Extra Practice Squeezing Theorem

○ *PROBLEM 1* : Compute $\lim_{x\to\infty} \frac{\sin x}{x}$.

$\forall x \in \mathbb{R}: \quad -1 \le \sin x \le 1$

$\forall x > 0: \quad -\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$

$\lim_{x\to\infty}\left(-\frac{1}{x}\right) = \lim_{x\to\infty}\left(\frac{1}{x}\right) = 0$

$\therefore$ (ST) $\lim_{x\to+\infty} \frac{\sin x}{x} = 0$

○ *PROBLEM 2* : Compute $\lim_{x\to\infty} \frac{2-\cos x}{x+3}$.

$\forall x \in \mathbb{R}: \quad -1 \le \cos x \le 1$

$1 \le 2 - \cos x \le 3$

$\forall x > -3: \quad \frac{1}{x+3} \le \frac{2-\cos x}{x+3} \le \frac{3}{x+3}$

$\lim_{x\to\infty} \frac{1}{x+3} = \lim_{x\to\infty} \frac{3}{x+3} = 0$

$\therefore$ (ST) $\lim_{x\to\infty} \frac{2-\cos x}{x+3} = 0$

○ *PROBLEM 3* : Compute $\lim_{x\to+\infty} \frac{\cos^2(2x)}{3-2x}$.

$\forall x \in \mathbb{R}: \quad 0 \le \cos^2(2x) \le 1$

$\forall x > \frac{3}{2} \quad \frac{1}{3-2x} \le \frac{\cos^2(2x)}{3-2x} \le 0$

$\lim_{x\to+\infty} \frac{1}{3-2x} = 0$

$\therefore$ (ST) $\lim \frac{\cos^2(2x)}{3-2x} = 0$

○ *PROBLEM 4* : Compute $\lim_{x\to 0^-} x^3 \cos\left(\frac{2}{x}\right)$.

$\forall x < 0: \quad -1 \le \cos\left(\frac{2}{x}\right) \le 1$

$x^3 \le x^3 \cos\left(\frac{2}{x}\right) \le -x^3$

$\lim_{x\to 0^-} x^3 = \lim_{x\to 0^-} (-x^3) = 0$

$\therefore$ (ST) $\lim_{x\to 0^-} x^3 \cos\left(\frac{2}{x}\right) = 0$

○ *PROBLEM 5* : Compute $\lim_{x\to\infty} \frac{x^2(2+\sin^2 x)}{x+100}$.

$\forall x > 0: \quad 0 \le \sin^2 x \le 1$

$2 \le 2 + \sin^2 x \le 3$

$\frac{2x^2}{x+100} \le \frac{x^2(2+\sin^2 x)}{x+100} \le \frac{3x^2}{x+100}$

$\lim_{x\to+\infty} \frac{2x^2}{x+100} = \lim_{x\to+\infty} \frac{3x^2}{x+100} = +\infty$

$\therefore \lim_{x\to+\infty} \frac{x^2(2+\sin^2 x)}{x+100} = +\infty$

○ *PROBLEM 6* : Compute $\lim_{x\to-\infty} \frac{5x^2 - \sin(3x)}{x^2+10}$.

$\frac{5x^2 - \sin(3x)}{x^2+10} = \frac{5x^2}{x^2+10} - \frac{\sin(3x)}{x^2+10}$

$\forall x \in \mathbb{R}: \quad -1 \le -\sin(3x) \le 1$

$\frac{-1}{x^2+10} \le -\frac{\sin(3x)}{x^2+10} \le \frac{1}{x^2+10}$

$\lim_{x\to-\infty} -\frac{1}{x^2+10} = \lim_{x\to-\infty} \frac{1}{x^2+10} = 0$

$\lim_{x\to-\infty} \frac{5x^2}{x^2+10} = 5$

$\therefore \lim_{x\to-\infty} \frac{5x^2 - \sin(3x)}{x^2+10} = 5$

○ *PROBLEM 7* : Compute $\lim_{x\to-\infty} \frac{x^2(\sin x + \cos^3 x)}{(x^2+1)(x-3)}$.

$\forall x \in \mathbb{R}: \quad -2 \le \sin x + \cos^3 x \le 2$

$\frac{-2x^2}{(x^2+1)(x-3)} \le \frac{x^2(\sin x + \cos^3 x)}{(x^2+1)(x-3)} \le \frac{2x^2}{(x^2+1)(x-3)}$

$\lim_{x\to-\infty} \frac{-2x^2}{(x^2+1)(x-3)} = \lim_{x\to-\infty} \frac{2x^2}{(x^2+1)(x-3)} = 0$

$\therefore$ (ST) $\lim_{x\to-\infty} \frac{x^2(\sin x + \cos^3 x)}{(x^2+1)(x-3)} = 0$

○ *PROBLEM 8* : Assume that $\lim_{\theta\to-1^-} f(\theta)$ exists and $\frac{\theta^2+\theta-2}{\theta+3} \le \frac{f(\theta)}{\theta^2} \le \frac{\theta^2+2\theta-1}{\theta+3}$. Find $\lim_{\theta\to-1^-} f(\theta)$.

$-1 \qquad 1 \qquad -1$

$\lim_{x\to -1^-} f(\theta) = -1$