1. Prove the identity.

(3 marks)

$$\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$$

Left side	Right side
$\tan^2\theta - \sin^2\theta = \frac{\sin^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{1}$	$\tan^2\theta \cdot \sin^2\theta = \frac{\sin^2\theta}{\cos^2\theta} \cdot \sin^2\theta$
$= \frac{\sin^2\theta - \sin^2\theta \cdot \cos^2\theta}{\cos^2\theta}$	$= \frac{\sin^4\theta}{\cos^2\theta} = L.S$
$= \frac{\sin^2\theta (1 - \cos^2\theta)}{\cos^2\theta}$	
$= \frac{\sin^2\theta \cdot \sin^2\theta}{\cos^2\theta}$	
$= \frac{\sin^4\theta}{\cos^2\theta}$	

2. Prove the identity.

(2 marks)

$$\frac{\sec\theta - \cos\theta}{\tan\theta} = \sin\theta$$

Left side	Right side
$\frac{\frac{1}{\cos\theta} - \cos\theta}{\frac{\sin\theta}{\cos\theta}}$	
$= \frac{\frac{1 - \cos^2\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$	
$= \frac{\sin^2\theta}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$	
$= \sin\theta$	
R.S ✓	

3. Prove the identity.

(2 marks)

$$\frac{\cos\theta + \sin\theta\tan\theta}{\sin\theta\sec\theta} = \csc\theta$$

Left side	Right side
$\dfrac{\cos\theta + \sin\theta \times \dfrac{\sin\theta}{\cos\theta}}{\sin\theta \times \dfrac{1}{\cos\theta}}$	
$= \dfrac{\dfrac{\cos^2\theta}{\cos\theta} + \dfrac{\sin^2\theta}{\cos\theta}}{\dfrac{\sin\theta}{\cos\theta}}$	
$= \dfrac{\dfrac{1}{\cos\theta}}{\dfrac{\sin\theta}{\cos\theta}}$	
$= \dfrac{1}{\sin\theta}$	
$= \csc\theta$	
R.S. ✓	

4. Prove the identity.

(2 marks)

$$\frac{1}{1+\sin\theta} = \sec^2\theta - \frac{\tan\theta}{\cos\theta}$$

Left Side	Right Side
	$\frac{1}{\cos^2\theta} - \frac{\frac{\sin\theta}{\cos\theta}}{\cos\theta}$
	$= \frac{1}{\cos^2\theta} - \frac{\sin\theta}{\cos^2\theta}$
	$= \frac{1-\sin\theta}{\cos^2\theta}$
	$= \frac{1-\sin\theta}{1-\sin^2\theta}$
	$= \frac{1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$
	$= \frac{1}{1+\sin\theta}$
	L.S ✓

5. Prove the following identity: **(2 marks)**

$$\sin\theta + \cos\theta \cot\theta = \csc\theta$$

Left side	Right side
$\sin\theta + \cos\theta \times \frac{\cos\theta}{\sin\theta}$	
$= \sin\theta + \frac{\cos^2\theta}{\sin\theta}$	
$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$	
$= \frac{1}{\sin\theta}$	
$= \csc\theta$ R.S. ✓	

6. Prove the following identity: **(3 marks)**

$$\frac{\cot\theta}{\csc\theta - 1} = \frac{\csc\theta + 1}{\cot\theta}$$

LEFT SIDE	RIGHT SIDE
$\dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta} - 1}$	$\dfrac{\frac{1}{\sin\theta} + 1}{\frac{\cos\theta}{\sin\theta}}$
$= \dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{1 - \sin\theta}{\sin\theta}}$	$= \dfrac{\frac{1 + \sin\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}}$
	$= \dfrac{1 + \sin\theta}{\cos\theta}$
$= \dfrac{\cos\theta}{1 - \sin\theta} \times \dfrac{1 + \sin\theta}{1 + \sin\theta}$	
$= \dfrac{\cos\theta(1 + \sin\theta)}{1 - \sin^2\theta}$	
$= \dfrac{\cos\theta(1 + \sin\theta)}{\cos^2\theta}$	
$= \dfrac{1 + \sin\theta}{\cos\theta}$ R.S ✓	

