6 Linear Functions

BUILDING ON

- graphing linear relations
- recognizing the properties of linear relations
- solving linear equations

BIG IDEAS

- The graph of a linear function is a non-vertical straight line with a constant slope.
- Certain forms of the equation of a linear function identify the slope and *y*-intercept of the graph or the slope and the coordinates of a point on the graph.

NEW VOCABULARY

slope
rise
run
negative reciprocals
slope-intercept form
slope-point form
general form

POTASH MINING *Saskatchewan currently provides almost $\frac{1}{4}$ of the world's potash, which is an ingredient of fertilizer. Sales data are used to predict the future needs for potash.*

6.1 Slope of a Line

LESSON FOCUS

Determine the slope of a line segment and a line.

Make Connections

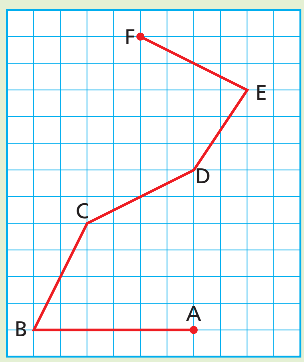
The town of Falher in Alberta is known as *la capitale du miel du Canada*, the Honey Capital of Canada. It has the 3-story slide in the photo above. How could you describe the steepness of the slide?

Construct Understanding

TRY THIS

Work with a partner.

This diagram shows different line segments on a square grid.



A. Think of a strategy to calculate a number to represent the steepness of each line segment.

B. Which is the steepest line segment? How does your number show that?

C. Which segment is the least steep? How does its number compare with the other numbers?

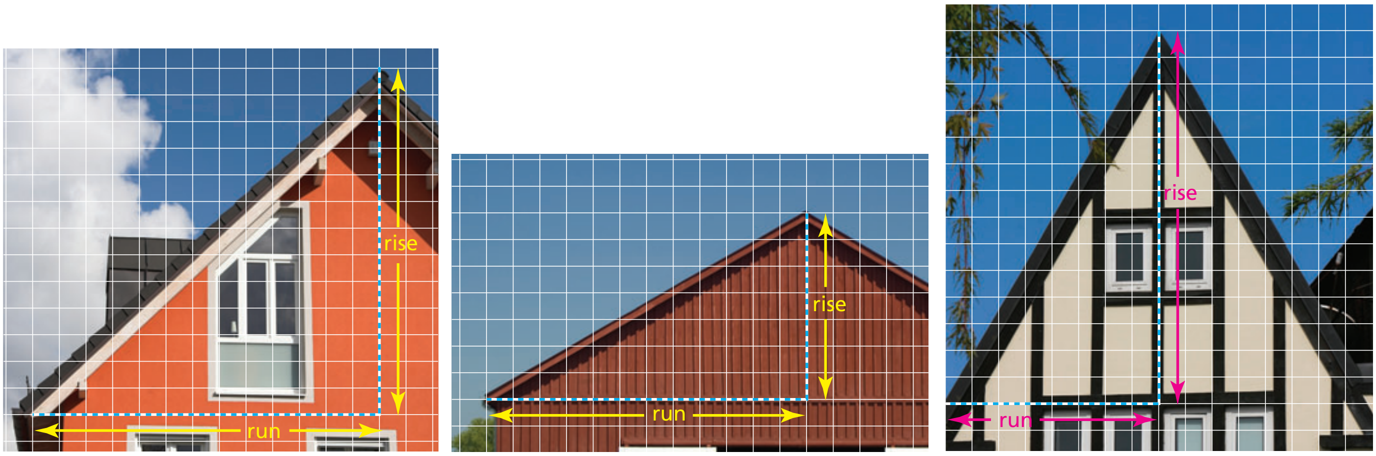
D. On a grid, draw a line segment that is steeper than segment CD, but not as steep as segment BC. Use your strategy to calculate a number to represent its steepness.

E. How are line segments CD and EF alike and different? How do the numbers for their steepnesses compare?

F. What number would you use to describe the steepness of a horizontal line?

Some roofs are steeper than others. Steeper roofs are more expensive to shingle.

Roof A **Roof B** **Roof C**



The steepness of a roof is measured by calculating its **slope.**

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

The **rise** is the vertical distance from the bottom of the edge of the roof to the top.
The **run** is the corresponding horizontal distance.
For each roof above, we count units to determine the rise and the run.

For Roof A

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{13}{13}$$

$$\text{Slope} = 1$$

For Roof B

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{7}{12}$$

$$\text{Slope} = 0.58\overline{3}$$

For Roof C

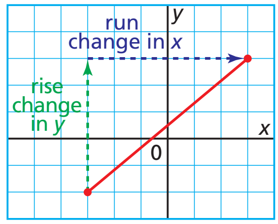
$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{14}{8}$$

$$\text{Slope} = 1.75$$

Roof C is the steepest because its slope is the greatest.
Roof B is the least steep because its slope is the least.

The slope of a line segment on a coordinate grid is the measure of its rate of change. From Chapter 5, recall that:



$$\text{Rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

$$\text{Rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

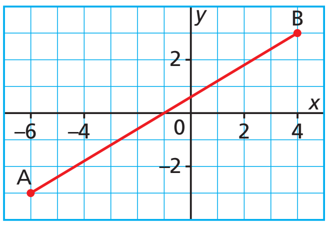
The change in y is the rise.
The change in x is the run.

So, slope $= \dfrac{\text{rise}}{\text{run}}$

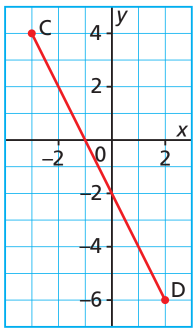
Example 1 Determining the Slope of a Line Segment

Determine the slope of each line segment.

a)



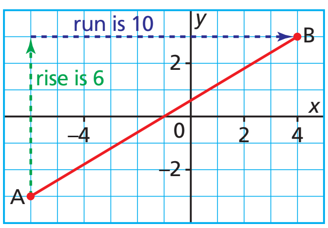
b)



SOLUTION

Count units to determine the rise and run.

a) From A to B, both x and y are increasing, so the rise is 6 and the run is 10.



$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{6}{10}$$ Write the fraction in simplest form.

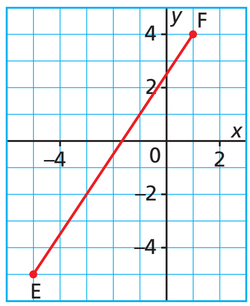
$$\text{Slope} = \frac{3}{5}$$

Line segment AB has slope $\frac{3}{5}$.

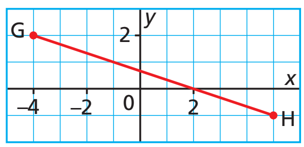
CHECK YOUR UNDERSTANDING

1. Determine the slope of each line segment.

a)

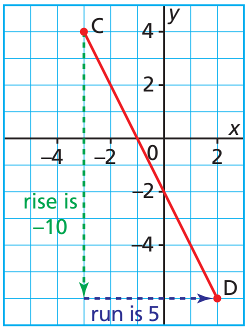


b)



[Answers: a) $\frac{3}{2}$ b) $-\frac{1}{3}$]

b) From C to D, y is decreasing, so the rise is -10;
x is increasing, so the run is 5.



$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{-10}{5}$$ Write the fraction in simplest form.

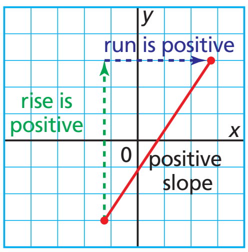
$$\text{Slope} = -2$$

Line segment CD has slope -2.

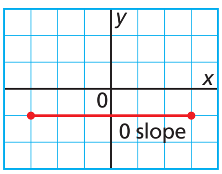
Why does calculating the slope of the line segment joining D to C produce the same result as calculating the slope of the segment from C to D?

Suppose the slope is an integer. How do you identify the rise and the run?

When a line segment goes up to the right, both y and x increase; both the rise and run are positive, so the slope of the segment is positive.



For a horizontal line segment, the change in y is 0 and x increases. The rise is 0 and the run is positive.



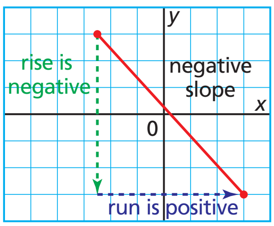
$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{0}{\text{run}}$$

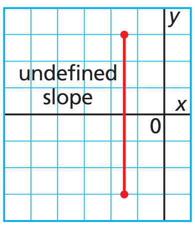
$$\text{Slope} = 0$$

So, any horizontal line segment has slope 0.

When a line segment goes down to the right, y decreases and x increases; the rise is negative and the run is positive, so the slope of the segment is negative.



For a vertical line segment, y increases and the change in x is 0. The rise is positive and the run is 0.



$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{\text{rise}}{0}$$

A fraction with denominator 0 is not defined.

So, any vertical line segment has a slope that is undefined.

For a vertical line segment, y could decrease and the rise would be negative.

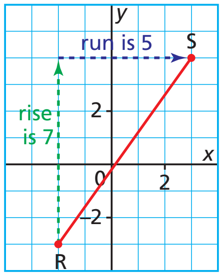
Example 2 Drawing a Line Segment with a Given Slope

Draw a line segment with each given slope.

a) $\frac{7}{5}$ b) $-\frac{3}{8}$

SOLUTION

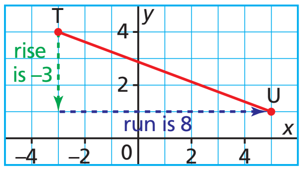
a) A line segment with slope $\frac{7}{5}$ has a rise of 7 and a run of 5. Choose any point R on a grid. From R, move 7 units up and 5 units right. Label the point S. Join RS.



Line segment RS has slope $\frac{7}{5}$.

b) The slope $-\frac{3}{8}$ can be written as $\frac{-3}{8}$.

A line segment with slope $\frac{-3}{8}$ has a rise of -3 and a run of 8. Choose any point T on a grid. From T, move 3 units down and 8 units right. Label the point U. Join TU.



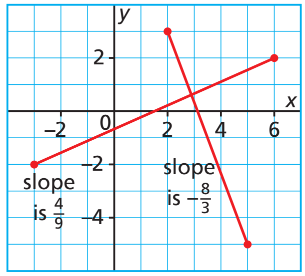
Line segment TU has slope $-\frac{3}{8}$.

CHECK YOUR UNDERSTANDING

2. Draw a line segment with each slope.

a) $\frac{4}{9}$ b) $-\frac{8}{3}$

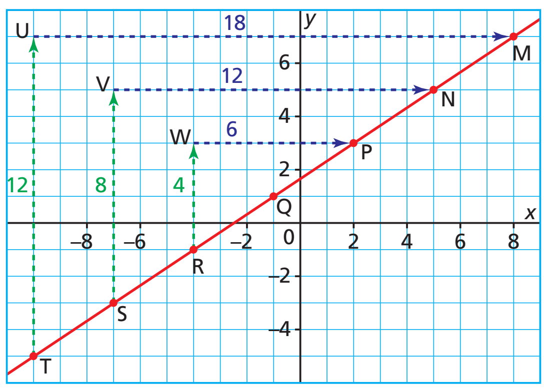
Sample Answers:



Why can we choose any point on the grid as one endpoint of the line segment?

Suppose the slope was written as $\frac{3}{-8}$. How would you draw the line segment?

We can show that the slopes of all segments of a line are equal. On line MT, vertical and horizontal segments are drawn for the rise and run. These segments form right triangles. Consider the lengths of the legs of these right triangles.



$\frac{TU}{UM} = \frac{12}{18}$ $\frac{SV}{VN} = \frac{8}{12}$ $\frac{RW}{WP} = \frac{4}{6}$

$\frac{TU}{UM} = \frac{2}{3}$ $\frac{SV}{VN} = \frac{2}{3}$ $\frac{RW}{WP} = \frac{2}{3}$

The lengths of the legs have the same ratio.
So, the triangles are similar.

Any right triangle drawn with its hypotenuse on line MT will have legs in the ratio $\frac{2}{3}$. So it does not matter which points we choose on the line; the slope of the line is the slope of any segment of the line. For example,

Slope of segment PQ $= \frac{2}{3}$

Slope of segment NR $= \frac{6}{9}$, or $\frac{2}{3}$

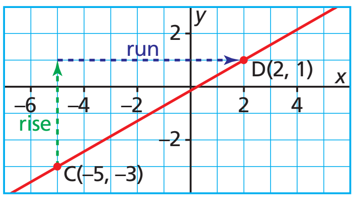
So, the slope of line MT is $\frac{2}{3}$.

Example 3 Determining Slope Given Two Points on a Line

Determine the slope of the line that passes through C(–5, –3) and D(2, 1).

SOLUTION

Sketch the line.
Subtract corresponding coordinates to determine the change in x and in y.



From C to D:
The rise is the change in y-coordinates.
Rise $= 1 - (-3)$
The run is the change in x-coordinates.
Run $= 2 - (-5)$

Slope of CD $= \frac{1 - (-3)}{2 - (-5)}$

Slope of CD $= \frac{4}{7}$

CHECK YOUR UNDERSTANDING

3. Determine the slope of the line that passes through E(4, –5) and F(8, 6).

[Answer: $\frac{11}{4}$]

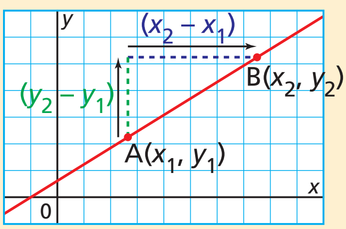
How could you use slope to verify that three points lie on the same line?

Example 3 leads to a formula we can use to determine the slope of any line.

Slope of a Line

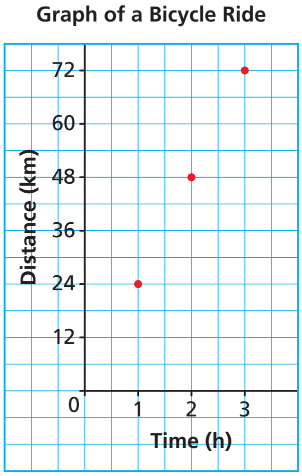
A line passes through $A(x_1, y_1)$ and $B(x_2, y_2)$.

Slope of line AB $= \frac{y_2 - y_1}{x_2 - x_1}$



Example 4 Interpreting the Slope of a Line

Yvonne recorded the distances she had travelled at certain times since she began her cycling trip along the Trans Canada Trail in Manitoba, from North Winnipeg to Grand Beach. She plotted these data on a grid.



a) What is the slope of the line through these points?

b) What does the slope represent?

c) How can the answer to part b be used to determine:

i) how far Yvonne travelled in $1\frac{3}{4}$ hours?

ii) the time it took Yvonne to travel 55 km?

SOLUTION

a) Choose two points on the line, such as P(1, 24) and Q(3, 72). Label the axes x and y. Use this formula:

$$\text{Slope of PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$

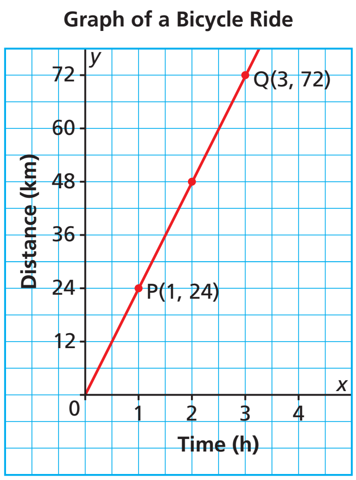
Substitute: $y_2 = 72$, $y_1 = 24$, $x_2 = 3$, and $x_1 = 1$

$$\text{Slope of PQ} = \frac{72 - 24}{3 - 1}$$

$$\text{Slope of PQ} = \frac{48}{2}$$

Slope of PQ = 24

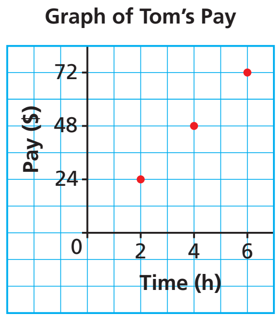
The slope of the line is 24.



b) The values of y are distances in kilometres.
The values of x are time in hours.
So, the slope of the line is measured in kilometres per hour;
this is Yvonne's average speed for her trip.
Yvonne travelled at an average speed of 24 km/h.

CHECK YOUR UNDERSTANDING

4. Tom has a part-time job. He recorded the hours he worked and his pay for 3 different days. Tom plotted these data on a grid.



a) What is the slope of the line through these points?

b) What does the slope represent?

c) How can the answer to part b be used to determine:

i) how much Tom earned in $3\frac{1}{2}$ hours?

ii) the time it took Tom to earn $30?

[Answers: a) 12 b) Tom's hourly rate of pay: $12/h c) i) $42 ii) $2\frac{1}{2}$ hours]

c) i) In 1 h, Yvonne travelled approximately 24 km.

So, in $1\frac{3}{4}$ hours, Yvonne travelled: $\left(1\frac{3}{4}\right)(24\text{ km}) = 42\text{ km}$

In $1\frac{3}{4}$ hours, Yvonne travelled approximately 42 km.

ii) Yvonne travelled approximately 24 km in 1 h, or 60 min.

To travel 1 km, Yvonne took: $\frac{60\text{ min}}{24} = 2.5\text{ min}$

So, to travel 55 km, Yvonne took:
55(2.5 min) = 137.5 min, or 2 h 17.5 min
Yvonne took approximately 2 h 20 min to travel 55 km.

