TUTORIAL PROBLEMS: INTEGRATION BY PARTS

Calc 101

1. $$\int x e^{3x}\,dx \underset{IBP}{=} \frac{1}{3} x e^{3x} - \frac{1}{3}\int e^{3x}\,dx$$

$u = x \to u' = 1$
$v' = e^{3x} \to v = \frac{1}{3}e^{3x}$

$$= \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + C$$

2. $$\int x^2 e^{-x}\,dx \underset{IBP}{=} -x^2 e^{-x} + 2\int x e^{-x}\,dx$$

$u = x^2 \to u' = 2x$
$v' = e^{-x} \to v = -e^{-x}$

$$\underset{IBP}{=} -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,dx\right)$$

$u = x \to u' = 1$
$v' = e^{-x} \to v = -e^{-x}$

$$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C$$

$$= -e^{-x}\left(x^2 + 2x + 2\right) + C$$

3. $$\int x \cos 2x \,.\, dx \underset{IBP}{=} \frac{1}{2} x \sin 2x - \frac{1}{2}\int \sin 2x\,dx$$

$u = x \to u' = 1$
$v' = \cos 2x \to v = \frac{1}{2}\sin 2x$

$$= \frac{1}{2} x \sin 2x + \frac{1}{4}\cos 2x + C$$

4. $$\int \theta^2 \sin\theta \,.\, d\theta \underset{IBP}{=} -\theta^2\cos\theta + 2\int \theta \,.\, \cos\theta \,.\, d\theta$$

$u = \theta^2 \to u' = 2\theta$
$v' = \sin\theta \to v = -\cos\theta$

$$\underset{IBP}{=} -\theta^2\cos\theta + 2\left(\theta\sin\theta - \int \sin\theta\,d\theta\right)$$

$u = \theta \to u' = 1$
$v' = \cos\theta \to v = \sin\theta$

$$= -\theta^2\cos\theta + 2\theta\sin\theta + 2\cos\theta + C$$

5. $$\int \sqrt{x}\ln x\,dx \underset{IBP}{=} \frac{2}{3} x\sqrt{x}\ln x - \frac{2}{3}\int \sqrt{x}\,dx$$

$u = \ln x \to u' = \frac{1}{x}$
$v' = \sqrt{x} \to v = \frac{2}{3}x^{3/2}$

$$= \frac{2}{3} x\sqrt{x}\ln x - \frac{4}{9} x\sqrt{x} + C$$

6. $\int \frac{\ln x}{\sqrt{x}}\,dx \underset{IBP}{=} 2\sqrt{x}\ln x - 2\int \frac{1}{\sqrt{x}}\,dx$

$u=\ln x \to u' = \frac{1}{x}$

$v' = \frac{1}{\sqrt{x}} \to v = 2\sqrt{x}$

$= 2\sqrt{x}\ln x - 4\sqrt{x} + C$

7. $\int e^{2x}.\cos 3x\,dx \underset{IBP}{=} \frac{1}{2}\cos 3x\, e^{2x} + \frac{3}{2}\int \sin 3x.\,e^{2x}\,dx$

$u = \cos 3x \to u' = -3\sin 3x$

$v' = e^{2x} \to v = \frac{1}{2}e^{2x}$

$\underset{IBP}{=} \frac{1}{2}\cos 3x\, e^{2x} + \frac{3}{2}\left(\frac{1}{2}\sin 3x\, e^{2x} - \frac{3}{2}\int \cos 3x\, e^{2x}\right)$

$u = \sin 3x \to u' = 3\cos 3x$

$v' = e^{2x} \to v = \frac{1}{2}e^{2x}$

$\therefore \quad I = \frac{1}{2}\cos 3x\, e^{2x} + \frac{3}{4}\sin 3x\, e^{2x} - \frac{9}{4} I$

$\frac{13}{4} I = \frac{1}{2}e^{2x}\left(\cos 3x + \frac{3}{2}\sin 3x\right)$

$I = \frac{2}{13}e^{2x}\left(\cos 3x + \frac{3}{2}\sin 3x\right)$

8. $\int \cos(\ln x)\,dx = \int \cos a .\, e^{a}\,da \underset{IBP}{=} \cos a\, e^{a} + \int e^{a}\sin a\,da$

$a = \ln x$

$da = \frac{1}{x}dx$

$u = \cos a \to u' = -\sin a$

$v' = e^{a} \to v = e^{a}$

$u = \sin a$

$v' = e^{a}$

$\underset{IBP}{=} \cos a\, e^{a} + \sin a\, e^{a} - \int e^{a}\cos a\,da$

$2I = e^{a}(\cos a + \sin a) + C$

$I = \frac{1}{2}e^{a}(\cos a + \sin a) + C$

$I = \frac{1}{2}x\left(\cos(\ln x) + \sin(\ln x)\right) + C$

9. $\int x\underbrace{\tan^2 x}_{\sec^2 x - 1}\,dx = \int x(\sec^2 x - 1)\,dx$

$= \int x\sec^2 x\,dx - \int x\,dx$

$u = x \to u' = 1$

$v' = \sec^2 x \to v = \tan x$

$\underset{IBP}{=} x\tan x - \int \tan x\,dx - \frac{1}{2}x^2$

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9 (cont.) $= x\tan x + \ln|\cos x| - \frac{1}{2}x^2 + C$

11_ $\int_0^1 x e^{-5x}\,dx \underset{IBP}{=} \left[-\frac{1}{5}x e^{-5x}\right]_0^1 + \frac{1}{5}\int_0^1 e^{-5x}\,dx$

