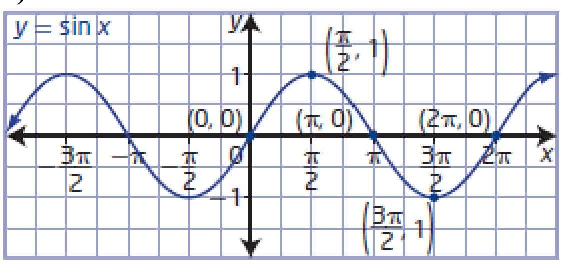
Chapter 5 Trigonometric Functions Graphs

Section 5.1 Graphing Sine and Cosine Functions

Section 5.1 Page 233 Question 1

a) One cycle of the sine function $y = \sin x$, from 0 to 2π, includes three x-intercepts, a maximum, and a minimum. These five key points divide the period into quarters: (0, 0), $\left(\frac{\pi}{2}, 1\right)$, (π, 0), $\left(\frac{3\pi}{2}, -1\right)$, and (2π, 0).

b)



c) The x-intercepts of the graph of $y = \sin x$ for $-2\pi \le x \le 2\pi$ are –2π, –π, 0, π, and 2π.

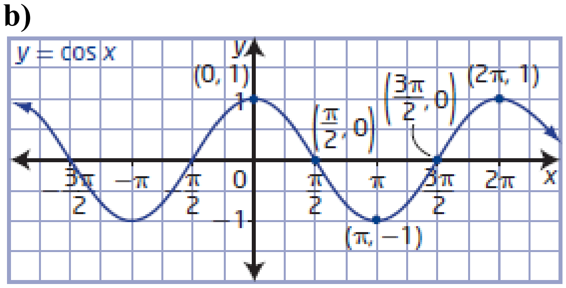
d) The y-intercept of the graph of $y = \sin x$ is 0.

e) For the graph of $y = \sin x$, the maximum value is 1 and the minimum value is –1.

Section 5.1 Page 233 Question 2

a) One cycle of the function $y = \cos x$, from 0 to 2π, includes two x-intercepts, two maximums, and a minimum. These five key points divide the period into quarters: (0, 1), $\left(\frac{\pi}{2}, 0\right)$, (π, –1), $\left(\frac{3\pi}{2}, 0\right)$, and (2π, 1).

b)



c) The x-intercepts of the graph of $y = \cos x$ for $-2\pi \le x \le 2\pi$ are $-\frac{3\pi}{2}$, $-\frac{\pi}{2}$, $\frac{\pi}{2}$, and $\frac{3\pi}{2}$.

d) The y-intercept of the graph of $y = \cos x$ is 1.

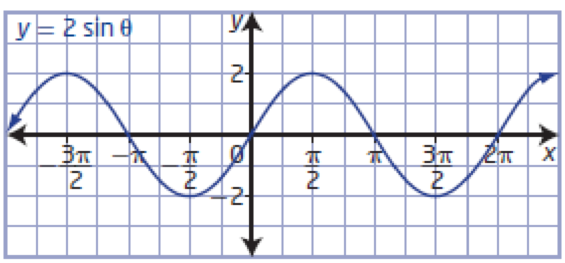
e) For the graph of $y = \cos x$, the maximum value is 1 and the minimum value is –1.

Section 5.1 Page 233 Question 3

Property	$y = \sin x$	$y = \cos x$
maximum	1	1
minimum	-1	-1
amplitude	1	1
period	2π	2π
domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
range	$\{y \mid -1 \le y \le 1, y \in R\}$	$\{y \mid -1 \le y \le 1, y \in R\}$
y-intercept	0	1
x-intercepts	$\pi n, n \in I$	$\frac{\pi}{2} + \pi n, n \in I$

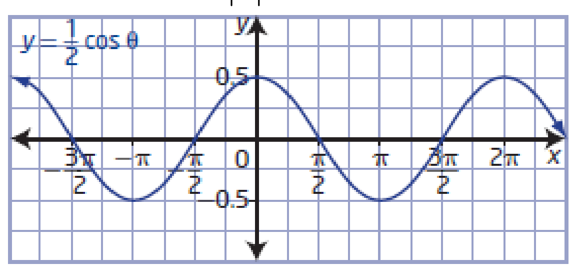
Section 5.1 Page 233 Question 4

a) For the function $y = 2 \sin \theta$, $a = 2$. The amplitude is $|2|$, or 2.