Discuss the Ideas

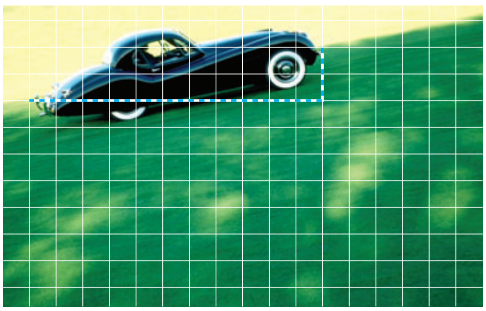
1. When you look at a line on a grid, how can you tell whether its slope is positive, negative, 0, or not defined? Give examples.
2. Why can you choose any 2 points on a line to determine its slope?
3. When you know the coordinates of two points E and F, and use the formula to determine the slope of EF, does it matter which point has the coordinates (x_1, y_1)? Explain.

Exercises

A

4. Determine the slope of the road in each photo.

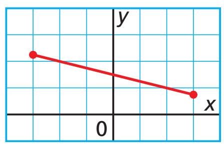
a)



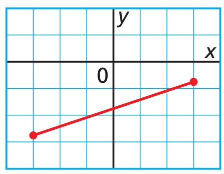
b)

5. For each line segment, is its slope positive, negative, zero, or not defined?

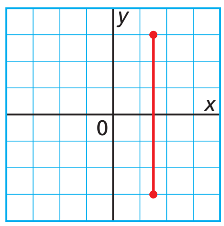
a)



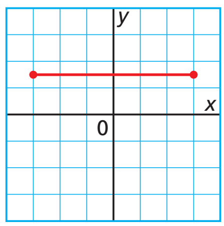
b)



c)

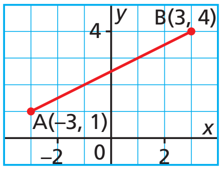


d)



6. For each line segment, determine its rise, run, and slope.

a)



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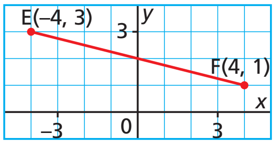
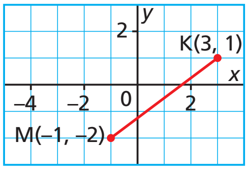
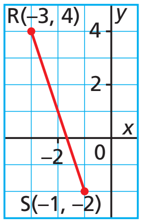


c)



d)



7. Determine the slope of each line described below.

a) As x increases by 1, y increases by 3.

b) As x increases by 2, y decreases by 7.

c) As x decreases by 4, y decreases by 2.

d) As x decreases by 2, y increases by 1.

8. Sketch a line whose slope is:

a) positive **b)** zero

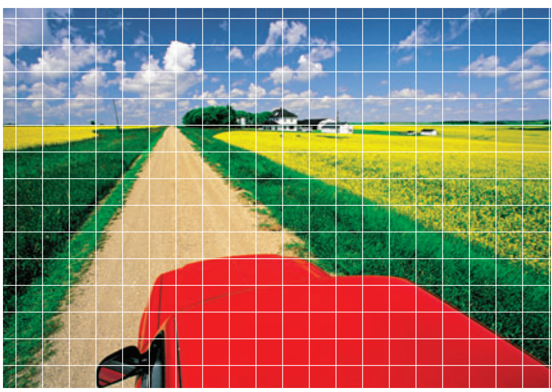
c) negative **d)** not defined

9. Draw a line segment that has one endpoint at the origin and whose slope is:

a) $\frac{2}{3}$ **b)** $-\frac{2}{5}$ **c)** 4 **d)** $-\frac{4}{3}$

10. To copy a picture by hand, an artist places a square grid over the picture. The artist then copies the image on a different grid, making sure corresponding grid squares match.

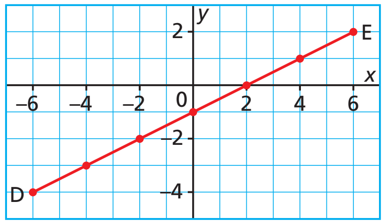
a) How would determining the slopes of lines in the picture help a person to copy the picture?



b) Copy the picture above, using the strategy you described in part a.

B

11. a) Choose two points on line segment DE. Use these two points to determine the slope of the line segment.



b) Choose two different points on segment DE and calculate its slope.

c) Compare the slopes you calculated in parts a and b. Explain the results.

12. a) Draw 2 different line segments with slope $\frac{7}{5}$.

b) How are the line segments in part a the same? How are they different?

13. a) Determine the slope of the line that passes through each pair of points.

i) P(1, 2) and Q(3, 6)

ii) S(0, 1) and T(8, 5)

iii) V(−1, 4) and R(3, −8)

iv) U(−12, −7) and W(−6, −5)

b) Explain what each slope tells you about the line.

14. a) On a grid, draw a line that passes through 3 points. Label the points C, D, and E.

b) Determine the slope of each segment.

i) CD **ii)** DE **iii)** CE

What do you notice?

15. a) A treadmill is set with a rise of 6 in. and a run of 90 in. What is the slope of the treadmill?

b) The treadmill is set at its maximum slope, 0.15. The run is 90 in. What is the rise?

16. A trench is to be dug to lay a drainage pipe. To ensure that the water in the pipe flows away, the trench must be dug so that it drops 1 in. for every 4 ft. measured horizontally.

a) What is the slope of the trench?

b) Suppose the trench drops $6\frac{1}{2}$ in. from beginning to end. How long is the trench measured horizontally?

c) Suppose the trench is 18 ft. long measured horizontally. By how much does it drop over that distance?

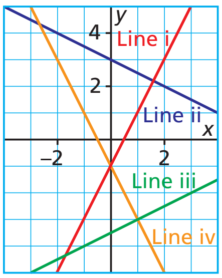
17. Match each line below with a slope. Explain your choices.

a) slope: –2

b) slope: $\frac{1}{2}$

c) slope: $-\frac{1}{2}$

d) slope: 2



18. **a)** Draw the line through each pair of points. Determine the slope of the line.

i) B(0, 3) and C(5, 0)

ii) D(0, –3) and C(5, 0)

iii) D(0, –3) and E(–5, 0)

iv) B(0, 3) and E(–5, 0)

b) How are the slopes of the lines in part a related?

19. **a)** Explain why the slope of a horizontal line is zero.

b) Explain why the slope of a vertical line is undefined.

20. Four students determined the slope of the line through B(6, −2) and C(−3, −5). Their answers were: 3, −3, $\frac{1}{3}$, and $-\frac{1}{3}$

a) Which number is correct for the slope of line BC? Give reasons for your choice.

b) For each incorrect answer, explain what the student might have done wrong to get that answer.

21. **a)** On a grid, sketch each line:

i) a line that has only one intercept

ii) a line that has two intercepts

iii) a line that has more intercepts than you can count

b) How many lines could you draw in each of part a? What is the slope of each line?

22. A hospital plans to build a wheelchair ramp. Its slope must be less than $\frac{1}{12}$. The entrance to the hospital is 70 cm above the ground. What is the minimum horizontal distance needed for the ramp? Justify your answer.

23. Draw the line through G(−5, 1) with each given slope. Write the coordinates of 3 other points on the line. How did you determine these points?

a) 4

b) −1

c) $-\frac{1}{3}$

d) $\frac{7}{4}$

24. **a)** For each line described below, is its slope positive, negative, zero, or undefined? Justify your answer.

i) The line has a positive x-intercept and a negative y-intercept.

ii) The line has a negative x-intercept and a positive y-intercept.

iii) Both intercepts are positive.

iv) The line has an x-intercept but does not have a y-intercept.

b) Sketch each line in part a.

25. Tess conducted an experiment where she determined the masses of different volumes of aluminum cubes. Here are her data:

Volume of Aluminum (cm^3)	Mass of Aluminum (g)
64	172.8
125	337.5
216	583.2

a) Graph these data on a grid.

b) Calculate the slope of the line through the points.

c) What does the slope represent?

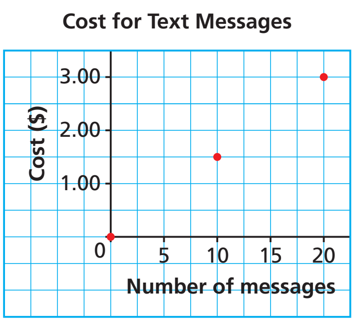
d) How could you use the slope to determine the mass of each volume of aluminum? Explain your strategy.

i) 50 cm^3 **ii)** 275 cm^3

e) What is the approximate volume of each mass of aluminum?

i) 100 g **ii)** 450 g

26. This graph shows the cost for text messages as a function of the number of text messages.



a) Why is a line not drawn through the points on the graph?

b) What is the cost for one text message? How do you know?

c) Determine the cost to send 33 text messages.

d) How many messages can be sent for $7.20?

e) What assumptions did you make when you completed parts c and d?

27. Charin saves the same amount of money each month. This table shows how his savings account balance is changing.

Months Saved	Account Balance ($)
2	145
5	280

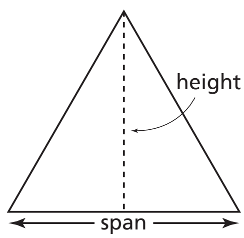
a) How much money does Charin save each month? How could you use the concept of slope to determine this?

b) Determine how much money Charin will have saved after 10 months.

c) Determine how much money Charin had in his account when he started saving money each month. Explain your strategy.

d) What assumptions did you make when you answered parts a to c?

28. *Pitch* is often used to measure the steepness of a roof.



a) For a *full pitch* roof, the height and span are equal. A full pitch roof has a span of 36 ft. What is the slope of this roof?

b) For a *one-third pitch* roof, the height is one-third the span. A one-third pitch roof has a span of 36 ft. What is the slope of this roof?

C

29. On July 23, 1983, a Boeing 767 travelling from Montreal to Edmonton ran out of fuel over Red Lake, Ontario, and the pilot had to glide to make an emergency landing in Gimli, Manitoba. When the plane had been fuelled, imperial units instead of metric units were used for the calculations of the volume of fuel needed. Suppose the plane glided to the ground at a constant speed. The altitude of the plane decreased from 7000 m to 5500 m in a horizontal distance of 18 km. The plane was at an altitude of 2600 m when it was 63 km away from Winnipeg. Could this plane reach Winnipeg? Explain.

30. Use grid paper.

a) Plot point O at the origin, point B(2, 4), and any point A on the positive x-axis.

b) Determine the slope of segment OB and $\tan \angle AOB$.

c) Repeat parts a and b for B(5, 2).

d) How is the slope of a line segment related to the tangent of the angle formed by the segment and the positive x-axis?

31. a) Construct an angle of 30° at the origin, with one arm along the positive x-axis. Determine the slope of the other arm of the angle.

b) Repeat part a for an angle of 60°.

c) For an angle with one arm horizontal, when the angle doubles does the slope of the other arm double? Justify your answer.

Reflect

Describe the types of slope a line may have. How is the slope of a line related to rate of change? Include examples in your explanation.

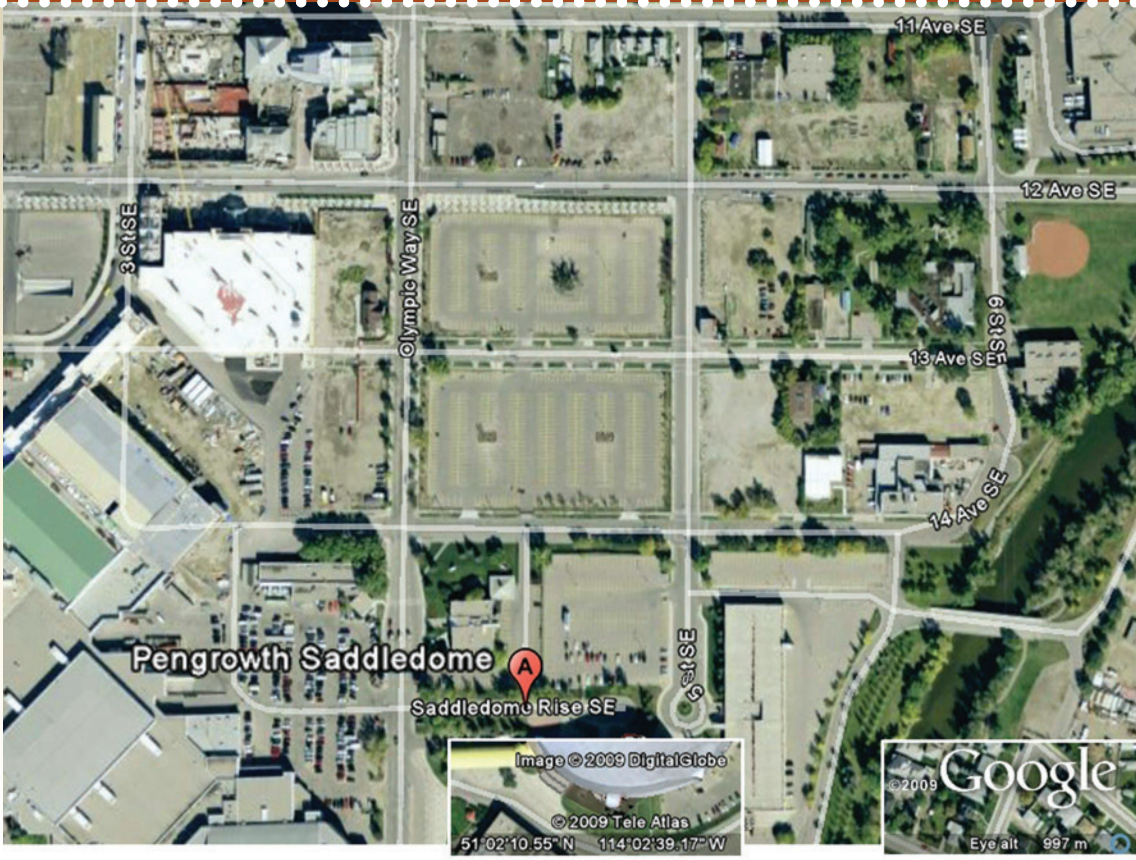
THE WORLD OF MATH

Profile: The Slope of a Road

The slope of a road is called the *grade* of the road, which is the fraction $\frac{\text{rise}}{\text{run}}$ expressed as a percent. When a grade is greater than 6%, a sign is erected by the side of the road to warn traffic travelling downhill. Trucks may have to gear down to travel safely. What are the rise and the run of a road with slope 6%?

6.2 Slopes of Parallel and Perpendicular Lines

LESSON FOCUS
Use slope to determine whether two lines are parallel or perpendicular.



This map of Calgary shows the area close to the Saddledome.

Make Connections

Look at the map above.
Which streets are parallel to 11th Avenue?
Which streets are perpendicular to 11th Avenue? How could you verify this?

Construct Understanding

TRY THIS

Work on your own.

You will need grid paper and a ruler.

A. On a coordinate grid, draw 2 squares with different orientations.

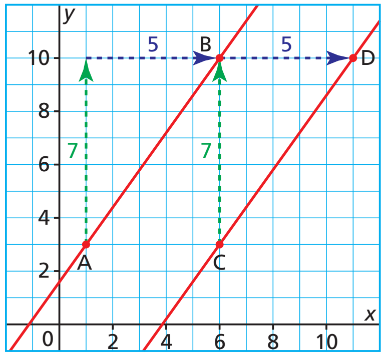
B. For each square, determine the slope of each side.
- What do you notice about the slopes of parallel line segments?
- What do you notice about the slopes of perpendicular line segments?

C. Compare your results with those of 3 classmates. Do the relationships you discovered in Step B seem to be true in general? Justify your answer.

When two lines have the same slope, congruent triangles can be drawn to show the rise and the run.
Lines that have the some slope are parallel.

Slope of AB $= \frac{7}{5}$

Slope of CD $= \frac{7}{5}$



Since the slope of AB is equal to the slope of CD, line AB is parallel to line CD.

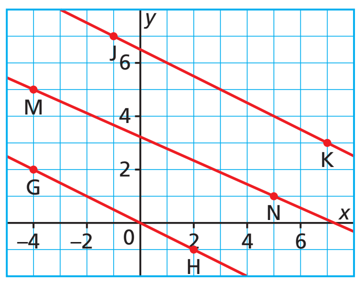
Example 1 Identifying Parallel Lines

Line GH passes through G(−4, 2) and H(2, −1). Line JK passes through J(−1, 7) and K(7, 3). Line MN passes through M(−4, 5) and N(5, 1). Sketch the lines. Are they parallel? Justify the answer.

SOLUTION

Use the formula for the slope of a line through points with coordinates (x_1, y_1) and (x_2, y_2):

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$



$$\text{Slope of GH} = \frac{-1 - 2}{2 - (-4)}$$

$$\text{Slope of GH} = \frac{-3}{6}, \text{ or } -\frac{1}{2}$$

$$\text{Slope of JK} = \frac{3 - 7}{7 - (-1)}$$

$$\text{Slope of JK} = \frac{-4}{8}, \text{ or } -\frac{1}{2}$$

$$\text{Slope of MN} = \frac{1 - 5}{5 - (-4)}$$

$$\text{Slope of MN} = \frac{-4}{9}, \text{ or } -\frac{4}{9}$$

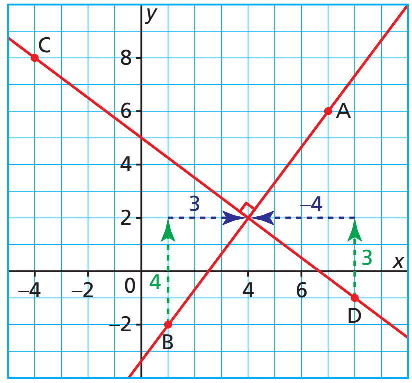
Since the slopes of GH and JK are equal, the two lines are parallel. Since the slope of MN is different from the slopes of GH and JK, MN is not parallel to those lines.

CHECK YOUR UNDERSTANDING

1. Line EF passes through E(−3, −2) and F(−1, 6). Line CD passes through C(−1, −3) and D(1, 7). Line AB passes through A(−3, 7) and B(−5, −2). Sketch the lines. Are they parallel? Justify your answer.

   [Answer: The slopes of the lines are not equal, so the lines are not parallel.]