$u = x \rightarrow u' = 1$
$v' = e^{-5x} \rightarrow -\frac{1}{5}e^{-5x}$

$= -\frac{1}{5}e^{-5} + \frac{1}{5}\left[-\frac{1}{5}e^{-5x}\right]_0^1$

$= -\frac{1}{5}e^{-5} - \frac{1}{25}e^{-5} + \frac{1}{25}$

$= -\frac{6}{25}e^{-5} + \frac{1}{25}$

12_ $\int_{\sqrt{e}}^{e} \frac{\ln x}{x^2}\,dx \underset{IBP}{=} \left[-\frac{1}{x}\ln x\right]_{\sqrt{e}}^{e} + \int_{\sqrt{e}}^{e} \frac{1}{x^2}\,dx$

$u = \ln x \rightarrow u' = \frac{1}{x}$
$v' = \frac{1}{x^2} \rightarrow v = -\frac{1}{x}$

$= -\frac{1}{e} + \frac{1}{2\sqrt{e}} + \left[-x^{-1}\right]_{\sqrt{e}}^{e}$

$= -\frac{1}{e} + \frac{1}{2\sqrt{e}} - \frac{1}{e} + \frac{1}{\sqrt{e}}$

$= -\frac{2}{e} + \frac{3}{2\sqrt{e}}$

13_ $\int \cos^5 x\,dx \underset{Red}{=} \frac{1}{5}\cos^4 x.\sin x + \frac{4}{5}\int \cos^3 x\,dx$

$\underset{Red}{=} \frac{1}{5}\cos^4 x.\sin x + \frac{4}{5}\left(\frac{1}{3}\cos^2 x.\sin x + \frac{2}{3}\int \cos x\,dx\right)$

$= \frac{1}{5}\cos^4 x.\sin x + \frac{4}{15}\cos^2 x.\sin x + \frac{8}{15}\sin x + C$

14_ $\int_0^{\pi/2} \cos^6 x\,dx = \left[\frac{1}{6}\cos^5 x.\sin x\right]_0^{\pi/2} + \frac{5}{6}\int_0^{\pi/2} \cos^4 x\,dx$

$= \frac{5}{6}\left(\left[\frac{1}{4}\cos^3 x.\sin x\right]_0^{\pi/2} + \frac{3}{4}\int_0^{\pi/2} \cos^2 x\,dx\right)$

$= \frac{5}{8}\left(\frac{1}{2}\cos x.\sin x\right]_0^{\pi/2} + \frac{1}{2}\int_0^{\pi/2} dx\Big)$

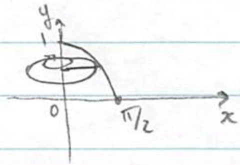
$= \frac{5}{16} \times \frac{\pi}{2} = \frac{5\pi}{32}$.

12

15. $\int x^n e^x\,dx = x^n e^x - n\int x^{n-1} e^x\,dx.$

$u = x^n \to u' = nx^{n-1}$
$v' = e^x \to v = e^x$

16.



$$V = 2\pi\int_0^{\pi/2} x.\cos x.\,dx$$

$u = x \to u' = 1$
$v' = \cos x \to v = \sin x$

$$\underset{IBP}{=} 2\pi\left(\left[x\sin x\right]_0^{\pi/2} - \int_0^{\pi/2}\sin x\,dx\right)$$

$$= \pi^2 + 2\pi\left[\cos x\right]_0^{\pi/2}$$

$$= \pi^2 - 2\pi$$

17. ⟶ $d = \int_0^{\pi} v(t)\,dt$

$$= \int_0^{\pi} t^3 \sin t\,dt$$

$u = t^3 \to u' = 3t^2$
$v' = \sin t \to v = -\cos t$

$$\underset{IBP}{=} -t^3\cos t\Big]_0^{\pi} + 3\int_0^{\pi} t^2\cos t\,dt$$

$u = t^2 \to u' = 2t$
$v' = \cos t \to v = \sin t$

$$\underset{IBP}{=} \pi^3 + 3\left(\left[t^2\sin t\right]_0^{\pi} - 2\int_0^{\pi} t\sin t\,dt\right)$$

$u = t \to u' = 1$
$v' = \sin t \to v = -\cos t$

$$\underset{IBP}{=} \pi^3 - 6\left(\left[-t\cos t\right]_0^{\pi} + \int_0^{\pi}\cos t\,dt\right)$$

$$= \pi^3 - 6\pi + \left[\sin t\right]_0^{\pi}$$

$$= \pi^3 - 6\pi$$

18. $\int x^2 e^{3x}\,dx = e^{3x}\left(\frac{1}{3}x^2 - \frac{2}{9}x + \frac{2}{27}\right) + C$

19. $\int_0^1 x e^{-\sqrt{x}}\,dx = \int_0^1 u^2 e^{-u} \times 2u\,du = \ldots = -\frac{32}{e} + 12$

$u = \sqrt{x}$

$x = u^2$

$dx = 2u\,du$

10 - $$\int \frac{xe^x}{(x+1)^2}\,dx \underset{IBP}{=} -\frac{xe^x}{x+1} + \int e^x\,dx = -\frac{xe^x}{x+1} + e^x + C$$

$$u = xe^x \rightarrow u' = e^x(x+1)$$

$$v' = \frac{1}{(x+1)^2} \rightarrow v = -\frac{1}{x+1}$$

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