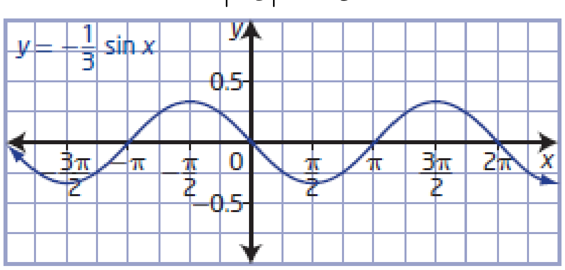
b) For the function $y = \frac{1}{2} \cos x$, $a = \frac{1}{2}$.

The amplitude is $\left|\frac{1}{2}\right|$, or $\frac{1}{2}$.

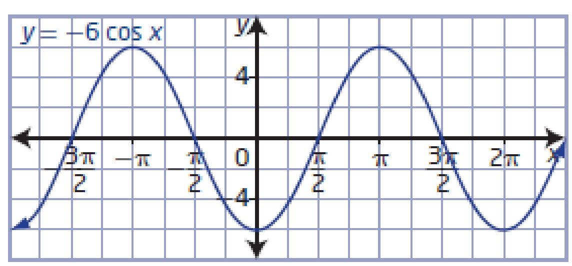


c) For the function $y = -\frac{1}{3} \sin x$, $a = -\frac{1}{3}$.

The amplitude is $\left|-\frac{1}{3}\right|$, or $\frac{1}{3}$.



d) For the function $y = -6 \cos \theta$, $a = -6$. The amplitude is $|-6|$, or 6.

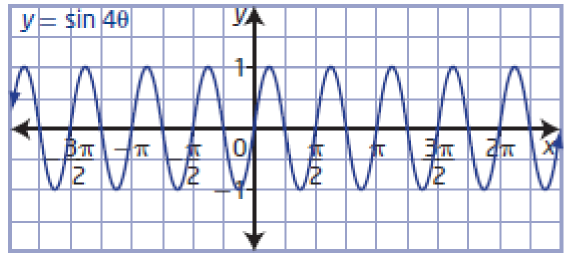


Section 5.1 Page 233 Question 5

a) For the function $y = \sin 4\theta$, $b = 4$.

$$\begin{aligned}\text{Period} &= \frac{360°}{|b|}\\ &= \frac{360°}{|4|}\\ &= 90°\end{aligned}$$

$$\begin{aligned}\text{Period} &= \frac{2\pi}{|b|}\\ &= \frac{2\pi}{|4|}\\ &= \frac{\pi}{2}\end{aligned}$$

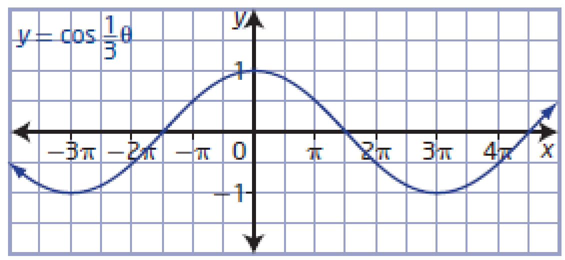


The period is 90° or $\frac{\pi}{2}$.

b) For the function $y = \cos \frac{1}{3}\theta$, $b = \frac{1}{3}$.

$$\begin{aligned}\text{Period} &= \frac{360°}{|b|}\\ &= \frac{360°}{\left|\frac{1}{3}\right|}\\ &= 1080°\end{aligned}$$

$$\begin{aligned}\text{Period} &= \frac{2\pi}{|b|}\\ &= \frac{2\pi}{\left|\frac{1}{3}\right|}\\ &= 6\pi\end{aligned}$$

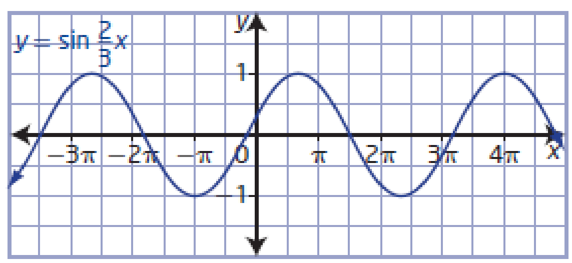


The period is 1080° or 6π.