Non-parallel lines in the same plane have different slopes. Perpendicular lines are not parallel, so they have different slopes.



Lines AB and CD are drawn perpendicular.

Slope of AB $= \frac{\text{rise}}{\text{run}}$ Slope of CD $= \frac{\text{rise}}{\text{run}}$

Slope of AB $= \frac{4}{3}$ Slope of CD $= \frac{3}{-4}$

Slope of CD $= -\frac{3}{4}$

Two real numbers, *a* and *b*, are **negative reciprocals** if $ab = -1$.

The rise of AB is the opposite of the run of CD.
The run of AB is equal to the rise of CD.
$-\frac{3}{4}$ is the *negative reciprocal* of $\frac{4}{3}$.

And, $\left(-\frac{3}{4}\right)\left(\frac{4}{3}\right) = -1$

The relationship between the slopes of AB and CD is true for any two oblique perpendicular lines. Horizontal and vertical lines are an exception.

The slope of a horizontal line is 0. The slope of a vertical line is $\frac{1}{0}$, which is not defined. So, the slopes of horizontal and vertical lines are not negative reciprocals.

Slopes of Perpendicular Lines

The slopes of two oblique perpendicular lines are negative reciprocals; that is, a line with slope a, $a \neq 0$, is perpendicular to a line with slope $-\frac{1}{a}$.

Example 2 Examining Slopes to Compare Lines

Line PQ passes through P(−7, 2) and Q(−2, 10).
Line RS passes through R(−3, −4) and S(5, 1).

a) Are these two lines parallel, perpendicular, or neither? Justify the answer.

b) Sketch the lines to verify the answer to part a.

CHECK YOUR UNDERSTANDING

2. Line ST passes through S(−2, 7) and T(2, −5). Line UV passes through U(−2, 3) and V(7, 6).

a) Are these two lines parallel, perpendicular, or neither? Justify your answer.

b) Sketch the lines to verify your answer to part a.

[Answer: a) The two lines are perpendicular.]

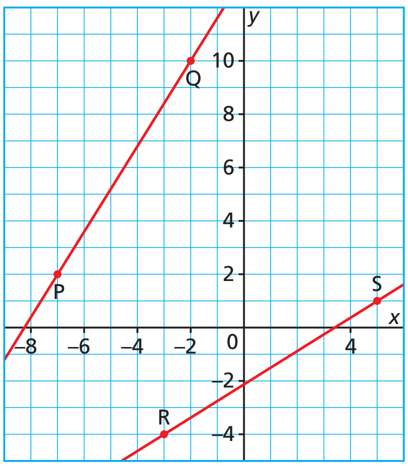
SOLUTION

a) Slope of PQ $= \dfrac{10 - 2}{-2 - (-7)}$ Slope of RS $= \dfrac{1 - (-4)}{5 - (-3)}$

Slope of PQ $= \dfrac{8}{5}$ Slope of RS $= \dfrac{5}{8}$

The two slopes are not equal, so the lines are not parallel.
The two slopes are reciprocals, but not negative reciprocals, so the lines are not perpendicular.
So, the two lines are neither parallel nor perpendicular.

b)



Example 3 Identifying a Line Perpendicular to a Given Line

a) Determine the slope of a line that is perpendicular to the line through E(2, 3) and F(−4, −1).

b) Determine the coordinates of G so that line EG is perpendicular to line EF.

SOLUTION

a) Determine the slope of EF.

Slope of EF $= \dfrac{-1 - 3}{-4 - 2}$

Slope of EF $= \dfrac{-4}{-6}$

Slope of EF $= \dfrac{2}{3}$

The slope of a line perpendicular to EF is the negative reciprocal of $\frac{2}{3}$, which is $-\frac{3}{2}$.

The slope of a line perpendicular to EF is $-\frac{3}{2}$.

(Solution continues.)

CHECK YOUR UNDERSTANDING

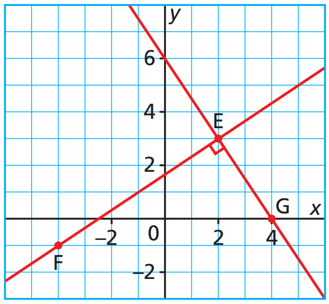
3. a) Determine the slope of a line that is perpendicular to the line through G(−2, 3) and H(1, −2).

b) Determine the coordinates of J so that line GJ is perpendicular to line GH.

[Answers: a) $\frac{3}{5}$ b) sample answer: J(3, 6)]

b) Draw line EF.

The slope of line EG is $-\frac{3}{2}$, so for each rise of -3 units, there is a run of 2 units. From point E, move 3 units down and 2 units right. Mark point G. Its coordinates are G(4, 0). Draw a line through EG. Line EG is perpendicular to line EF.

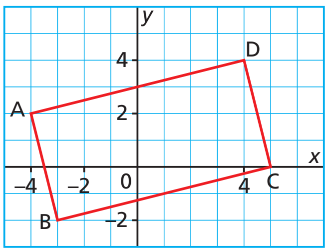


Why do the slopes of oblique perpendicular lines have opposite signs?

What are some other possible coordinates for G?

Example 4 Using Slope to Identify a Polygon

ABCD is a parallelogram. Is it a rectangle? Justify the answer.



SOLUTION

A parallelogram has opposite sides equal.
It is a rectangle if its angles are right angles.
To check whether ABCD is a rectangle, determine whether two intersecting sides are perpendicular.
Determine whether AB is perpendicular to BC.
From the diagram, the rise from A to B is -4 and the run is 1.

Slope of AB $= \frac{-4}{1}$

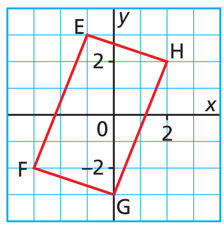
From the diagram, the rise from B to C is 2 and the run is 8.

Slope of BC $= \frac{2}{8}$, or $\frac{1}{4}$

Since the slopes of AB and BC are negative reciprocals, AB and BC are perpendicular.
This means that $\angle$ABC is a right angle and that ABCD is a rectangle.

CHECK YOUR UNDERSTANDING

4. EFGH is a parallelogram. Is it a rectangle? Justify your answer.



[Answer: No, EFGH is not a rectangle.]

Why didn't we need to check that all the angles of parallelogram ABCD were right angles?

Why didn't we write the slope of AB as -4?

Discuss the Ideas

1. How do you determine whether two lines are parallel?

2. How do you determine whether two lines are perpendicular?

Exercises

A

3. The slopes of lines are given below. For each line, what is the slope of a parallel line?

a) $\frac{4}{5}$ **b)** $-\frac{4}{3}$

c) 3 **d)** 0

4. The slopes of lines are given below. For each line, what is the slope of a perpendicular line?

a) $\frac{7}{6}$ **b)** $-\frac{5}{8}$

c) 9 **d)** -5

5. The slopes of two lines are given. Are the two lines parallel, perpendicular, or neither?

a) 4, 4 **b)** $\frac{1}{6}$, 6

c) $\frac{7}{8}, -\frac{7}{8}$ **d)** $\frac{1}{10}, -10$

6. The slopes of lines are given below. What is the slope of a line that is:

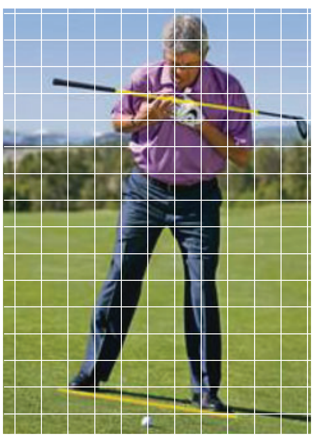
i) parallel to each given line?

ii) perpendicular to each given line?

a) $-\frac{4}{9}$ **b)** 5 **c)** $\frac{7}{3}$ **d)** -4

B

7. This golfer is checking his set-up position by holding his club to his chest and looking to see whether it is parallel to an imaginary line through the tips of his shoes.



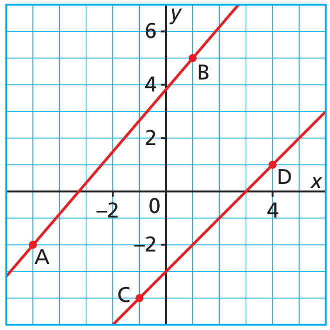
Is this golfer set up correctly? How did you find out?

8. For each grid below:

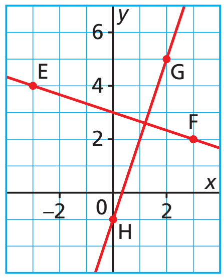
i) Write the coordinates of the 2 labelled points on each line.

ii) Are the two lines parallel, perpendicular, or neither? Justify your answer.

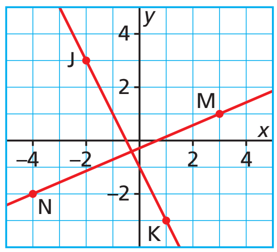
a)



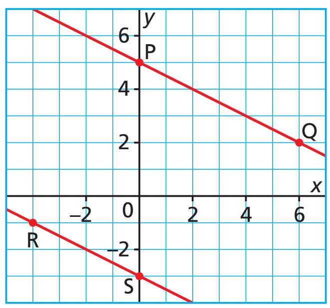
b)



c)



d)



9. The coordinates of the endpoints of segments are given below. Are the two line segments parallel, perpendicular, or neither? Justify your answer.

a) S(-4, -1), T(-1, 5) and U(1, 1), V(5, -1)

b) B(-6, -2), C(-3, 3) and D(2, 0), E(5, 5)

c) N(-6, 2), P(-3, -4) and Q(1, -3), R(3, 4)

d) G(-2, 5), H(4, 1) and J(1, -4), K(7, 0)

10. How are the lines in each pair related? Justify your answer.

a) DE has an x-intercept of 4 and a y-intercept of -6.
FG has an x-intercept of -6 and a y-intercept of 4.

b) HJ has an x-intercept of -2 and a y-intercept of 3.
KM has an x-intercept of -9 and a y-intercept of 6.

11. A line passes through A$(-3, -2)$ and B$(1, 4)$.

a) On a grid, draw line AB and determine its slope.

b) Line CD is parallel to AB. What is the slope of CD?

c) Point C has coordinates $(-1, -1)$. Determine two sets of possible coordinates for D. Why might your answers be different from those of a classmate?

d) Line AE is perpendicular to AB. What is the slope of AE?

e) Determine two sets of possible coordinates for E.

12. A line passes through A$(5, -2)$ and B$(3, 2)$.

a) Draw line AB on a grid and determine its slope.

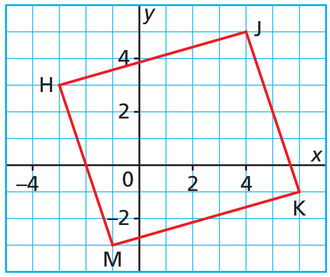
b) Line CD is parallel to AB. What is the slope of CD?

c) Given that Q$(1, -4)$ lies on CD, draw line CD. Determine the coordinates of its x- and y-intercepts.

d) Line EF is perpendicular to AB. What is the slope of EF?

e) Given that R$(-4, -4)$ lies on EF, draw line EF. Determine the coordinates of its x- and y-intercepts.

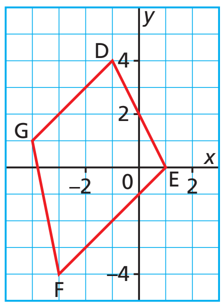
13. HJKM is a quadrilateral.



a) Is HJKM a parallelogram? Justify your answer.

b) Is HJKM a rectangle? Justify your answer.

14. Which type of quadrilateral is DEFG? Justify your answer.



15. QRST is a rectangle with Q$(-2, 4)$ and R$(1, 1)$. Do you have enough information to determine the coordinates of S and T? Explain.

16. The coordinates of the vertices of $\triangle$ABC are A$(-3, 1)$, B$(6, -2)$, and C$(3, 4)$. How can you tell that $\triangle$ABC is a right triangle?

17. The coordinates of the vertices of $\triangle$DEF are D$(-3, -2)$, E$(1, 4)$, and F$(4, 2)$. Is $\triangle$DEF a right triangle? Justify your answer.

18. Draw a triangle on a grid.

a) Determine the slope of each side of the triangle.

b) Join the midpoints of the sides. Determine the slope of each new line segment formed.

c) What relationship do you notice between the slopes in parts a and b?

19. ABCD is a parallelogram. Three vertices have coordinates A(−4, 3), B(2, 4), and C(4, 0).
a) Is ABCD a rectangle? Justify your answer.
b) Determine the coordinates of D. Explain your answer.
c) What other strategy could you use to determine the coordinates of D? Explain.

20. The coordinates of two of the vertices of △RST are R(−3, 4) and S(0, −2).
Determine possible coordinates for T so that △RST is a right triangle. Explain your strategy.

C

21. On a grid, draw several different rhombuses. Use slopes to determine the relationship between the diagonals.

22. Determine the value of c so that the line segment with endpoints B(2, 2) and C(9, 6) is parallel to the line segment with endpoints D(c, −7) and E(5, −3).

23. Given A(3, 5), B(7, 10), C(0, 2), and D(1, a), determine the value of a for which:
a) Line AB is parallel to line CD.
b) Line AB is perpendicular to line CD.

24. **a)** On grid paper, construct a square with side length 4 units and one vertex at the origin. Verify that the diagonals of this square are perpendicular.
b) Repeat part a for a square with side length a units.

Reflect

What have you learned about perpendicular lines and parallel lines? Include examples in your answer.

THE WORLD OF MATH

Historical Moment: Agnes Martin

Agnes Martin was born in Macklin, Saskatchewan, and lived from 1912 to 2004. She was an artist who used parallel lines and grids in her artwork. Before Agnes began a painting, she calculated the distances between pairs of parallel lines or bands. She then drew each line by hand, using a string stretched tightly across the surface to guide her, and a ruler to draw the line.

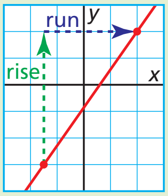
CHECKPOINT 1

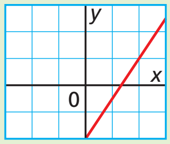
Connections

Definition

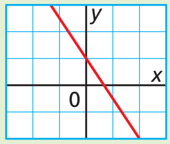
The slope of a line is the measure of its rate of change.

$\text{Slope} = \dfrac{\text{rise}}{\text{run}}$

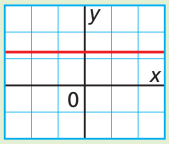




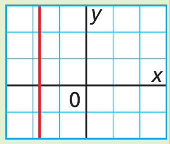
positive slope



negative slope



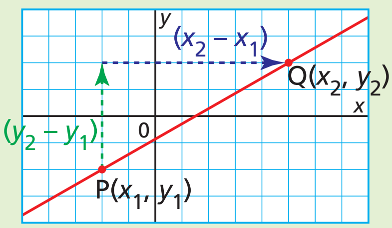
0 slope



undefined slope

The slope of a line through $P(x_1, y_1)$ and $Q(x_2, y_2)$ is:

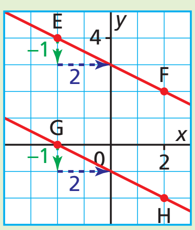
$\text{Slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$



Two lines are parallel when they have equal slopes.

Slope of EF $= -\dfrac{1}{2}$

Slope of GH $= -\dfrac{1}{2}$

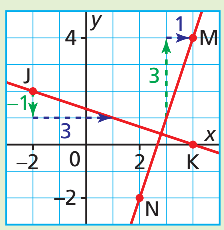


Two lines are perpendicular when their slopes are negative reciprocals.

Slope of MN = 3

Slope of JK $= -\dfrac{1}{3}$

$(3)\left(-\dfrac{1}{3}\right) = -1$



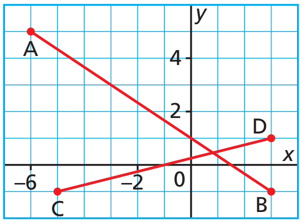
Concept Development

- **In Lesson 6.1**
  – You defined the slope of a line segment and the slope of a line as rate of change.
  – You determined the slope of a line segment and the slope of a line from measurements of the rise and run.
  – You showed that the slope of a line is equal to the slope of any segment of the line.
  – You determined the slope of a line segment given the coordinates of the endpoints of the segment, and the slope of a line given the coordinates of two points on the line.
  – You explained the meaning of the slope of a horizontal line and a vertical line.
  – You drew a line, given its slope and a point on the line.
  – You determined the coordinates of a point on a line, given its slope and another point on the line.
  – You solved contextual problems involving slope.
- **In Lesson 6.2**
  – You generalized and applied rules for determining whether two lines are parallel or perpendicular.
  – You drew lines that were parallel or perpendicular to a given line.

Assess Your Understanding

6.1

1. Determine the slopes of line segments AB and CD.

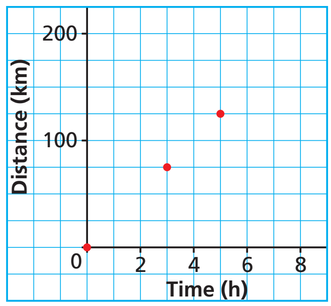


2. Determine the slope of the line that passes through each pair of points.
   a) Q(−2, 5) and R(2, −10)
   b) an x-intercept of 3 and a y-intercept of −5

3. Why can the slope of a line be determined by using any two points on the line?

4. Jordan recorded the distances he had travelled at certain times since he began his snowmobile trip along the Overland Trail from Whitehorse to Dawson in the Yukon. He plotted these data on a grid.
   a) What is the slope of the line through these points? What does it represent?
   b) How far did Jordan travel in $1\frac{1}{4}$ hours?
   c) How long did it take Jordan to travel 65 km?

