

Worksheet 1. What You Need to Know About Motion Along the x -axis (Part 1)

In discussing motion, there are three closely related concepts that you need to keep straight. These are:

If $x(t)$ represents the position of a particle along the x -axis at any time t , then the following statements are true.

1. “Initially” means when _____ = 0.
2. “At the origin” means _____ = 0.
3. “At rest” means _____ = 0.
4. If the velocity of the particle is positive, then the particle is moving to the _____.
5. If the velocity of the particle is _____, then the particle is moving to the left.
6. To find average velocity over a time interval, divide the change in _____ by the change in time.
7. Instantaneous velocity is the velocity at a single moment (instant!) in time.
8. If the acceleration of the particle is positive, then the _____ is increasing.
9. If the acceleration of the particle is _____, then the velocity is decreasing.
10. In order for a particle to change direction, the _____ must change signs.
11. One way to determine total distance traveled over a time interval is to find the sum of the absolute values of the differences in position between all resting points. Here’s an example: If the position of a particle is given by:

$$x(t) = \frac{1}{3}t^3 - t^2 - 3t + 4,$$

find the total distance traveled on the interval $0 \leq t \leq 6$.

Worksheet 1. Solutions and Notes for Students

The three concepts are as follows.

Position: $x(t)$ —determines where the particle is located on the x -axis at a given time t

Velocity: $v(t) = x'(t)$ —determines how fast the position is changing at a time t as well as the direction of movement

Acceleration: $a(t) = v'(t) = x''(t)$ —determines how fast the velocity is changing at time t ; the sign indicates if the velocity is increasing or decreasing

The true statements are as follows.

1. “Initially” means when **time, t** = 0.
2. “At the origin” means **position, $x(t)$** = 0.
3. “At rest” means **velocity, $v(t)$** = 0.
4. If the velocity of the particle is positive, then the particle is moving to the **right**.
5. If the velocity of the particle is **negative**, then the particle is moving to the left.
6. To find average velocity over a time interval, divide the change in **position** by the change in time.
7. Instantaneous velocity is the velocity at a single moment (instant!) in time.
8. If the acceleration of the particle is positive, then the **velocity** is increasing.
9. If the acceleration of the particle is **negative**, then the velocity is decreasing.
10. In order for a particle to change direction, the **velocity** must change signs.
11. First, find the times at which $x'(t) = v(t) = 0$. That would be $t = -1$ (which is out of our interval) and $t = 3$. Next, evaluate the position at the end points and at each of the “resting” points. The particle moved to the left 9 units and then to the right by 27 units for a total distance traveled of 36 units. Point out to students how this is similar to a closed interval test, where you have to determine function values at the end points as well as at any critical points found.

t	$x(t)$
0	4
3	-5
6	22