c) For the function $y = \sin \frac{2}{3}x$, $b = \frac{2}{3}$.

$$\begin{aligned}\text{Period} &= \frac{360°}{|b|}\\ &= \frac{360°}{\left|\frac{2}{3}\right|}\\ &= 540°\end{aligned}$$

$$\begin{aligned}\text{Period} &= \frac{2\pi}{|b|}\\ &= \frac{2\pi}{\left|\frac{2}{3}\right|}\\ &= 3\pi\end{aligned}$$

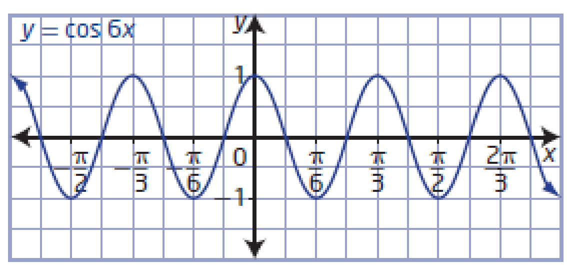


The period is 540° or 3π.

d) For the function $y = \cos 6x$, $b = 6$.

$$\begin{aligned}\text{Period} &= \frac{360°}{|b|}\\ &= \frac{360°}{|6|}\\ &= 60°\end{aligned}$$

$$\begin{aligned}\text{Period} &= \frac{2\pi}{|b|}\\ &= \frac{2\pi}{|6|}\\ &= \frac{\pi}{3}\end{aligned}$$



The period is 60° or $\frac{\pi}{3}$.

Section 5.1 Page 233 Question 6

a) For the function $y = 3 \cos x$, $a = 3$ and $b = 1$. The graph of this cosine function will have an amplitude of 3 and a period of 2π: choice **A**.

b) For the function $y = \cos 3x$, $a = 1$ and $b = 3$. The graph of this cosine function will have an amplitude of 1 and a period of $\frac{2\pi}{3}$: choice **D**.

c) For the function $y = -\sin x$, $a = -1$ and $b = 1$. The graph of this sine function will have an amplitude of 1, be reflected in the x-axis, and have a period of 2π: choice **C**.

d) For the function $y = -\cos x$, $a = -1$ and $b = 1$. The graph of this cosine function will have an amplitude of 1, be reflected in the x-axis, and have a period of 2π: choice **B**.

Section 5.1 Page 234 Question 7

a) For the function $y = 3 \sin x$, $a = 3$. The amplitude is $|3|$, or 3. The graph of this function is related to the graph of $y = \sin x$ by a vertical stretch by a factor of 3.

b) For the function $y = -5 \sin x$, $a = -5$. The amplitude is $|-5|$, or 5. The graph of this function is related to the graph of $y = \sin x$ by a vertical stretch by a factor of 5 and a reflection in the x-axis.

c) For the function $y = 0.15 \sin x$, $a = 0.15$. The amplitude is $|0.15|$, or 0.15. The graph of this function is related to the graph of $y = \sin x$ by a vertical stretch by a factor of 0.15.

d) For the function $y = -\frac{2}{3} \sin x$, $a = -\frac{2}{3}$. The amplitude is $\left|-\frac{2}{3}\right|$, or $\frac{2}{3}$. The graph of this function is related to the graph of $y = \sin x$ by a vertical stretch by a factor of $\frac{2}{3}$ and a reflection in the x-axis.

Section 5.1 Page 234 Question 8

a) For the function $y = \cos 2x$, $b = 2$.

$$\begin{aligned} \text{Period} &= \frac{360°}{|b|} \\ &= \frac{360°}{|2|} \\ &= 180° \end{aligned}$$

The graph of this function is related to the graph of $y = \cos x$ by a horizontal stretch by a factor of $\frac{1}{2}$.

b) For the function $y = \cos(-3x)$, $b = -3$.

$$\begin{aligned}\text{Period} &= \frac{360°}{|b|}\\ &= \frac{360°}{|-3|}\\ &= 120°\end{aligned}$$

The graph of this function is related to the graph of $y = \cos x$ by a horizontal stretch by a factor of $\frac{1}{3}$ and a reflection in the y-axis.

c) For the function $y = \cos \frac{1}{4}x$, $b = \frac{1}{4}$.

$$\begin{aligned}\text{Period} &= \frac{360°}{|b|}\\ &= \frac{360°}{\left|\frac{1}{4}\right|}\\ &= 1440°\end{aligned}$$

The graph of this function is related to the graph of $y = \cos x$ by a horizontal stretch by a factor of 4.

d) For the function $y = \cos \frac{2}{3}x$, $b = \frac{2}{3}$.

$$\begin{aligned}\text{Period} &= \frac{360°}{|b|}\\ &= \frac{360°}{\left|\frac{2}{3}\right|}\\ &= 540°\end{aligned}$$

The graph of this function is related to the graph of $y = \cos x$ by a horizontal stretch by a factor of $\frac{3}{2}$.