Jordan's Snowmobile Journey



6.2

5. Draw lines with the given slopes. Are the lines parallel, perpendicular, or neither? Justify your answers.
   a) $\frac{2}{5}, \frac{5}{2}$ **b)** $-\frac{1}{4}, 4$ **c)** $\frac{9}{7}, \frac{18}{14}$

6. A line passes through D(−6, −1) and E(2, 5).
   a) Determine the coordinates of two points on a line that is parallel to DE.
   b) Determine the coordinates of two points on a line that is perpendicular to DE.
   Describe the strategies you used to determine the coordinates.

7. The vertices of a triangle have coordinates A(−1, 5), B(−5, −6), and C(3, 1). Is △ABC a right triangle? Justify your answer.

8. Two vertices of right △MNP have coordinates M(−3, 6) and P(3, −3). Point N lies on an axis. Determine two possible sets of coordinates for N. Explain your strategy.

MATH LAB

6.3 Investigating Graphs of Linear Functions

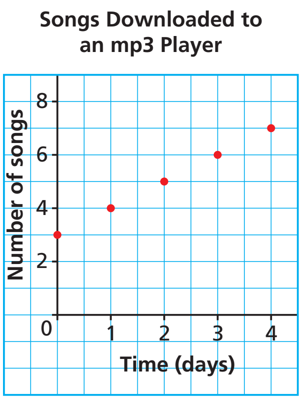
LESSON FOCUS

Investigate the relationship between the graph and the equation of a linear function.

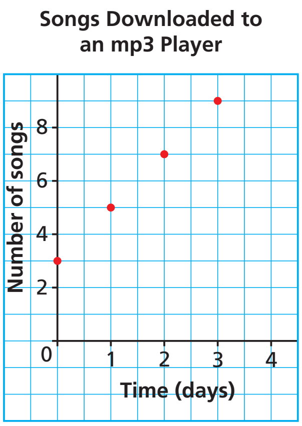
Make Connections

Alimina purchased an mp3 player and downloaded 3 songs.
Each subsequent day, she downloads 2 songs.
Which graph represents this situation? Explain your choice.

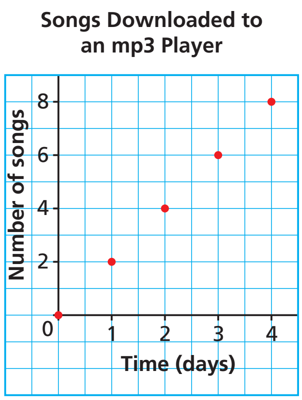
Graph A



Graph B



Graph C



Construct Understanding

TRY THIS

Work with a partner.

Use a graphing calculator or a computer with graphing software.

A. Graph $y = mx + 6$ for different values of m.
Include values of m that are negative and 0.
Use a table to record your results.

Equation	Value of m	Sketch of the Graph	Slope of the Graph	x-intercept	y-intercept
$y = x + 6$	1				

B. How does changing the value of m change the appearance of the graph?
What does m represent?

C. Graph $y = 2x + b$ for different values of b.
Include values of b that are negative and 0.
Use a table to record your results.

Equation	Value of b	Sketch of the Graph	Slope of the Graph	x-intercept	y-intercept
$y = 2x + 6$	6				

D. How does changing the value of b change the appearance of the graph?
What does b represent?

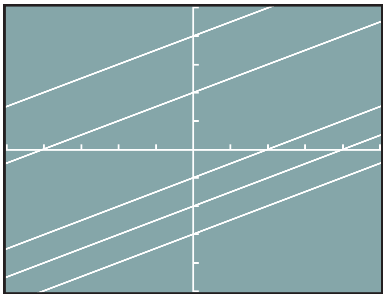
E. Predict the appearance of the graph of $y = -2x + 4$.
Verify your prediction by graphing.

Suppose you are given the graph of a linear function. How could you use what you learned in this lesson to determine an equation for that function?

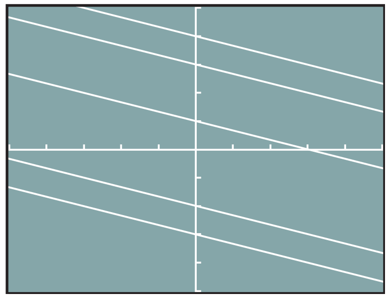
Assess Your Understanding

1. In the screens below, each mark on the x-axis and y-axis represents 1 unit. What is the equation of each line?

a) The slope of each line is $\frac{1}{2}$.



b) The slope of each line is $-\frac{1}{3}$.



2. A linear function is written in the form $y = mx + b$. Use your results from *Try This* to suggest what the numbers m and b represent. Explain how you could use this information to graph the function.

3. Describe the graph of the linear function whose equation is $y = -3x + 6$. Draw this graph without using technology.

4. **a)** Predict what will be common about the graphs of these equations.

i) $y = x - 1$ **ii)** $y = 2x - 1$

iii) $y = -3x - 1$ **iv)** $y = -2x - 1$

b) Graph the equations to check your prediction.

5. **a)** Predict what will be common about the graphs of these equations.

i) $y = x - 3$ **ii)** $y = x - 2$

iii) $y = x$ **iv)** $y = x + 3$

b) Graph the equations to check your prediction.

6. Graph each equation on grid paper without using a table of values. Describe your strategy.

a) $y = 3x + 5$ **b)** $y = -3x + 5$

c) $y = 3x - 5$ **d)** $y = -3x - 5$

7. In Lesson 5.6, question 12, page 309, the cost, C dollars, to rent a hall for a banquet is given by the equation $C = 550 + 15n$, where n represents the number of people attending the banquet.

a) Graph this equation on grid paper.

b) Compare the equation above with the equation $y = mx + b$. What do m and b represent in this context?

6.4 Slope-Intercept Form of the Equation for a Linear Function

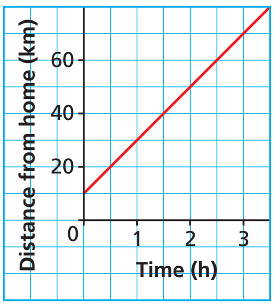
LESSON FOCUS

Relate the graph of a linear function to its equation in slope-intercept form.

Make Connections

This graph shows a cyclist's journey where the distance is measured from her home.

Graph of a Bicycle Journey



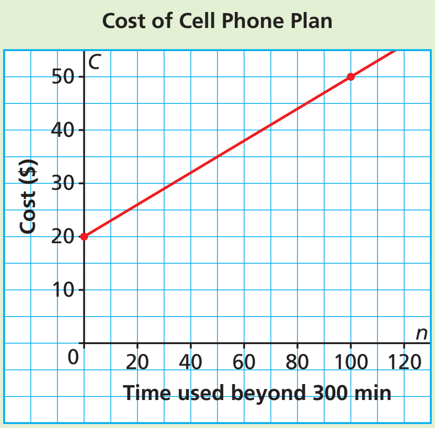
What does the vertical intercept represent?
What does the slope of the line represent?

Construct Understanding

THINK ABOUT IT

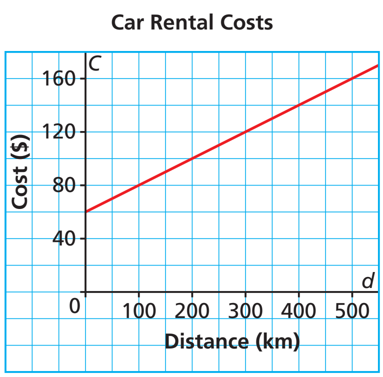
Work with a partner.

A cell phone plan charges a monthly fee that covers the costs of the first 300 min of phone use. This graph represents the cost of the plan based on the time beyond 300 min.



How do you know this is the graph of a linear function?
What does the slope of the graph represent?
Write an equation to describe this function.
Verify that your equation is correct.

In Chapter 5, Lesson 5.6, we described a linear function in different ways. The linear function below represents the cost of a car rental.



An equation of the function is:
$C = 0.20d + 60$

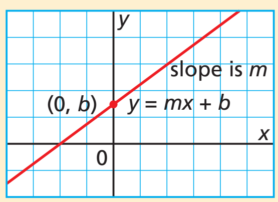
The number, 0.20, is the rate of change, or the slope of the graph. This is the cost in dollars for each additional 1 km driven.

The number, 60, is the vertical intercept of the graph. This is the cost in dollars that is independent of the distance driven – the initial cost for renting the car.

In general, any linear function can be described in **slope-intercept form.**

Slope-Intercept Form of the Equation of a Linear Function

The equation of a linear function can be written in the form $y = mx + b$, where m is the slope of the line and b is its y-intercept.



Example 1 Writing an Equation of a Linear Function Given Its Slope and *y*-Intercept

The graph of a linear function has slope $\frac{3}{5}$ and y-intercept -4.
Write an equation for this function.

SOLUTION

Use:

$y = mx + b$ Substitute: $m = \frac{3}{5}$ and $b = -4$

$y = \frac{3}{5}x - 4$

An equation for this function is: $y = \frac{3}{5}x - 4$

CHECK YOUR UNDERSTANDING

1. The graph of a linear function has slope $-\frac{7}{3}$ and y-intercept 5. Write an equation for this function.

[Answer: $y = -\frac{7}{3}x + 5$]

Can you write an equation for a linear function when you know its slope and *x*-intercept? How would you do it?

Example 2 Graphing a Linear Function Given Its Equation in Slope-Intercept Form

Graph the linear function with equation: $y = \frac{1}{2}x + 3$

SOLUTION

Compare: $y = \frac{1}{2}x + 3$

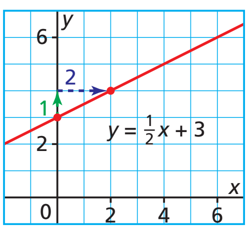
with: $y = mx + b$

The slope of the graph is $\frac{1}{2}$.

The y-intercept is 3, with coordinates (0, 3).
On a grid, plot a point at (0, 3).
The slope of the line is:

$\frac{\text{rise}}{\text{run}} = \frac{1}{2}$

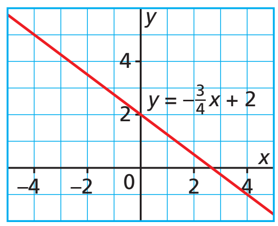
So, from (0, 3), move 1 unit up and 2 units right, then mark a point.
Draw a line through the points.



CHECK YOUR UNDERSTANDING

2. Graph the linear function with equation: $y = -\frac{3}{4}x + 2$

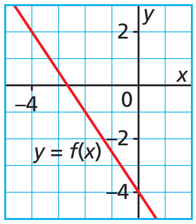
Answer:



What other strategy could you use to graph this linear function?

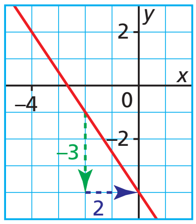
Example 3 Writing the Equation of a Linear Function Given Its Graph

Write an equation to describe this function.
Verify the equation.



SOLUTION

Use the equation: $y = mx + b$
To write the equation of a linear function, determine the slope of the line, m, and its y-intercept, b.
The line intersects the y-axis at -4; so, $b = -4$.
From the graph, the rise is -3 when the run is 2.



So, $m = \frac{-3}{2}$, or $-\frac{3}{2}$

Substitute for m and b in $y = mx + b$.

$y = -\frac{3}{2}x - 4$

An equation for the function is: $y = -\frac{3}{2}x - 4$

To verify the equation, substitute the coordinates of a point on the line into the equation. Choose the point $(-2, -1)$.

Substitute $x = -2$ and $y = -1$ into the equation: $y = -\frac{3}{2}x - 4$

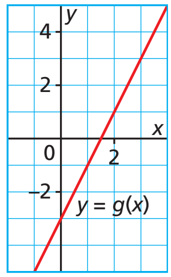
L. S. $= y$
$= -1$

R. S. $= -\frac{3}{2}x - 4$
$= -\frac{3}{2}(-2) - 4$
$= 3 - 4$
$= -1$

Since the left side is equal to the right side, the equation is correct.

CHECK YOUR UNDERSTANDING

3. Write an equation to describe this function. Verify the equation.



[Answer: $y = 2x - 3$]

Can the graph of a linear function be described by more than one equation of the form $y = mx + b$? Explain.

THE WORLD OF MATH

Historical Moment: Why Is *m* Used to Represent Slope?

Some historians have researched the works of great mathematicians from many different countries over the past few hundred years to try to answer this question. Others have attempted to identify words that could be used to refer to the slope of a line. The choice of the letter *m* may come from the French word *monter,* which means to climb. However, the French mathematician René Descartes did not use *m* to represent slope. At this time, historians cannot answer this question; it remains a mystery.

Example 4 Using an Equation of a Linear Function to Solve a Problem

The student council sponsored a dance. A ticket cost $5 and the cost for the DJ was $300.

a) Write an equation for the profit, P dollars, on the sale of t tickets.

b) Suppose 123 people bought tickets. What was the profit?

c) Suppose the profit was $350. How many people bought tickets?

d) Could the profit be exactly $146? Justify the answer.

SOLUTION

a) The profit is: income − expenses
When t tickets are sold, the income is: $5t$ dollars
The expenses are $300.
So, an equation is: $P = 5t - 300$

b) Use the equation:

$P = 5t - 300$ — Substitute: $t = 123$

$P = 5(123) - 300$ — Simplify.

$P = 615 - 300$

$P = 315$

The profit was $315.

c) Use the equation:

$P = 5t - 300$ — Substitute: $P = 350$

$350 = 5t - 300$ — Collect like terms.

$350 + 300 = 5t - 300 + 300$

$650 = 5t$ — Solve for t.

$$\frac{650}{5} = \frac{5t}{5}$$

$130 = t$

One hundred thirty people bought tickets.

d) Use the equation:

$P = 5t - 300$ — Substitute: $P = 146$

$146 = 5t - 300$ — Simplify.

$146 + 300 = 5t - 300 + 300$

$446 = 5t$ — Solve for t.

$$\frac{446}{5} = \frac{5t}{5}$$

$89.2 = t$

Since the number of tickets sold is not a whole number, the profit cannot be exactly $146.

CHECK YOUR UNDERSTANDING

4. To join the local gym, Karim pays a start-up fee of $99, plus a monthly fee of $29.

a) Write an equation for the total cost, C dollars, for n months at the gym.

b) Suppose Karim went to the gym for 23 months. What was the total cost?

c) Suppose the total cost was $505. For how many months did Karim use the gym?

d) Could the total cost be exactly $600? Justify your answer.

[Answers: a) $C = 29n + 99$ b) $766 c) 14 months d) no]

Suppose you graphed the linear relation. What would the slope and vertical intercept be?

Discuss the Ideas

1. When a real-world situation can be modelled by a linear function, what do the slope and vertical intercept usually represent?

2. When you are given the graph of a linear function, how can you determine an equation that represents that function?

3. When you are given an equation of a linear function in slope-intercept form, how can you quickly sketch the graph?

Exercises

A

4. For each equation, identify the slope and y-intercept of its graph.
   a) $y = 4x - 7$ b) $y = x + 12$
   c) $y = -\frac{4}{9}x + 7$ d) $y = 11x - \frac{3}{8}$
   e) $y = \frac{1}{5}x$ f) $y = 3$

5. Write an equation for the graph of a linear function that:
   a) has slope 7 and y-intercept 16
   b) has slope $-\frac{3}{8}$ and y-intercept 5
   c) passes through H(0, -3) and has slope $\frac{7}{16}$
   d) has y-intercept -8 and slope $-\frac{6}{5}$
   e) passes through the origin and has slope $-\frac{5}{12}$

6. Graph the line with each y-intercept and slope.
   a) y-intercept is 1, slope is $\frac{1}{2}$
   b) y-intercept is -5, slope is 2
   c) y-intercept is 4, slope is $-\frac{2}{3}$
   d) y-intercept is 0, slope is $\frac{4}{3}$

B

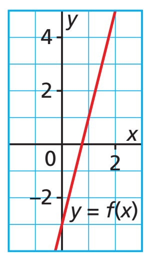
7. Graph each equation on grid paper. Explain the strategy you used.
   a) $y = 2x - 7$ b) $y = -x + 3$
   c) $y = -\frac{1}{4}x + 5$ d) $y = \frac{5}{2}x - 4$
   e) $V = -100t + 6000$ f) $C = 10n + 95$

8. For a service call, an electrician charges an \$80 initial fee, plus \$50 for each hour she works.
   a) Write an equation to represent the total cost, C dollars, for t hours of work.
   b) How would the equation change if the electrician charges \$100 initial fee plus \$40 for each hour she works?

9. The total fee for withdrawing money at an ATM in a foreign country is a \$3.50 foreign cash withdrawal fee, plus a 2% currency conversion fee. Write an equation to represent the total fee, F dollars, for withdrawing d dollars.

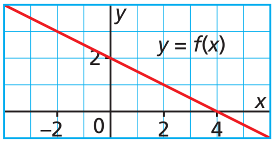
10. Use a graphing calculator or a computer with graphing software. Graph each equation. Explain the strategy you used. Sketch or print the graph.
    a) $f(x) = -\frac{3}{13}x + \frac{4}{11}$ b) $g(x) = 3.75x - 2.95$
    c) $C(n) = 0.45n + 25.50$ d) $F(c) = \frac{9}{5}c + 32$

11. A student said that the equation of this graph is $y = -3x + 4$.
    a) What mistakes did the student make?
    b) What is the equation of the graph?

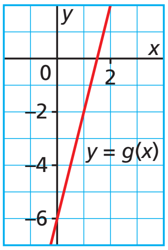


12. For each graph that follows:
    i) Determine its slope and y-intercept.
    ii) Write an equation to describe the graph, then verify the equation.
    iii) Use the equation to calculate the value of y when $x = 10$.