7. Prove the identity: **(3 marks)**

$$\frac{\sin\theta + \tan\theta}{1+\cos\theta} = \frac{\sin 2\theta}{2\cos^2\theta}$$

LEFT SIDE	RIGHT SIDE
$\dfrac{\sin\theta + \dfrac{\sin\theta}{\cos\theta}}{1+\cos\theta}$	$\dfrac{2\sin\theta\cos\theta}{2\cos^2\theta}$
$= \dfrac{\dfrac{\sin\theta\cdot\cos\theta + \sin\theta}{\cos\theta}}{1+\cos\theta}$	$= \tan\theta$
$= \dfrac{\dfrac{\sin\theta(\cos\theta+1)}{\cos\theta}}{1+\cos\theta}$	
$= \dfrac{\sin\theta(\cos\theta+1)}{\cos\theta} \times \dfrac{1}{1+\cos\theta}$	
$= \tan\theta$ R.S. ✓	

8. Prove the identity: **(2 marks)**

$$\frac{\csc\theta}{\tan\theta+\cot\theta}=\cos\theta$$

Left Side	Right Side
$\dfrac{\frac{1}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}$	
$=\dfrac{\frac{1}{\sin\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\cdot\sin\theta}}$	
$=\frac{1}{\sin\theta}\times\frac{\cos\theta\cdot\sin\theta}{1}$	
$=\cos\theta$ RS ✓	

9. Prove the identity. **(2 marks)**

$$\frac{1-\cos\theta}{\sin^2\theta} = \frac{1}{1+\cos\theta}$$

Left Side	Right Side
$\frac{1-\cos\theta}{1-\cos^2\theta}$	
$= \frac{(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$	
$= \frac{1}{1+\cos\theta}$ R.S ✓	

10. Prove the identity: **(3 marks)**

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

LEFT SIDE	RIGHT SIDE
$\frac{2\sin\theta.\cos\theta}{\cos\theta} + \frac{1-2\sin^2\theta}{\sin\theta}$	
$= \frac{2\sin^2\theta}{\sin\theta} + \frac{1-2\sin^2\theta}{\sin\theta}$	
$= \frac{1}{\sin\theta}$	
$= \csc\theta$ R.S ✓	

11. Prove:

$$\frac{\sin 2\theta}{2-2\cos^2\theta} = \cot\theta$$

(3 marks)

Left side	Right side
$\frac{2\sin\theta.\cos\theta}{2(1-\cos^2\theta)}$ $= \frac{\sin\theta.\cos\theta}{\sin^2\theta}$ $= \cot\theta$ R.S ✓	

12. Prove: **(4 marks)**

$$\frac{\sin\theta\cos\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\tan\theta}$$

LEFT SIDE	RIGHT SIDE
$\frac{\sin\theta.\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}$	$\frac{1-\cos\theta}{\frac{\sin\theta}{\cos\theta}}$
$= \frac{\sin\theta.\cos\theta(1-\cos\theta)}{1-\cos^2\theta}$	$= \frac{\cos\theta(1-\cos\theta)}{\sin\theta}$
$= \frac{\sin\theta.\cos\theta(1-\cos\theta)}{\sin^2\theta}$	
$= \frac{\cos\theta(1-\cos\theta)}{\sin\theta}$	
R S ✓	

13. Prove the identity:

(3 marks)

$$\csc^2 x + \sec^2 x \quad = \quad \csc^2 x \sec^2 x$$

LEFT SIDE	RIGHT SIDE
$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$	$\frac{1}{\sin^2 x} \times \frac{1}{\cos^2 x}$
$= \frac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}$	
$= \frac{1}{\sin^2 x \cos^2 x}$	

