

Fractions Review

I – Changing a denominator:

If you multiply or divide the numerator and denominator of a fraction by the same number (not 0), you get an equivalent fraction.

Examples: $\frac{15}{21} = \frac{5}{7}$ ← I divided "top" and "bottom" by 3

$\frac{2}{3} = \frac{10}{15}$ ← I multiplied "top" and "bottom" by 5

II – Mixed Numbers – Improper Fractions – Decimal Notation:

A fraction is called *improper* when its numerator is greater than its denominator.

That's when the value of the fraction is greater than 1.

When a fraction is improper, we can choose to write it as a mixed number.

Example:

numerator → $\frac{9}{7} = 1\frac{2}{7}$ ← mixed number (more than 1)
 denominator →

Transforming a mixed number into an improper fraction:

$$3\frac{5}{6} = \frac{3 \times 6 + 5}{6} = \frac{23}{6}$$

$$2\frac{1}{3} = \frac{2 \times 3 + 1}{3} = \frac{7}{3}$$

Transforming an improper fraction into a mixed number:

$$\frac{17}{3} = 5\frac{2}{3} \quad 17 \div 3 \approx 5.6\bar{6} \quad \frac{17}{3} - 5 = \frac{17}{3} - \frac{15}{3} = \frac{2}{3}$$

$$\frac{9}{4} = 2\frac{1}{4} \quad \frac{9}{4} \approx 2.25 \quad \frac{9}{4} - 2 = \frac{9}{4} - \frac{8}{4} = \frac{1}{4}$$

When you transform a fraction into a decimal form, very often you don't have an exact value anymore (only an approximation)

Examples: $\frac{7}{4} = 1.75$ (still exact)

$\frac{2}{3} \approx 0.67$ (not exact)

Mathematicians usually prefer exact values and more specifically, the improper form...

Note, some calculators can transform decimal numbers (that are exact) into fractions

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III – Simplifying Fractions

A fraction is simplified when the numerator and denominator don't have a common factor, which means that there is no equivalent fraction that could have smaller denominator and numerator.

To show our work when simplifying a fraction, we can decompose the numerator and denominator into multiplication that display what can be divided.

Examples:

$$\frac{21}{14} = \frac{\cancel{7} \times 3}{\cancel{7} \times 2} = \frac{3}{2}$$

$$\frac{35}{40} = \frac{\cancel{5} \times 7}{\cancel{5} \times 8} = \frac{7}{8}$$

$$\frac{66}{99} = \frac{\cancel{33} \times 2}{\cancel{33} \times 3} = \frac{2}{3}$$

I would like you to make an effort to show your work this way, in order to build good habits for the future...

NOTE: An integer can always be seen as a fraction (with denominator 1)

$$\text{examples: } -3 = -\frac{3}{1} \quad \text{or } 5 = \frac{5}{1}$$

IV – Multiplying Fractions

To multiply with fractions, you don't need the same numerator. You just multiply the numerators together, and the denominators together, and you try to simplify as you go instead of at the end...

Examples:

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

$$\frac{2}{3} \times 5 = \frac{2}{3} \times \frac{5}{1} = \frac{10}{3}$$

$$\frac{3}{4} \times \frac{8}{9} = \frac{\cancel{3} \times \cancel{4} \times 2}{\cancel{4} \times \cancel{3} \times 3} = \frac{2}{3}$$

$$\frac{16}{5} \times \frac{5}{8} = \frac{\cancel{8} \times 2 \times \cancel{5}}{\cancel{8} \times 5} = 2$$

$$\frac{7}{6} \times 6 = \frac{7}{\cancel{6}} \times \frac{\cancel{6}}{1} = 7$$

Note, to multiply you need to transform the mixed numbers into improper fractions:

$$3\frac{1}{4} \times \frac{2}{3} = \frac{13}{4} \times \frac{2}{3} = \frac{13 \times \cancel{2}}{\cancel{2} \times 2 \times 3} = \frac{13}{6}$$

V – Adding and subtracting Fractions

To add or subtract fractions, you need to write them on the same denominator.
It is a good idea to try to use the Least Common Denominator...

Examples:

$$\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$$

$$\frac{11}{14} - \frac{5}{21} = \frac{33}{42} - \frac{10}{42} = \frac{23}{42}$$

$$7 - 2\frac{3}{4} = \frac{28}{4} - \frac{11}{4} = \frac{17}{4}$$

$$\frac{16}{25} + \frac{14}{15} = \frac{48}{75} + \frac{70}{75} = \frac{118}{75}$$

VI – Division with Fractions

Dividing can be replaced by multiplying by the reciprocal...

Vocabulary: The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$
The reciprocal of 3 is $\frac{1}{3}$
The reciprocal of $-\frac{4}{3}$ is $-\frac{3}{4}$

Examples:

$$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$$

$$\frac{\frac{3}{4}}{\frac{2}{5}} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$$

$$\frac{3}{\frac{4}{5}} = 3 \times \frac{5}{4} = \frac{15}{4}$$

$$\frac{\frac{5}{2}}{3} = \frac{5}{2} \times \frac{1}{3} = \frac{5}{6}$$