calculator use is permitted in this section

11. The graph shows f', the derivative of the function f. The domain of the function f is the interval $[0, 20]$.

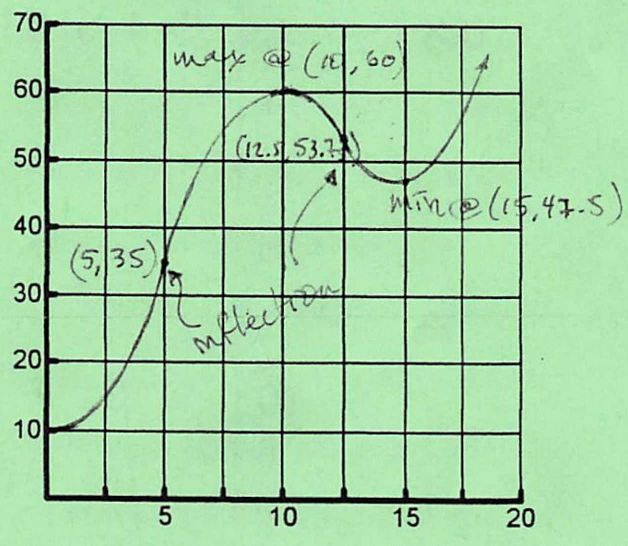



```
WINDOW
 Xmin=-3
 Xmax=20
 Xscl=5
 Ymin=-5
 Ymax=15
 Yscl=5
 Xres=1
```

a) For what value(s) of x, $0 < x < 20$, does f have a relative maximum? Justify your answer.
b) For what value(s) of x is the graph of f concave down? Justify your answer.
c) If $f(0) = 10$, sketch a graph of the function f on the axes provided. List the coordinates of all critical points and inflection points. **[6]**

a) max when f' changes from positive to negative $\Rightarrow x = 10$

b) concave down when $f'' < 0$
$(5, 12.5)$
concave up when $f'' > 0$
$(0, 5) \cup (12.5, 20)$



c) I. Since the slope of f' on $[0, 5)$ is $2x$ we know that $f(x) = x^2$ with a vert. translation of 10
so $f(x) = x^2 + 10$
$\therefore f(5) = (5)^2 + 10 = 35$

II. On $(5, 12.5)$ f' has slope of $-2x$ so $f(x)$ looks like $y = -x^2$
$\therefore$ f has a max 25 units up from $(5, 35)$ at $x = 10$
$\therefore$ local max @ $(10, 60)$

III. f has another pt. of inflection @ $x = 12.5$ (2.5 units to the right of $(10, 60)$ so $(2.5)^2 = 6.25$ units down
$\therefore$ inflection @ $(12.5, 53.75)$

IV. Since f' is locally symmetric about $x = 12.5$, $f(x)$ is also symmetric but opening up
$\therefore$ the local min @ $x = 15$ is another 6.25 units down from pt. of inflection $\therefore$ local min @ $(15, 47.5)$

Given the following functions of f'(x), sketch possible graphs of f(x)