14. Prove the identity: **(3 marks)**

$$\frac{1}{\sec\theta + \tan\theta} = \frac{1-\sin\theta}{\cos\theta}$$

LEFT SIDE	RIGHT SIDE

$$\frac{1}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{1}{\frac{1+\sin\theta}{\cos\theta}}$$

$$= \frac{\cos\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$ RS ✓

15. Prove the identity: **(3 marks)**

$$\frac{\cos 2\theta}{\sin \theta} = \frac{\cot^2 \theta - 1}{\csc \theta}$$

LEFT SIDE	RIGHT SIDE
	$\dfrac{\dfrac{\cos^2\theta}{\sin^2\theta} - 1}{\dfrac{1}{\sin\theta}}$
	$= \dfrac{\dfrac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}}{\dfrac{1}{\sin\theta}}$
	$= \dfrac{\cos^2\theta - \sin^2\theta}{\sin^2\theta} \times \dfrac{\sin\theta}{1}$
	$= \dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta}$
	$= \dfrac{\cos 2\theta}{\sin\theta}$ L.S ✓

16. Prove the identity: **(4 marks)**

$$\frac{\cot\theta - 1}{1 - \tan\theta} = \frac{\csc\theta}{\sec\theta}$$

LEFT SIDE	RIGHT SIDE
$\dfrac{\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\sin\theta}}{1 - \frac{\sin\theta}{\cos\theta}}$	$\dfrac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}}$
$= \dfrac{\frac{\cos\theta - \sin\theta}{\sin\theta}}{\frac{\cos\theta - \sin\theta}{\cos\theta}}$	$= \dfrac{\cos\theta}{\sin\theta}$
$= \dfrac{\cos\theta}{\sin\theta}$	L.S ✓

17. Prove the identity: **(4 marks)**

$$(1-\sin\theta)(\sec\theta+\tan\theta) = \frac{1}{\sec\theta}$$

LEFT SIDE	RIGHT SIDE
$(1-\sin\theta)\left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)$	$=\cos\theta$ L.S ✓
$=\frac{(1-\sin\theta)(1+\sin\theta)}{\cos\theta}$	
$=\frac{1-\sin^2\theta}{\cos\theta}$	
$=\frac{\cos^2\theta}{\cos\theta}$	
$=\cos\theta$	

18. Prove the identity:

(5 marks)

$$\sin 2x(\tan x + \cot x) = 2$$

LEFT SIDE	RIGHT SIDE
$2\sin x \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$	
$= 2\sin x \cos x \left(\frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x}\right)$	
$= 2$	
RS ✓	

19. Prove the identity: **(5 marks)**

$$\frac{\cot\theta}{\sin\theta - \csc\theta} = -\sec\theta$$

LEFT SIDE	RIGHT SIDE
$\dfrac{\dfrac{\cos\theta}{\sin\theta}}{\sin\theta - \dfrac{1}{\sin\theta}}$	
$= \dfrac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\sin^2\theta - 1}{\sin\theta}}$	
$= \dfrac{\cos\theta}{\sin\theta} \times \dfrac{\sin\theta}{-\cos^2\theta}$	
$= -\dfrac{1}{\cos\theta}$	
$= -\sec\theta$ RS ✓	

20. Prove: **(5 marks)**

$$\frac{\sin 2x}{1+\cos 2x} = \frac{\sec^2 x - 1}{\tan x}$$

LEFT SIDE	RIGHT SIDE
$\frac{2\sin x \cdot \cos x}{2\cos^2 x}$	$\frac{\frac{1}{\cos^2 x} - 1}{\frac{\sin x}{\cos x}}$
$= \frac{\sin x}{\cos x}$ R.S. ✓	$= \frac{\frac{1-\cos^2 x}{\cos^2 x}}{\frac{\sin x}{\cos x}}$
	$= \frac{\sin^2 x}{\cos^2 x} \times \frac{\cos x}{\sin x}$
	$= \frac{\sin x}{\cos x}$

21. Prove: **(5 marks)**

$$\frac{2\cos x + 2\cos^2 x}{\sin 2x} = \frac{\sin x}{1-\cos x}$$

LEFT SIDE	RIGHT SIDE
$\frac{2\cos x(1+\cos x)}{2\sin x \cdot \cos x}$	$\frac{\sin x}{1-\cos x} \times \frac{1+\cos x}{1+\cos x}$
$= \frac{1+\cos x}{\sin x}$	$= \frac{\sin x(1+\cos x)}{1-\cos^2 x}$
	$= \frac{\sin x(1+\cos x)}{\sin^2 x}$
	$= \frac{1+\cos x}{\sin x}$ R.S. ✓