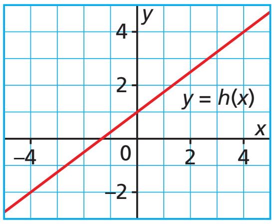
a)



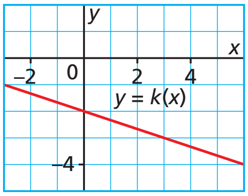
b)



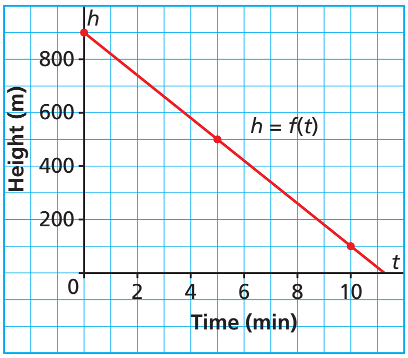
c)



d)



13. This graph represents the height of a float plane above a lake as the plane descends to land.



a) Determine the slope and the h-intercept. What do they represent?

b) Write an equation to describe the graph, then verify the equation.

c) Use the equation to calculate the value of h when $t = 5.5$ min.

d) Suppose the plane began its descent at 700 m and it landed after 8 min.

  i) How would the graph change?

  ii) How would the equation change?

14. An online music site charges a one-time membership fee of \$20, plus \$0.80 for every song that is downloaded.

a) Write an equation for the total cost, C dollars, for downloading n songs.

b) Jacques downloaded 109 songs. What was the total cost?

c) Michelle paid a total cost of \$120. How many songs did she download?

15. a) How can you use the slope-intercept form of an equation, $y = mx + b$, to graph the horizontal line $y = 2$?

b) How can you graph the vertical line $x = 2$? Explain your answers.

16. Alun has a part-time job working as a bus boy at a local restaurant. He earns \$34 a night plus 5% of the tips.

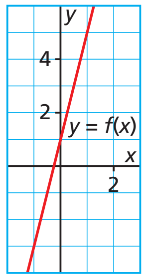
a) Write an equation for Alun's total earnings, E dollars, when the tips are t dollars.

b) What will Alun earn when the tips are \$400? Explain your strategy.

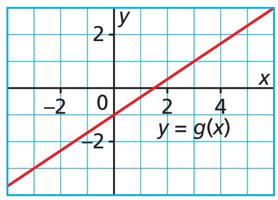
c) What were the nightly tips when Alun earned \$64? Explain your strategy.

17. Which equation matches each given graph? Justify your choice.

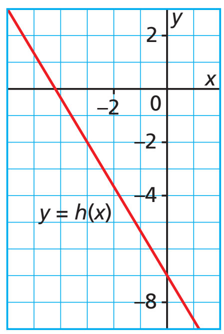
a) $y = x + 4$
$y = 4x + 1$
$y = x - 4$
$y = -4x + 1$



b) $y = \frac{3}{2}x - 1$
$y = -\frac{2}{3}x + 1$
$y = \frac{2}{3}x - 1$
$y = -x + \frac{2}{3}$



c) $y = \frac{5}{3}x + 7$
$y = -\frac{3}{5}x - 7$
$y = -7x - \frac{5}{3}$
$y = -\frac{5}{3}x - 7$

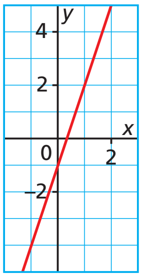


18. Match each equation with its graph. How did you decide on the equation for each graph?

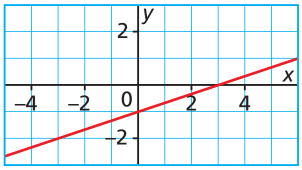
a) $y = 2x - 1$ b) $y = 3x - 1$

c) $y = -x - 1$ d) $y = \frac{1}{3}x - 1$

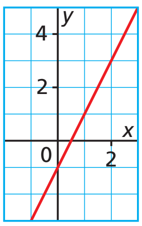
Graph A



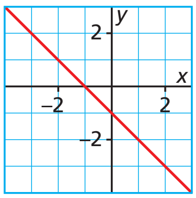
Graph B



Graph C



Graph D

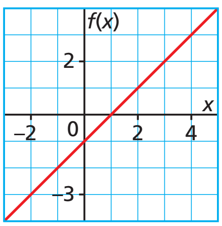


19. Match each equation with its graph. Compare the graphs. What do you notice?

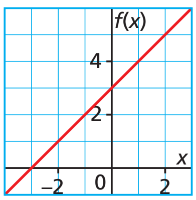
a) $f(x) = -x - 4$ b) $f(x) = -x + 1$

c) $f(x) = x + 3$ d) $f(x) = x - 1$

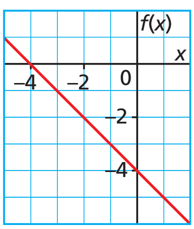
Graph A



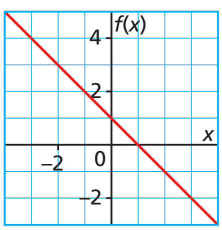
Graph B



Graph C



Graph D



20. Identify the graph below that corresponds to each given slope and y-intercept.

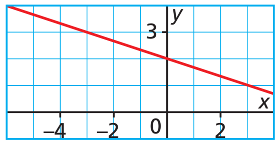
a) slope 3; y-intercept 2

b) slope $\frac{1}{3}$; y-intercept -2

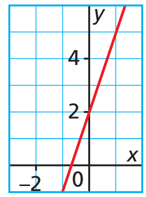
c) slope -3; y-intercept -2

d) slope $-\frac{1}{3}$, y-intercept 2

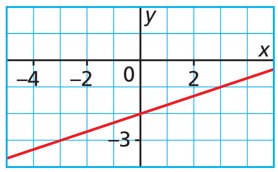
Graph A



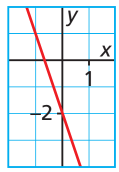
Graph B



Graph C



Graph D



21. Consider these equations:

$y = -5x - 7$, $y = 5x + 15$,
$y = \frac{1}{5}x + 9$, $y = -\frac{1}{5}x + 15$,
$y = \frac{1}{5}x + 21$, $y = -5x + 13$,
$y = 5x + 24$, $y = -\frac{1}{5}x$

Which equations represent parallel lines? Perpendicular lines? How do you know?

C

22. Write an equation of a linear function that has y-intercept 4 and x-intercept 3. Describe the steps you used to determine the equation.

23. An equation of a line is $y = \frac{5}{3}x + c$. Determine the value of c when the line passes through the point F(4, -6). Describe your strategy.

24. An equation of a line is $y = mx - \frac{7}{8}$. Determine the value of m when the line passes through the point E(-3, 5).

Reflect

How do the values of m and b in the linear equation $y = mx + b$ relate to the graph of the corresponding linear function? Include an example.

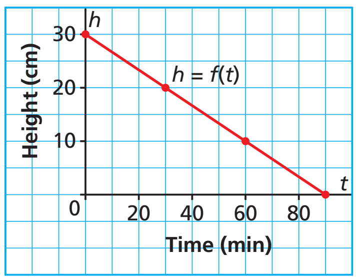
6.5 Slope-Point Form of the Equation for a Linear Function

LESSON FOCUS

Relate the graph of a linear function to its equation in slope-point form.

Make Connections

This graph shows the height of a candle as it burns.
How would you write an equation to describe this line?
Suppose you could not identify the h-intercept.
How could you write an equation for the line?

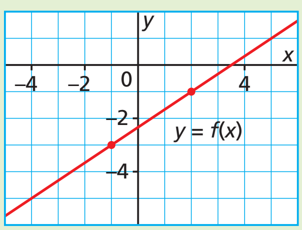


Construct Understanding

THINK ABOUT IT

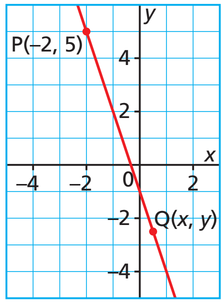
Work with a partner.

Determine an equation for this line.
How many different ways can you do this?
Compare your equations and strategies.
Which strategy is more efficient?



When we know the slope of a line and the coordinates of a point on the line, we use the property that the slope of a line is constant to determine an equation for the line.

This line has slope -3 and passes through P$(-2, 5)$. We use any other point Q(x, y) on the line to write an equation for the slope, m:



$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{y-5}{x-(-2)}$$

$$m = \frac{y-5}{x+2}$$ Substitute: $m = -3$

$$-3 = \frac{y-5}{x+2}$$ Multiply each side by $(x + 2)$.

$$-3(x+2) = (x+2)\left(\frac{y-5}{x+2}\right)$$ Simplify.

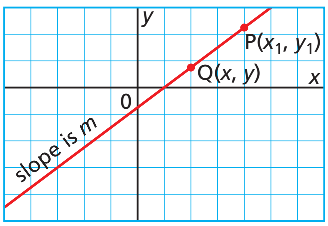
$$-3(x+2) = y-5$$

$$y-5 = -3(x+2)$$

This equation is called the **slope-point form;** both the slope and the coordinates of a point on the line can be identified from the equation.

We can use this strategy to develop a formula for the slope-point form for the equation of a line.

This line has slope m and passes through the point P(x_1, y_1).
Another point on the line is Q(x, y).



The slope, m, of the line is:

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{y-y_1}{x-x_1}$$ Multiply each side by $(x - x_1)$.

$$m(x-x_1) = (x-x_1)\left(\frac{y-y_1}{x-x_1}\right)$$ Simplify.

$$m(x-x_1) = y-y_1$$

$$y-y_1 = m(x-x_1)$$

Slope-Point Form of the Equation of a Linear Function

The equation of a line that passes through $P(x_1, y_1)$ and has slope m is:
$y - y_1 = m(x - x_1)$

Example 1 Graphing a Linear Function Given Its Equation in Slope-Point Form

a) Describe the graph of the linear function with this equation:

$$y - 2 = \frac{1}{3}(x + 4)$$

b) Graph the equation.

SOLUTION

a) Compare the given equation with the equation in slope-point form.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{3}(x + 4)$$

To match the slope-point form, rewrite the given equation so the operations are subtraction.

$$y - 2 = \frac{1}{3}[x - (-4)]$$

$$y - y_1 = m(x - x_1)$$

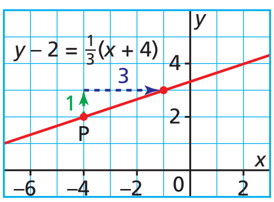
So, $y_1 = 2$

$m = \frac{1}{3}$

$x_1 = -4$

The graph passes through $(-4, 2)$ and has slope $\frac{1}{3}$.

b) Plot the point $P(-4, 2)$ on a grid and use the slope of $\frac{1}{3}$ to plot another point. Draw a line through the points.



CHECK YOUR UNDERSTANDING

1. a) Describe the graph of the linear function with this equation:

$$y + 1 = -\frac{1}{2}(x - 2)$$

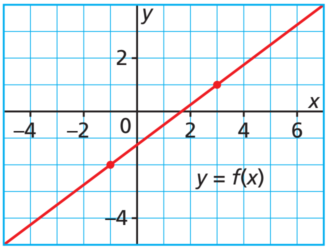
b) Graph the equation.

[Answer: a) slope $-\frac{1}{2}$; passes through $(2, -1)$]

Example 2 Writing an Equation Using a Point on the Line and Its Slope

a) Write an equation in slope-point form for this line.

b) Write the equation in part a in slope-intercept form. What is the *y*-intercept of this line?

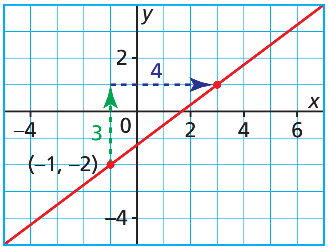


SOLUTION

a) Identify the coordinates of one point on the line and calculate the slope.
The coordinates of one point are (–1, –2).
To calculate the slope, *m*, use:

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{3}{4}$$



Use the slope-point form of the equation.

$y - y_1 = m(x - x_1)$ Substitute: $y_1 = -2$, $x_1 = -1$, and $m = \frac{3}{4}$

$$y - (-2) = \frac{3}{4}[x - (-1)]$$

$$y + 2 = \frac{3}{4}(x + 1)$$

In slope-point form, the equation of the line is:

$$y + 2 = \frac{3}{4}(x + 1)$$

b) $y + 2 = \frac{3}{4}(x + 1)$ Remove brackets.

$y + 2 = \frac{3}{4}x + \frac{3}{4}$ Solve for *y*.

$y = \frac{3}{4}x + \frac{3}{4} - 2$ Simplify.

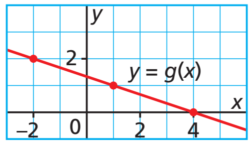
$$y = \frac{3}{4}x - \frac{5}{4}$$

In slope-intercept form, the equation of the line is: $y = \frac{3}{4}x - \frac{5}{4}$

From the equation, the *y*-intercept is $-\frac{5}{4}$.

CHECK YOUR UNDERSTANDING

2. a) Write an equation in slope-point form for this line.



b) Write the equation in part a in slope-intercept form. What is the *y*-intercept of this line?

[Answers: a) sample answer: $y - 1 = -\frac{1}{3}(x - 1)$

b) $y = -\frac{1}{3}x + \frac{4}{3}$; $\frac{4}{3}$]

The coordinates of another point on the line are (3, 1). Show that these coordinates produce the same equation in slope-intercept form.

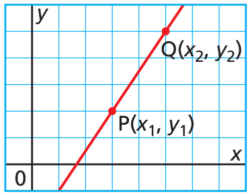
Explain how the general expression for the slope of a line can help you remember the equation $y - y_1 = m(x - x_1)$.

We can use the coordinates of two points that satisfy a linear function, $P(x_1, y_1)$ and $Q(x_2, y_2)$, to write an equation for the function.

We write the slope of the graph of the function in two ways:

$$m = \frac{y - y_1}{x - x_1} \quad \text{and} \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

So, an equation is: $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$



Example 3 Writing an Equation of a Linear Function Given Two Points

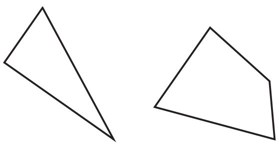
The sum of the angles, *s* degrees, in a polygon is a linear function of the number of sides, *n*, of the polygon. The sum of the angles in a triangle is 180°. The sum of the angles in a quadrilateral is 360°.

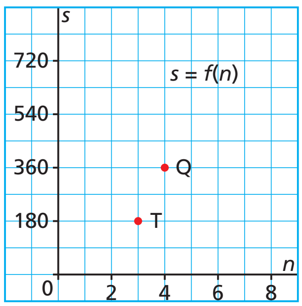
a) Write a linear equation to represent this function.

b) Use the equation to determine the sum of the angles in a dodecagon.

SOLUTION

a) $s = f(n)$, so two points on the graph have coordinates T(3, 180) and Q(4, 360)





Use this form for the equation of a linear function:

$$\frac{s - s_1}{n - n_1} = \frac{s_2 - s_1}{n_2 - n_1}$$ Substitute: $s_1 = 180$, $n_1 = 3$, $s_2 = 360$, and $n_2 = 4$

$$\frac{s - 180}{n - 3} = \frac{360 - 180}{4 - 3}$$ Simplify.

$$\frac{s - 180}{n - 3} = 180$$ Multiply each side by $(n - 3)$.

$$(n - 3)\left(\frac{s - 180}{n - 3}\right) = 180(n - 3)$$

$$s - 180 = 180(n - 3)$$ This is the slope-point form of the equation.

$$s - 180 = 180n - 540$$ Simplify.

$$s = 180n - 360$$ This is the slope-intercept form of the equation.

(Solution continues.)

CHECK YOUR UNDERSTANDING

3. A temperature in degrees Celsius, *c*, is a linear function of the temperature in degrees Fahrenheit, *f*. The boiling point of water is 100°C and 212°F. The freezing point of water is 0°C and 32°F.

a) Write a linear equation to represent this function.

b) Use the equation to determine the temperature in degrees Celsius at which iron melts, 2795°F.

[Answers: a) $c - 100 = \frac{5}{9}(f - 212)$, or $c = \frac{5}{9}f - \frac{160}{9}$ b) 1535°C]

Why is it possible for equations of a linear function to look different but still represent the same function?

b) A dodecagon has 12 sides.
Use:
$s = 180n - 360$ — Substitute: $n = 12$
$s = 180(12) - 360$
$s = 1800$
The sum of the angles in a dodecagon is 1800°.

In part b, why does it make sense to use the slope-intercept form instead of the slope-point form?

Example 4 Writing an Equation of a Line That Is Parallel or Perpendicular to a Given Line

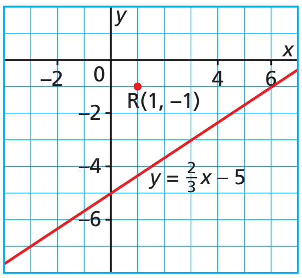
Write an equation for the line that passes through R(1, −1) and is:

a) parallel to the line $y = \frac{2}{3}x - 5$

b) perpendicular to the line $y = \frac{2}{3}x - 5$

SOLUTION

Sketch the line with equation: $y = \frac{2}{3}x - 5$, and mark a point at R(1, −1).



Compare the equation:
$y = \frac{2}{3}x - 5$ with the equation:
$y = mx + b$
The slope of the line is $\frac{2}{3}$.

a) Any line parallel to $y = \frac{2}{3}x - 5$ has slope $\frac{2}{3}$.

The required line passes through R(1, −1).

Use:

$y - y_1 = m(x - x_1)$ — Substitute: $y_1 = -1$, $x_1 = 1$, and $m = \frac{2}{3}$

$y - (-1) = \frac{2}{3}(x - 1)$ — Simplify.

$y + 1 = \frac{2}{3}(x - 1)$

The line that is parallel to the line $y = \frac{2}{3}x - 5$ and passes through R(1, −1) has equation: $y + 1 = \frac{2}{3}(x - 1)$

CHECK YOUR UNDERSTANDING

4. Write an equation for the line that passes through S(2, −3) and is:

a) parallel to the line $y = 3x + 5$

b) perpendicular to the line $y = 3x + 5$

[Answers: a) $y + 3 = 3(x - 2)$
b) $y + 3 = -\frac{1}{3}(x - 2)$]

What other strategies could you use to write an equation for each line?

Write each equation in slope-intercept form.

b) Any line perpendicular to $y = \frac{2}{3}x - 5$ has a slope that is the negative reciprocal of $\frac{2}{3}$; that is, its slope is $-\frac{3}{2}$.

The required line passes through R(1, −1).

Use:

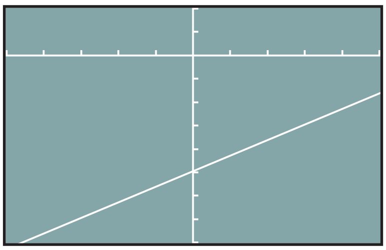
$$y - y_1 = m(x - x_1)$$ Substitute: $y_1 = -1$, $x_1 = 1$, and $m = -\frac{3}{2}$

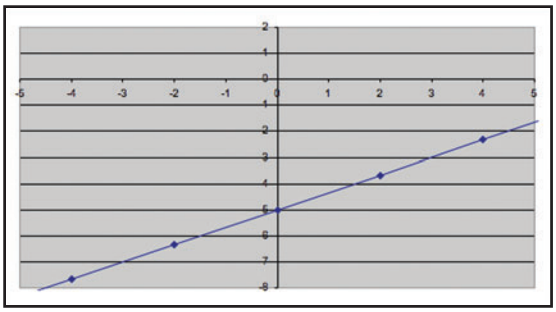
$$y - (-1) = -\frac{3}{2}(x - 1)$$ Simplify.

$$y + 1 = -\frac{3}{2}(x - 1)$$

The line that is perpendicular to the line $y = \frac{2}{3}x - 5$ and passes through the point R(1, −1) has equation: $y + 1 = -\frac{3}{2}(x - 1)$

To graph the equation of a linear function using technology, the equation needs to be rearranged to isolate y on the left side of the equation; that is, it must be in the form $y = f(x)$. So, if an equation is given in slope-point form, it must be rearranged before graphing. Here is the graph from *Example 4*, part a, on a graphing calculator and on a computer with graphing software.





Discuss the Ideas

1. How does the fact that the slope of a line is constant lead to the slope-point form of the equation of a line?
2. How can you use the slope-point form of the equation of a line to sketch a graph of the line?
3. How can you determine the slope-point form of the equation of a line given a graph of the line?

Exercises

A

4. For each equation, identify the slope of the line it represents and the coordinates of a point on the line.

a) $y - 5 = -4(x - 1)$
b) $y + 7 = 3(x - 8)$
c) $y + 11 = (x + 15)$
d) $y = 5(x - 2)$
e) $y + 6 = \frac{4}{7}(x + 3)$
f) $y - 21 = -\frac{8}{5}(x + 16)$

5. Write an equation for the graph of a linear function that:

a) has slope -5 and passes through P(-4, 2)
b) has slope 7 and passes through Q(6, -8)
c) has slope $-\frac{3}{4}$ and passes through R(7, -5)
d) has slope 0 and passes through S(3, -8)

6. Graph each line.

a) The line passes through T(-4, 1) and has slope 3.
b) The line passes through U(3, -4) and has slope -2.
c) The line passes through V(2, 3) and has slope $-\frac{1}{2}$.
d) The line has x-intercept -5 and slope $\frac{3}{4}$.

B

7. Describe the graph of the linear function with each equation, then graph the equation.

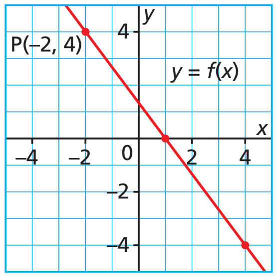
a) $y + 2 = -3(x - 4)$
b) $y + 4 = 2(x + 3)$
c) $y - 3 = (x + 5)$
d) $y = -(x - 2)$

8. A line passes through D(-3, 5) and has slope -4.

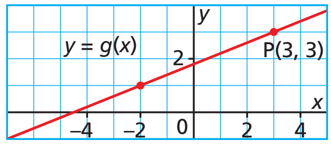
a) Why is $y - 5 = -4(x + 3)$ an equation of this line?
b) Why is $y = -4x - 7$ an equation of this line?

9. a) For each line, write an equation in slope-point form.

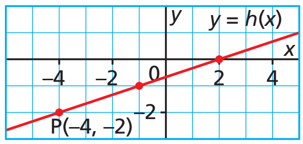
i)



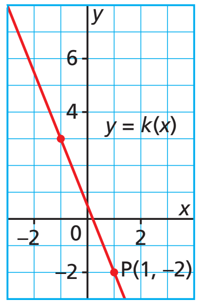
ii)



iii)



iv)



b) Write each equation in part a in slope-intercept form, then determine the x- and y-intercepts of each graph.

10. The speed of sound in air is a linear function of the air temperature. When the air temperature is 10°C, the speed of sound is 337 m/s. When the air temperature is 30°C, the speed of sound is 349 m/s.

a) Write a linear equation to represent this function.
b) Use the equation to determine the speed of sound when the air temperature is 0°C.

11. Write an equation for the line that passes through each pair of points. Write each equation in slope-point form and in slope-intercept form.

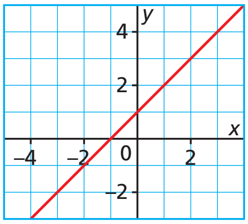
a) B(-2, -5) and C(1, 1)
b) Q(-4, 7) and R(5, -2)
c) U(-3, -7) and V(2, 8)
d) H(-7, -1) and J(-5, -5)

12. Which equation matches each graph? Describe each graph in terms of its slope and y-intercept.

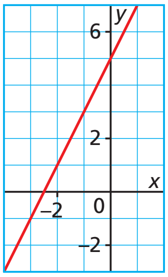
a) $y + 3 = 2(x - 1)$ **b)** $y - 3 = (x - 2)$

c) $y - 3 = 2(x + 1)$ **d)** $y + 3 = -(x + 2)$

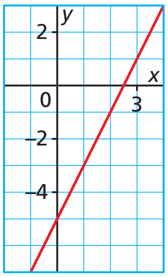
Graph A



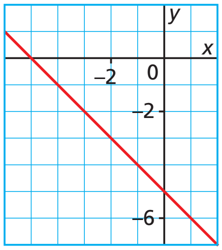
Graph B



Graph C



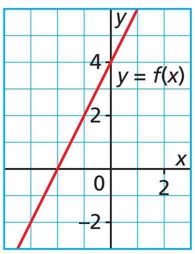
Graph D



13. How does the graph of $y + y_1 = m(x + x_1)$ compare with the graph of $y - y_1 = m(x - x_1)$? Include examples in your explanation.

14. Match each graph with its equation. Justify your choice.

a)



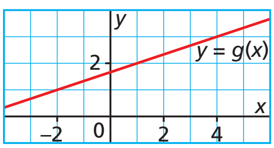
$y + 1 = 2(x - 2)$

$y + 2 = 2(x - 1)$

$y - 2 = 2(x + 1)$

$y + 1 = -2(x - 2)$

b)



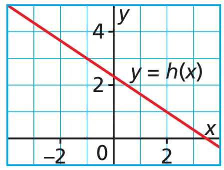
$y - 1 = \frac{1}{3}(x - 2)$

$y + 2 = \frac{1}{3}(x + 1)$

$y - 1 = 3(x - 2)$

$y - 2 = \frac{1}{3}(x - 1)$

c)



$y - 1 = \frac{2}{3}(x - 2)$

$y - 1 = \frac{3}{2}(x - 2)$

$y - 1 = -\frac{2}{3}(x - 2)$

$y - 2 = -\frac{2}{3}(x - 1)$

15. Use a graphing calculator or a computer with graphing software. Graph each equation. Sketch or print the graph. Write instructions that another student could follow to get the same display.

a) $y + \frac{2}{7} = \frac{3}{8}(x - 5)$

b) $y - \frac{10}{3} = -\frac{2}{9}(x + 11)$

c) $y + 1.4 = 0.375(x + 4)$

d) $y - 2.35 = -0.5(x - 6.3)$

16. Chloé conducted a science experiment where she poured liquid into a graduated cylinder, then measured the mass of the cylinder and liquid. Here are Chloé's data.

Volume of Liquid (mL)	Mass of Cylinder and Liquid (g)
10	38.9
20	51.5

a) When these data are graphed, what is the slope of the line and what does it represent?

b) Choose variables to represent the volume of the liquid, and the mass of the cylinder and liquid. Write an equation that relates these variables.

c) Use your equation to determine the mass of the cylinder and liquid when the volume of liquid is 30 mL.

d) Chloé forgot to record the mass of the empty graduated cylinder. Determine this mass. Explain your strategy.

17. In 2005, the Potash Corporation of Saskatchewan sold 8.2 million tonnes of potash. In 2007, due to increased demand, the corporation sold 9.4 million tonnes. Assume the mass of potash sold is a linear function of time.

a) Write an equation that describes the relation between the mass of potash and the time in years since 2005. Explain your strategy.

b) Predict the sales of potash in 2010 and 2015. What assumptions did you make?

18. In Alberta, the student population in francophone schools from January 2001 to January 2006 increased by approximately 198 students per year. In January 2003, there were approximately 3470 students enrolled in francophone schools.

a) Write an equation in slope-point form to represent the number of students enrolled in francophone schools as a function of the number of years after 2001.

b) Use the equation in part a to estimate the number of students in francophone schools in January 2005. Use a different strategy to check your answer.

19. A line passes through $G(-3, 11)$ and $H(4, -3)$.

a) Determine the slope of line GH.

b) Write an equation for line GH using point G and the slope.

c) Write an equation for line GH using point H and the slope.

d) Verify that the two equations are equivalent. What strategy did you use? What different strategy could you have used to verify that the equations are equivalent?

20. a) Write an equation for the line that passes through $D(-5, -3)$ and is:

i) parallel to the line $y = -\frac{4}{3}x + 1$

ii) perpendicular to the line $y = -\frac{4}{3}x + 1$

b) Compare the equations in part a. How are they alike? How are they different?

21. Write an equation for the line that passes through $C(1, -2)$ and is:

a) parallel to the line $y = 2x + 3$

b) perpendicular to the line $y = 2x + 3$

22. Write an equation for the line that passes through $E(2, 6)$ and is:

a) parallel to the line $y - 3 = -\frac{5}{2}(x + 2)$

b) perpendicular to the line $y - 3 = -\frac{5}{2}(x + 2)$

How do you know your equations are correct?

23. Write an equation for each line.

a) The line has x-intercept 4 and is parallel to the line with equation $y = \frac{3}{5}x - 7$.

b) The line passes through $F(4, -1)$ and is perpendicular to the line that has x-intercept -3 and y-intercept 6.

24. Two perpendicular lines intersect on the y-axis. One line has equation $y - 3 = \frac{2}{9}(x + 5)$. What is the equation of the other line?

25. Two perpendicular lines intersect at $K(-2, -5)$. One line has equation $y = -\frac{5}{3}x - \frac{25}{3}$. What is the equation of the other line?

C

26. Two perpendicular lines intersect at $M(3, 5)$. What might their equations be? How many possible pairs of equations are there?

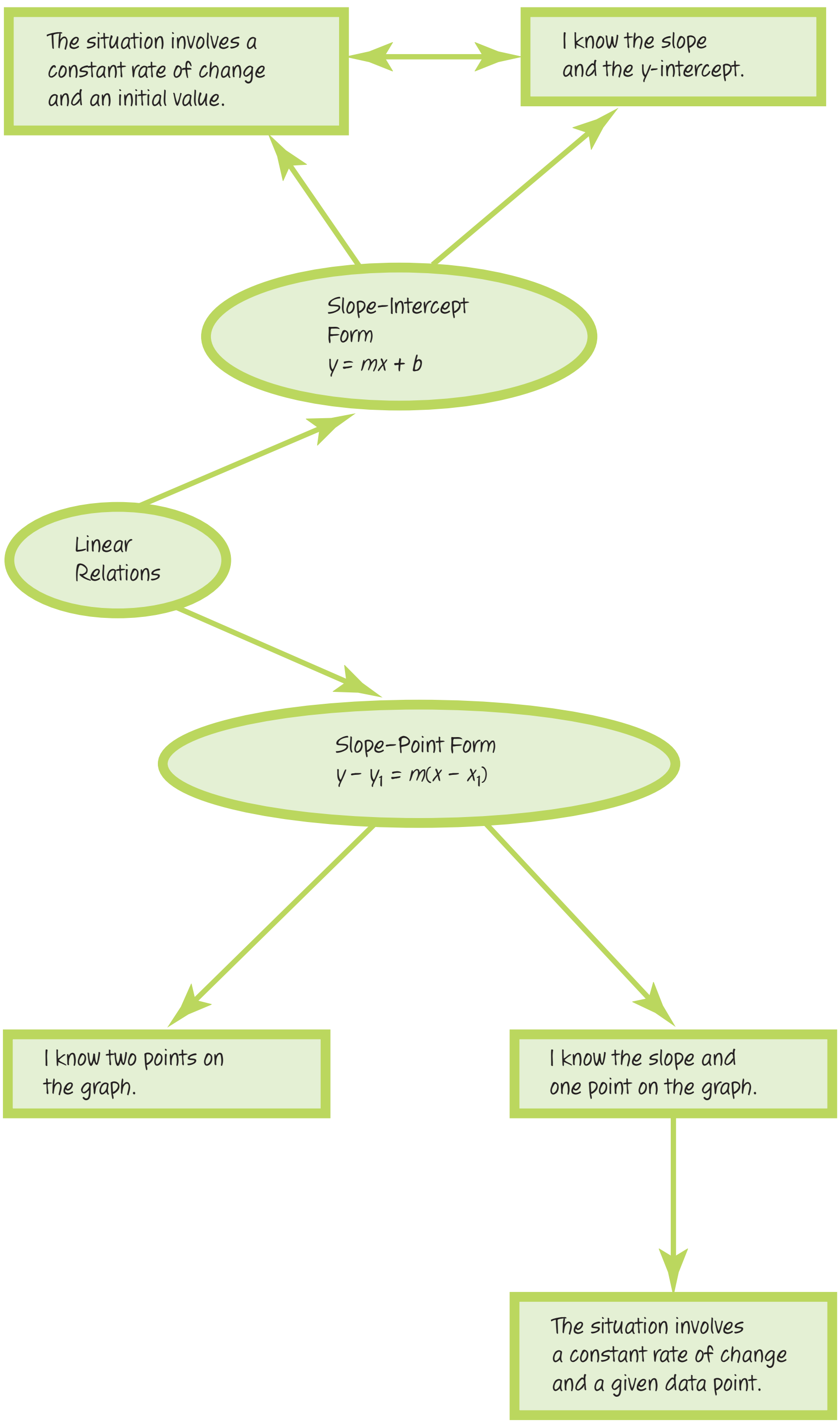
27. The slope-intercept form of the equation of a line is a special case of the slope-point form of the equation, where the point is at the y-intercept. Use the slope-point form to show that a line with slope m and intersecting the y-axis at b has equation $y = mx + b$.

Reflect

How is the slope-point form of the equation of a line different from the slope-intercept form? How would you use each form to graph a linear function? Include examples in your explanation.

CHECKPOINT 2

Connections



Concept Development

- **In Lesson 6.3**
  – You used technology to explore how changes in the constants m and b in the equation $y = mx + b$ affect the graph of the function.

- **In Lesson 6.4**
  – You used the slope and y-intercept of the graph of a linear function to write the equation of the function in slope-intercept form.
  – You graphed a linear function given its equation in slope-intercept form.
  – You used the graph of a linear function to write an equation for the function in slope-intercept form.

- **In Lesson 6.5**
  – You developed the slope-point form of the equation of a linear function.
  – You graphed a linear function given its equation in slope-point form.
  – You wrote the equation of a linear function after determining the slope of its graph and the coordinates of a point on its graph.
  – You wrote the equation of a linear function given the coordinates of two points on its graph.
  – You rewrote the equation of a linear function from slope-point form to slope-intercept form.

Assess Your Understanding

6.3

1. For the equation $y = \frac{3}{2}x - 4$:

a) Use a graphing calculator or a computer with graphing software to graph it.

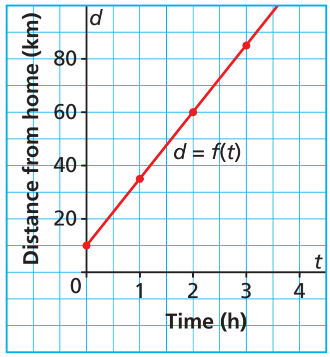
b) Explain how to change the equation so the line will have a greater slope, then a lesser slope. Make the change.

c) Explain how to change the equation so the line will have a greater y-intercept, then a lesser y-intercept. Make the change.

Sketch or print each graph.

6.4

2. This graph represents Eric's snowmobile ride.



a) Determine the slope and d-intercept. What does each represent?

b) Write an equation to represent the graph, then verify the equation.

c) Use the equation to answer each question below.

i) How far was Eric from home after he had travelled $2\frac{1}{4}$ hours?

ii) How long did it take Eric to travel 45 km from home?

6.5

3. Graph each line. Explain your strategy. Label each line with its equation.

a) $y + 2 = 3(x - 4)$

b) $y - 2 = -\frac{1}{2}(x - 6)$

c) The line passes through D$(-4, 7)$ and E$(6, -1)$.

d) The line passes through F$(4, -3)$ and is perpendicular to the line with equation $y + 4 = 2(x + 2)$.

e) The line passes through G$(-7, -2)$ and is parallel to the line that has x-intercept 5 and y-intercept 3.

4. A line has slope 2 and y-intercept 3.

a) Write an equation for this line using the slope-intercept form.

b) Write an equation for the line using the slope-point form.

c) Compare the two equations. How are they alike? How are they different?

6.6 General Form of the Equation for a Linear Relation

LESSON FOCUS

Relate the graph of a linear function to its equation in general form.

Make Connections

A softball team may field any combination of 9 female and male players. There must be at least one female and one male on the field at any time. What are the possible combinations for female and male players on the field?

Construct Understanding

TRY THIS

Work with a partner.

Holly works in a furniture plant. She takes 30 min to assemble a table and 15 min to assemble a chair. Holly works 8 h a day, not including meals and breaks.

A. Make a table of values for the possible numbers of tables and chairs that Holly could assemble in one day.

Number of Tables	Number of Chairs

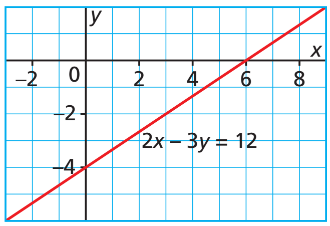
B. Graph the data. Use graphing technology if it is available. Describe the graph. What type of relation have you graphed? How do you know?

C. What do the intercepts represent?

D. Choose variables to represent the number of tables and number of chairs. Write an equation for your graph.

E. Suppose you interchanged the columns in the table, then graphed the data. How would the graph change? How would the equation change?

This graph is described by the equation $2x - 3y = 12$.



The equation $2x - 3y = 12$ is written in *standard form.*
The coefficients and constant terms are integers.
The x- and y-terms are on the left side of the equation, and the constant term is on the right side.
We may move the constant term to the left side of the equation:

$$\begin{aligned} 2x - 3y &= 12 \\ 2x - 3y - 12 &= 12 - 12 \\ 2x - 3y - 12 &= 0 \end{aligned}$$

The equation is now in **general form.**

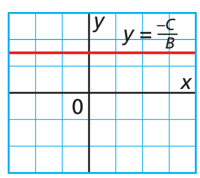
What values of A, B, C, would produce a vertical line? A horizontal line?

General Form of the Equation of a Linear Relation

$Ax + By + C = 0$ is the general form of the equation of a line, where A is a whole number, and B and C are integers.

Consider what happens to the general form of the equation in each of the following cases:

- When $A = 0$:
  $Ax + By + C = 0$ becomes
  $By + C = 0$ Solve for y.
  $By = -C$ Divide each side by B.
  $y = \frac{-C}{B}$

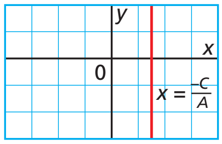


$\frac{-C}{B}$ is a constant, and the graph of $y = \frac{-C}{B}$ is a horizontal line.

- When $B = 0$:

$Ax + By + C = 0$ becomes

$$Ax + C = 0 \quad \text{Solve for } x.$$
$$Ax = -C \quad \text{Divide each side by } A.$$
$$x = \frac{-C}{A}$$



$\frac{-C}{A}$ is a constant, and the graph of $x = \frac{-C}{A}$ is a vertical line.

Example 1 Rewriting an Equation in General Form

Write each equation in general form.

a) $y = -\frac{2}{3}x + 4$ **b)** $y - 1 = \frac{3}{5}(x + 2)$

SOLUTION

a)

$$y = -\frac{2}{3}x + 4 \quad \text{Multiply each side by 3.}$$
$$3y = 3\left(-\frac{2}{3}x + 4\right) \quad \text{Remove the brackets.}$$
$$3y = 3\left(-\frac{2}{3}x\right) + 3(4)$$
$$3y = -2x + 12 \quad \text{Collect all the terms on the left side of the equation.}$$
$$2x + 3y - 12 = 0 \quad \text{This is the general form of the equation.}$$

b)

$$y - 1 = \frac{3}{5}(x + 2) \quad \text{Multiply each side by 5.}$$
$$5(y - 1) = 5\left(\frac{3}{5}\right)(x + 2) \quad \text{Remove the brackets.}$$
$$5y - 5 = 3(x + 2)$$
$$5y - 5 = 3x + 6 \quad \text{Collect like terms.}$$
$$5y = 3x + 11 \quad \text{Collect all the terms on the right side of the equation.}$$
$$0 = 3x - 5y + 11$$

The general form of the equation is: $3x - 5y + 11 = 0$

CHECK YOUR UNDERSTANDING

1. Write each equation in general form.

a) $y = -\frac{1}{4}x + 3$

b) $y + 2 = \frac{3}{2}(x - 4)$

[Answers: a) $x + 4y - 12 = 0$
b) $3x - 2y - 16 = 0$]

For the two equations, why were the terms collected on different sides of the equation?

When an equation of a line is written in general form, can all the terms be positive? Can all the terms be negative? Explain.

Example 2 Graphing a Line in General Form

a) Determine the x- and y-intercepts of the line whose equation is: $3x + 2y - 18 = 0$

b) Graph the line.

c) Verify that the graph is correct.

SOLUTION

a) To determine the x-intercept:

$$3x + 2y - 18 = 0 \qquad \text{Substitute: } y = 0$$
$$3x + 2(0) - 18 = 0 \qquad \text{Solve for } x.$$
$$3x = 18$$
$$\frac{3x}{3} = \frac{18}{3}$$
$$x = 6$$

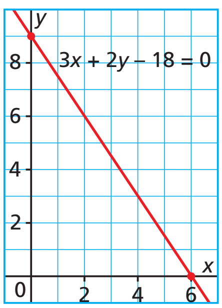
The x-intercept is 6 and is described by the point (6, 0).

To determine the y-intercept:

$$3x + 2y - 18 = 0 \qquad \text{Substitute: } x = 0$$
$$3(0) + 2y - 18 = 0 \qquad \text{Solve for } y.$$
$$2y = 18$$
$$\frac{2y}{2} = \frac{18}{2}$$
$$y = 9$$

The y-intercept is 9 and is described by the point (0, 9).

b) On a grid, plot the points that represent the intercepts. Draw a line through the points.



CHECK YOUR UNDERSTANDING

2. a) Determine the x- and y-intercepts of the line whose equation is: $x + 3y + 9 = 0$

   b) Graph the line.

   c) Verify that the graph is correct.

   [Answer: a) −9, −3]

Why is it a good idea to check that the graph is correct when you use the intercepts to draw the graph?

c) The point T(2, 6) appears to be on the graph.
Verify that T(2, 6) satisfies the equation.
Substitute $x = 2$ and $y = 6$ in the equation $3x + 2y - 18 = 0$.

$$\begin{aligned} \text{L.S.} &= 3x + 2y - 18 \\ &= 3(2) + 2(6) - 18 \\ &= 6 + 12 - 18 \\ &= 0 \end{aligned} \qquad \text{R.S.} = 0$$

Since the left side is equal to the right side, the point satisfies the equation and the graph is probably correct.

Example 3 Determining the Slope of a Line Given Its Equation in General Form

Determine the slope of the line with this equation:
$3x - 2y - 16 = 0$

SOLUTION

Rewrite the equation in slope-intercept form.

$$\begin{aligned} 3x - 2y - 16 &= 0 && \text{Solve for } y\text{. Subtract } 3x \text{ from each side.} \\ -2y - 16 &= -3x && \text{Add 16 to each side.} \\ -2y &= -3x + 16 && \text{Divide each side by } -2. \\ y &= \frac{-3x}{-2} + \frac{16}{-2} \\ y &= \frac{3}{2}x - 8 \end{aligned}$$

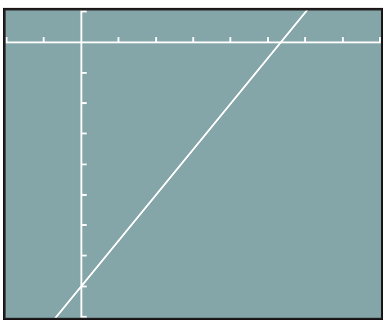
From the equation, the slope of the line is $\frac{3}{2}$.

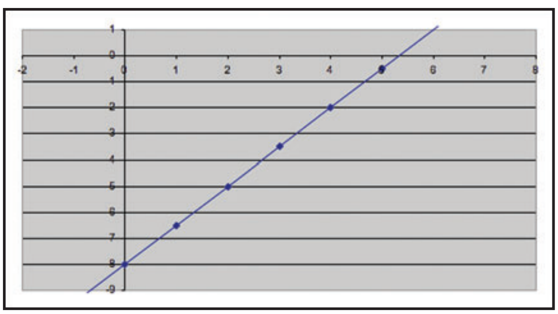
CHECK YOUR UNDERSTANDING

3. Determine the slope of the line with this equation:
$5x - 2y + 12 = 0$

[Answer: $\frac{5}{2}$]

If an equation is given in general form, it must be rearranged to the form $y = f(x)$ before graphing using technology. Here is the graph from *Example 3* on a graphing calculator and on a computer with graphing software.





Example 4 Determining an Equation from a Graph of Generated Data

Peanuts cost \$2 per 100 g and raisins cost \$1 per 100 g.
Devon has \$10 to purchase both these items.

a) Generate some data for this relation.

b) Graph the data.

c) Write an equation for the relation in general form.

d) **i)** Will Devon spend exactly \$10 if she buys 300 g of peanuts and 400 g of raisins?

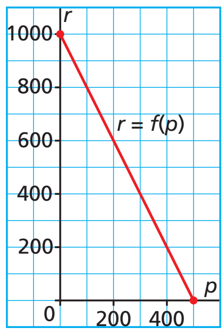
ii) Will Devon spend exactly \$10 if she buys 400 g of peanuts and 300 g of raisins?

Use the graph and the equation to justify the answers.

SOLUTION

a) If Devon buys only peanuts at \$2 for 100 g, she can buy 500 g.
If Devon buys only raisins at \$1 for 100 g, she can buy 1000 g.
If Devon buys 200 g of peanuts, they cost \$4; so she can buy 600 g of raisins for \$6.

Mass of Peanuts, p (g)	Mass of Raisins, r (g)
500	0
0	1000
200	600



b) Join the points because Devon can buy any mass of items she likes.

c) Use the coordinates of two points on the line: (500, 0) and (0, 1000)
Use the slope-point form with these coordinates:

$$\frac{r - r_1}{p - p_1} = \frac{r_2 - r_1}{p_2 - p_1}$$

Substitute: $r_1 = 0$, $p_1 = 500$, $r_2 = 1000$, and $p_2 = 0$

$$\frac{r - 0}{p - 500} = \frac{1000 - 0}{0 - 500}$$

$$\frac{r}{p - 500} = -2$$

Multiply each side by $(p - 500)$.

$$r = -2(p - 500)$$

$$r = -2p + 1000$$

Collect all the terms on the left side of the equation.

$$2p + r - 1000 = 0$$

CHECK YOUR UNDERSTANDING

4. Akeego is making a ribbon shirt. She has 60 cm of ribbon that she will cut into 5 pieces with 2 different lengths: 2 pieces have the same length and the remaining 3 pieces also have equal lengths.

a) Generate some data for this relation showing the possible lengths of the pieces.

b) Graph the data.

c) Write an equation for the relation in general form.

d) **i)** Can each of 2 pieces be 18 cm long and each of 3 pieces be 3 cm long?

ii) Can each of 2 pieces be 3 cm long and each of 3 pieces be 18 cm long?

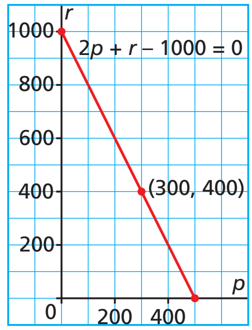
Use the graph and the equation to justify your answers.

[Sample Answers: a) (2, 27), (4, 24), (6, 21) c) $3x + 2y - 60 = 0$
d) i) no ii) yes]

What other strategies could you use to determine the equation of the line?

Suppose you interchanged the coordinates and graphed $p = f(r)$. How would the graph change? How would the equation change?

d) i) Use the graph to determine whether Devon will spend exactly \$10 if she buys 300 g of peanuts and 400 g of raisins.
300 g of peanuts and 400 g of raisins are represented by the point (300, 400).
Plot this point on the grid. Since this point lies on the line, Devon can buy these masses of peanuts and raisins.

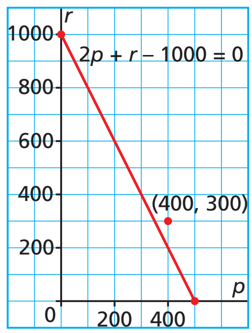


Check whether the point (300, 400) satisfies the equation:
$2p + r - 1000 = 0$
Substitute: $p = 300$ and $r = 400$

$$\begin{aligned} \text{L.S.} &= 2p + r - 1000 \qquad\qquad \text{R.S.} = 0 \\ &= 2(300) + 400 - 1000 \\ &= 0 \end{aligned}$$

Since the left side is equal to the right side, the point (300, 400) does satisfy the equation.

ii) Use the graph to determine whether Devon will spend exactly \$10 if she buys 400 g of peanuts and 300 g of raisins.
400 g of peanuts and 300 g of raisins are represented by the point (400, 300). Plot this point on the grid.
Since this point does not lie on the line, Devon cannot buy these masses of peanuts and raisins.



Check whether the point (400, 300) satisfies the equation:
$2p + r - 1000 = 0$
Substitute: $p = 400$ and $r = 300$

$$\begin{aligned} \text{L.S.} &= 2p + r - 1000 \qquad\qquad \text{R.S.} = 0 \\ &= 2(400) + 300 - 1000 \\ &= 100 \end{aligned}$$

Since the left side is not equal to the right side, the point (400, 300) does not satisfy the equation.

Discuss the Ideas

1. What steps would you use to sketch the graph of a linear relation in general form?

2. Is it easier to graph a linear relation with its equation in general form or slope-intercept form? Use examples to support your opinion.

3. An equation in general form may be rewritten in slope-intercept form. How is this process like solving a linear equation?

Exercises

A

4. In which form is each equation written?

a) $8x - 3y = 52$ **b)** $9x + 4y + 21 = 0$

c) $y = 4x + 7$ **d)** $y - 3 = 5(x + 7)$

5. Determine the x-intercept and the y-intercept for the graph of each equation.

a) $8x - 3y = 24$ **b)** $7x + 8y = 56$

c) $4x - 11y = 88$ **d)** $2x - 9y = 27$

6. Write each equation in general form.

a) $4x + 3y = 36$ **b)** $2x - y = 7$

c) $y = -2x + 6$ **d)** $y = 5x - 1$

7. Graph each line.

a) The x-intercept is 2 and the y-intercept is -3.

b) The x-intercept is -6 and the y-intercept is 2.

B

8. a) Explain how you can tell that each equation is not written in general form.

i) $-2x + 3y + 42 = 0$

ii) $4y - 5x = 100$

iii) $\frac{1}{2}x - \frac{1}{2}y + 1 = 0$

iv) $5y + 9x - 20 = 0$

b) Write each equation in part a in general form.

9. For each equation below:

i) Determine the x- and y-intercepts of the graph of the equation.

ii) Graph the equation.

iii) Verify that the graph is correct.

a) $3x - 4y = 24$ **b)** $6x - 5y = -60$

c) $3x - 2y = 24$ **d)** $5x - y = 10$

10. Two numbers, f and s, have a sum of 12.

a) Generate some data for this relation.

b) Graph the data. Should you join the points? Explain.

c) Write an equation in general form to relate f and s.

d) Use the graph to list 6 pairs of integers that have a sum of 12.

11. Rebecca makes and sells Nanaimo bars. She uses pans that hold 12 bars or 36 bars. Rebecca uses these pans to fill an order for 504 Nanaimo bars.

a) Generate some data for this relation, then graph the data.

b) Choose letters to represent the variables, then write an equation for the relation.

12. Write each equation in slope-intercept form.

a) $4x + 3y - 24 = 0$ **b)** $3x - 8y + 12 = 0$

c) $2x - 5y - 15 = 0$ **d)** $7x + 3y + 10 = 0$

13. Determine the slope of the line with each equation. Which strategy did you use each time?

a) $4x + y - 10 = 0$ **b)** $3x - y + 33 = 0$

c) $5x - y + 45 = 0$ **d)** $10x + 2y - 16 = 0$

14. Graph each equation on grid paper. Which strategy did you use each time?

a) $x - 2y + 10 = 0$ **b)** $2x + 3y - 15 = 0$

c) $7x + 4y + 4 = 0$ **d)** $6x - 10y + 15 = 0$

15. A pipe for a central vacuum is to be 96 ft. long. It will have s pipes each 6 ft. long and e pipes each 8 ft. long. This equation describes the relation: $6s + 8e = 96$

a) Suppose 4 pieces of 6-ft. pipe are used. How many pieces of 8-ft. pipe are needed?

b) Suppose 3 pieces of 8-ft. pipe are used. How many pieces of 6-ft. pipe are needed?

c) Could 3 pieces of 6-ft. pipe be used? Justify your answer.

d) Could 4 pieces of 8-ft. pipe be used? Justify your answer.

16. Pascal saves loonies and toonies. The value of his coins is \$24.

a) Generate some data for this relation.

b) Graph the data. Should you join the points? Explain.

c) Write an equation to relate the variables. Justify your choice for the form of the equation.

d) i) Could Pascal have 6 toonies and 8 loonies?

ii) Could Pascal have 6 loonies and 8 toonies?

Use the graph and the equation to justify your answers.

17. Use a graphing calculator or a computer with graphing software. Graph each equation. Sketch or print the graph.

a) $x - 22y - 15 = 0$ b) $15x + 13y - 29 = 0$

c) $33x + 2y + 18 = 0$ d) $34x - y + 40 = 0$

18. Write each equation in general form.

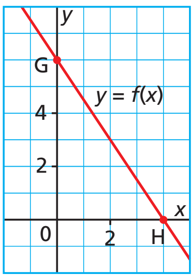
a) $y = \frac{1}{3}x - 4$ b) $y - 2 = \frac{1}{3}(x + 5)$

c) $y + 3 = -\frac{1}{4}(x - 1)$ d) $y = -\frac{3}{2}x + \frac{4}{3}$

19. Choose one equation from question 18. Write it in 2 different forms. Graph the equation in each of its 3 forms. Compare the graphs.

20. Describe the graph of $Ax + By + C = 0$, when $C = 0$. Include a sketch in your answer.

21. a) How are the x- and y-intercepts of this line related to the slope of the line? Justify your answer.

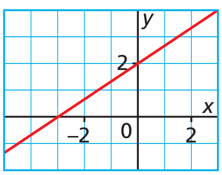


b) Is the relationship in part a true for all lines? Explain how you know.

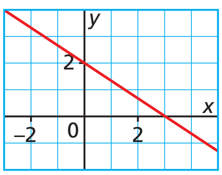
22. Match each equation with its graph. Justify your answer.

a) $2x + 3y - 6 = 0$ b) $2x - 3y + 6 = 0$

Graph A



Graph B



23. a) Why can't you use intercepts to graph the equation $4x - y = 0$?

b) Use a different strategy to graph the equation. Explain your steps.

24. Which equations below are equivalent? How did you find out?

a) $y = 3x + 6$ b) $2x - 3y - 3 = 0$

c) $y - 2 = \frac{2}{3}(x - 2)$ d) $3x - y - 6 = 0$

e) $y = \frac{2}{3}x - 1$ f) $y - 3 = 3(x - 3)$

g) $y - 1 = \frac{2}{3}(x - 3)$ h) $y + 3 = 3(x - 1)$

25. a) Write the equation of a linear function in general form that would be difficult to graph by determining its intercepts. Why is it difficult?

b) Use a different strategy to graph your equation. How did your strategy help you graph the equation?

C

26. If an equation of a line cannot be written in general form, the equation does not represent a linear function. Write each equation in general form, if possible. Indicate whether each equation represents a linear function.

a) $\frac{x}{4} + \frac{y}{3} = 1$

b) $y = \frac{10}{x}$

c) $y = 2x(x + 4)$

d) $y = \frac{x + y}{4} + 2$

27. Suppose you know the x- and y-intercepts of a line. How can you write an equation to describe the line without determining the slope of the line? Use the line with x-intercept 5 and y-intercept -3 to describe your strategy.

28. The general form for the equation of a line is: $Ax + By + C = 0$

a) Write an expression for the slope of the line in terms of A, B, and C.

b) Write an expression for the y-intercept in terms of A, B, and C.

Reflect

Describe a situation that can be most appropriately modelled with the equation of a linear relation in general form. Show that different forms of this equation represent the same graph.

STUDY GUIDE

CONCEPT SUMMARY

Big Ideas	Applying the Big Ideas
■ The graph of a linear function is a non-vertical straight line with a constant slope.	This means that: ■ The slope of a line is equal to the slope of any segment of the line. ■ When we know the slope of a line, we also know the slope of a parallel line and a perpendicular line.
■ Certain forms of the equation of a linear function identify the slope and y-intercept of the graph, or the slope and the coordinates of a point on the graph.	■ When the equation is written in the form $y = mx + b$, the slope of the line is m and its y-intercept is b. ■ When the equation is written in the form $y - y_1 = m(x - x_1)$, the slope of the line is m and the coordinates of a point on the line are (x_1, y_1). ■ Any equation can be written in the general form $Ax + By + C = 0$, where A is a whole number, and B and C are integers.

Reflect on the Chapter

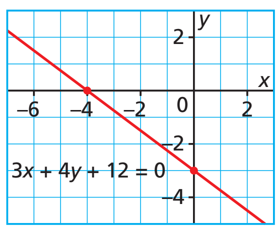
- What information do you need to know about a linear function to be able to write an equation to describe it? Include examples in your explanation.
- For each form of the equation of a linear function, describe how you would graph the function.

THE WORLD OF MATH

Careers: Marketing

Marketing involves understanding consumers' needs and buying habits. For a company to be successful, it must ensure that its product meets consumers' needs and can be produced and sold at prices that ensure the company makes a profit. To understand the market, research is conducted, then data are analyzed and used to make predictions. Often, these data will be used to produce linear models to solve problems.

SKILLS SUMMARY

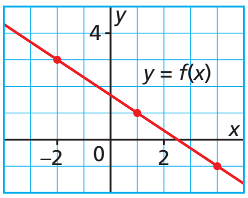
Skill	Description	Example
Determine the slope of a line and identify parallel lines and perpendicular lines. [6.1, 6.2]	A line that passes through $P(x_1, y_1)$ and $Q(x_2, y_2)$ has slope, m, where: $m = \frac{y_2 - y_1}{x_2 - x_1}$	For $P(-2, 4)$ and $Q(2, -1)$: $m = \frac{-1 - 4}{2 - (-2)}$, or $-\frac{5}{4}$ The slope of a line parallel to PQ is $-\frac{5}{4}$. The slope of a line perpendicular to PQ is $\frac{4}{5}$.
Write the equation of a line in slope-intercept form. [6.4]	A line with slope, m, and y-intercept, b, has equation: $y = mx + b$	For a line with slope $-\frac{5}{4}$ and y-intercept 3, an equation is: $y = -\frac{5}{4}x + 3$
Write the equation of a line in slope-point form. [6.5]	A line with slope, m, and passing through $P(x_1, y_1)$, has equation: $y - y_1 = m(x - x_1)$	A line with slope $\frac{4}{5}$ and passing through $P(-2, 4)$ has equation: $y - 4 = \frac{4}{5}(x - (-2))$, or $y - 4 = \frac{4}{5}(x + 2)$
Graph a linear relation in general form. [6.6]	The general form of the equation is: $Ax + By + C = 0$ Determine intercepts by substituting: $x = 0$, and solving for y, then $y = 0$, and solving for x. Plot points at the intercepts, then draw a line through the points.	A line has equation: $3x + 4y + 12 = 0$ For the y-intercept: $3(0) + 4y + 12 = 0$ $4y = -12$ $y = -3$ For the x-intercept: $3x + 4(0) + 12 = 0$ $3x = -12$ $x = -4$ 

REVIEW

6.1

1. Determine the slope of each line.

a)



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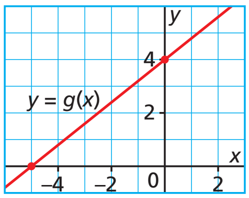


2. For each line described below, is its slope positive, negative, zero, or undefined? Justify your answer.

a) As x increases by 3, y decreases by 2.

b) The line has a negative x-intercept and a negative y-intercept.

c) The line has a y-intercept but does not have an x-intercept.

3. A line passes through $A(-3, 1)$. For each slope given below:

i) Sketch the line through A with that slope.

ii) Write the coordinates of three other points on the line.

a) -1 **b)** $\frac{1}{4}$ **c)** $-\frac{3}{2}$

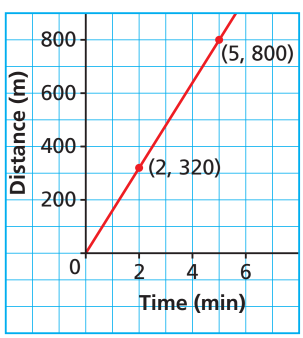
4. Determine the slope of a line that passes through each pair of points. What strategy did you use?

a) $B(-6, 8)$ and $C(-1, -2)$

b) $D(-3, 7)$ and $E(5, -5)$

5. Gabrielle likes to jog and has a pedometer to measure how far she runs. She checks her pedometer periodically and records its readings. Gabrielle plotted these data on a grid.

Graph of Gabrielle's Run



a) What is the slope of the line and what does it represent?

b) How is slope related to rate of change?

c) Assume Gabrielle continues to run at the same rate.

i) How far did Gabrielle jog in 4 min?

ii) How long will it take Gabrielle to jog 1000 m?

6.2

6. The slope of line FG is given. What is the slope of a line that is:

i) parallel to FG? **ii)** perpendicular to FG?

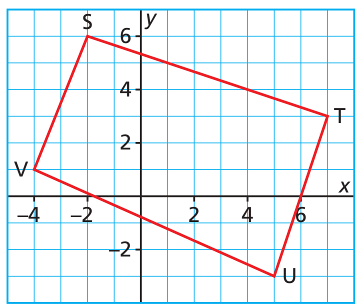
a) 3 **b)** $-\frac{6}{5}$ **c)** $\frac{11}{8}$ **d)** 1

7. The coordinates of two points on two lines are given. Are the two lines parallel, perpendicular, or neither? Justify your choice.

a) $H(-3, 3)$, $J(-1, 7)$ and $K(-1, 2)$, $M(5, -1)$

b) $N(-4, -2)$, $P(-1, 7)$ and $Q(2, 5)$, $R(4, -1)$

8. Is quadrilateral STUV a parallelogram? Justify your answer.



9. Triangle ABC has vertices $A(-1, -1)$, $B(2, 5)$, and $C(6, 3)$. Is $\triangle ABC$ a right triangle? Justify your answer.

6.3

10. Sketch graphs to help explain what happens to the graph of $y = 3x + 4$ when:

a) the coefficient of x increases by 1 each time until the coefficient is 6

b) the constant term decreases by 1 each time until it is -4

6.4

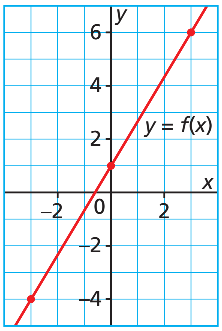
11. For each equation, identify the slope and y-intercept of its graph, then draw the graph.

a) $y = -3x + 4$ **b)** $y = \frac{3}{4}x - 2$

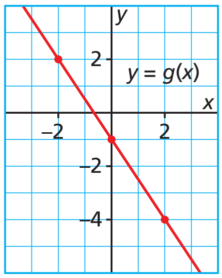
12. For each graph below:

i) Determine its slope and y-intercept.
ii) Write an equation that describes the graph.
iii) Verify your equation.

a)



b)

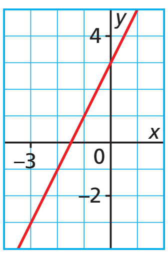


13. Match each equation with its graph. Explain your strategy.

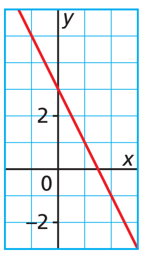
a) $y = \frac{1}{2}x - 3$ **b)** $y = -3x - 2$

c) $y = 2x + 3$ **d)** $y = -2x + 3$

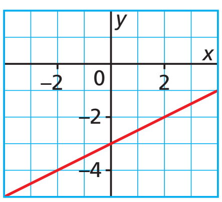
Graph A



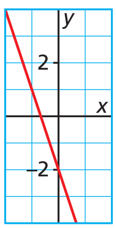
Graph B



Graph C



Graph D



14. Mason had \$40 in his bank account when he started to save \$15 each week.

a) Write an equation to represent the total amount, A dollars, he had in his account after w weeks.

b) After how many weeks did Mason have \$355 in his account?

c) Suppose you graphed the equation you wrote in part a. What would the slope and the vertical intercept of the graph represent?

15. Consider the graph of $y = \frac{4}{7}x - 5$.

a) Write 2 equations that describe 2 different lines that are parallel to this line. How do you know all 3 lines are parallel?

b) Write 2 equations that describe 2 different lines that are perpendicular to this line. How do you know that the 2 new lines are perpendicular to the original line?

6.5

16. Line DE passes through F$(-2, 3)$ and is perpendicular to the line described by the equation $y = 2x + 1$. Write an equation for line DE.

17. For each equation below:

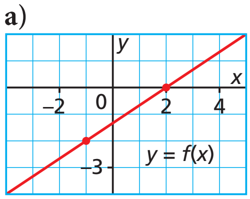
i) Identify the slope of its graph and the coordinates of a point on the graph.
ii) Graph the equation.
iii) Choose a different point on the graph, then write its equation in a different way.

a) $y + 4 = 2(x + 3)$

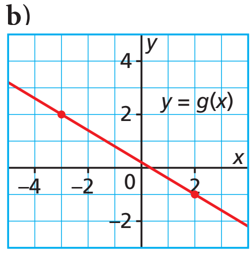
b) $y - 1 = -\frac{1}{3}(x - 4)$

18. Write an equation for each graph. Describe your strategy. Verify that the equation is correct.

a)



b)



19. a) Write an equation for the line that passes through each pair of points. Describe your strategy.
   i) $G(-3, -7)$ and $H(1, 5)$
   ii) $J(-3, 3)$ and $K(5, -1)$
   b) Use each equation you wrote in part a to determine the coordinates of another point on each line.

20. Two families went on a traditional nuuchahnulth dugout canoe tour in Tofino harbour, B.C. One family paid \$220 for 5 people. The other family paid \$132 for 3 people.
   a) Choose variables, then write an equation for the cost as a function of the number of people. Explain your strategy.
   b) What is the cost per person? How can you determine this from the equation?
   c) A third family paid \$264. How many people went on the tour?

6.6

21. a) Why is each equation not in general form?
   i) $4y - 5x - 40 = 0$ ii) $\frac{1}{3}x + y = 4$
   iii) $y - 2 = \frac{1}{3}(x + 4)$ iv) $y = \frac{1}{5}x + 3$
   b) Write each equation in part a in general form.

22. a) Graph each equation. Describe the strategies you used.
   i) $3x - 4y - 24 = 0$ ii) $x - 3y + 12 = 0$
   b) What is the slope of each line in part a? How did you determine the slopes?

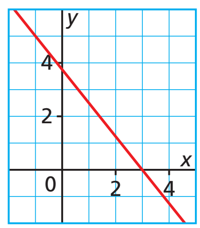
23. Write the equation of a line in general form that you could not easily graph by using intercepts. Choose another strategy to graph the equation, and explain why you used that strategy.

24. The difference between two numbers, g and l, is 6.
   a) Generate some data for this relation, then graph the data.
   b) Write an equation in general form to relate g and l.
   c) Use the graph to list 5 pairs of numbers that have a difference of 6.

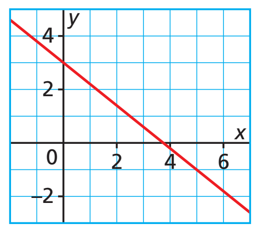
25. Which equations are equivalent? How did you determine your answers?
   a) $y = \frac{2}{5}x + 1$ b) $y - 3 = \frac{2}{5}(x - 4)$
   c) $y - 1 = \frac{2}{5}(x - 1)$ d) $y - 3 = \frac{2}{5}(x - 5)$
   e) $2x - 5y + 7 = 0$ f) $2x - 5y - 5 = 0$

26. Match each equation with its graph below. Justify each choice.
   a) $y = -\frac{4}{5}x + 3$ b) $y - 3 = -\frac{4}{5}(x + 3)$
   c) $5x + 4y - 15 = 0$

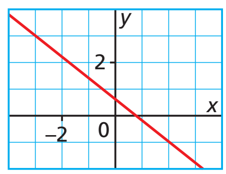
Graph A



Graph B



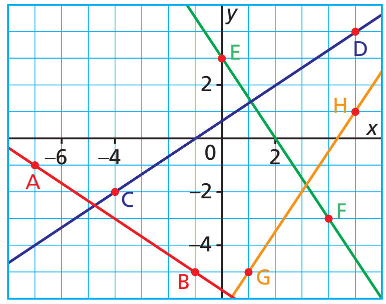
Graph C



27. Max babysits for 2 families. One family pays him \$5 an hour, the other family pays \$4 an hour. Last week, Max earned \$60.
   a) Generate some data for this relation, then graph the data.
   b) Write an equation for the relation. Explain what each variable represents.

28. A video store charges \$5 to rent a new release and \$3 to rent an older movie. Kylie spent \$45 renting movies last month.
   a) Generate some data for this relation, graph the data, then write an equation.
   b) i) Could Kylie have rented 5 new releases and 6 old movies?
      ii) Could Kylie have rented 6 new releases and 5 old movies?
   Justify your answers.

PRACTICE TEST



For questions 1 and 2, choose the correct answer: A, B, C, or D

1. Which line at the right has slope $-\frac{3}{2}$?

A. AB **B.** CD **C.** EF **D.** GH

2. Which line at the right has equation $2x - 3y + 2 = 0$?

A. AB **B.** CD **C.** EF **D.** GH

3. a) Graph each line. Explain your strategies.

i) $y = -\frac{3}{2}x + 5$ **ii)** $y - 3 = \frac{1}{3}(x + 2)$ **iii)** $3x - 4y - 12 = 0$

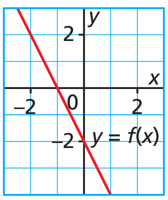
b) Determine an equation of the line that is parallel to the line with equation $y = -\frac{3}{2}x + 5$, and passes through A(6, 2). Explain how you know your equation is correct.

c) Determine an equation of the line that is perpendicular to the line with equation $y - 3 = \frac{1}{3}(x + 2)$, and passes through B(−1, 2). Write the new equation in general form.

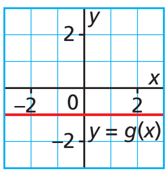
d) Determine the coordinates of a point P on the line with equation $3x - 4y - 12 = 0$. Do not use an intercept. Write an equation of the line that passes through P and Q(1, 5). Write the new equation in slope-intercept form.

4. Write the equation of each line in the form that you think best describes the line. Justify your choice.

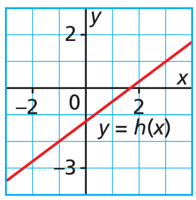
a)



b)



c)



5. Sophia is planning the graduation banquet. The caterer charges a fixed amount plus an additional charge for each person who attends. The banquet will cost \$11 250 if 600 people attend and \$7650 if 400 people attend.

a) Suppose 340 people attend the banquet. What will the total cost be?

b) The total cost was \$9810. How many people attended the banquet?

c) What strategies did you use to answer parts a and b?