Section 5.1 Page 234 Question 9

a) For the function $y = 2 \sin x$, $a = 2$ and $b = 1$. The amplitude is $|2|$, or 2.

$$\begin{aligned}\text{Period} &= \frac{360°}{|b|}\\ &= \frac{360°}{|1|}\\ &= 360°\end{aligned} \qquad \begin{aligned}\text{Period} &= \frac{2\pi}{|b|}\\ &= \frac{2\pi}{|1|}\\ &= 2\pi\end{aligned}$$

b) For the function $y = -4 \sin 2x$, $a = -4$ and $b = 2$. The amplitude is $|-4|$, or 4.

$$\text{Period} = \frac{360°}{|b|} = \frac{360°}{|2|} = 180°$$

$$\text{Period} = \frac{2\pi}{|b|} = \frac{2\pi}{|2|} = \pi$$

c) For the function $y = \frac{5}{3} \sin\left(-\frac{2}{3}x\right)$, $a = \frac{5}{3}$ and $b = -\frac{2}{3}$. The amplitude is $\left|\frac{5}{3}\right|$, or $\frac{5}{3}$.

$$\text{Period} = \frac{360°}{|b|} = \frac{360°}{\left|-\frac{2}{3}\right|} = 540°$$

$$\text{Period} = \frac{2\pi}{|b|} = \frac{2\pi}{\left|-\frac{2}{3}\right|} = 3\pi$$

d) For the function $y = 3 \sin \frac{1}{2}x$, $a = 3$ and $b = \frac{1}{2}$. The amplitude is $|3|$, or 3.

$$\text{Period} = \frac{360°}{|b|} = \frac{360°}{\left|\frac{1}{2}\right|} = 720°$$

$$\text{Period} = \frac{2\pi}{|b|} = \frac{2\pi}{\left|\frac{1}{2}\right|} = 4\pi$$

Section 5.1 Page 234 Question 10

a) Use Amplitude = $\frac{\text{maximum value} - \text{minimum value}}{2}$.

The amplitude of graph A is $\frac{2-(-2)}{2}$, or 1, and the period is 4π.

The amplitude of graph B is $\frac{0.5-(-0.5)}{2}$, or 0.5, and the period is π.

b) Graph A has the pattern of a sine curve. Since the amplitude is 1, $a = 1$. Using the period of 4π and choosing b to be positive

$$\text{Period} = \frac{2\pi}{|b|}$$
$$4\pi = \frac{2\pi}{|b|}$$
$$b = \frac{1}{2}$$
So, the equation of the function in the form $y = a \sin bx$ is $y = \sin \frac{1}{2}x$.

Graph B has the pattern of a cosine curve. Since the amplitude is 0.5, $a = 0.5$. Using the period of π and choosing b to be positive

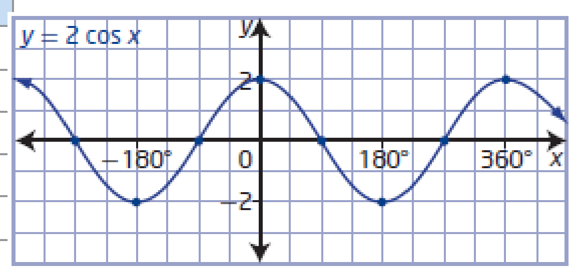
$$\text{Period} = \frac{2\pi}{|b|}$$
$$\pi = \frac{2\pi}{|b|}$$
$$b = 2$$
So, the equation of the function in the form $y = a \cos bx$ is $y = 0.5 \cos 2x$.

c) Since graph passes through (0, 0), the sine function is the better choice. Since graph B passes through (0, 1), the cosine function is the better choice.

Section 5.1 Page 234 Question 11

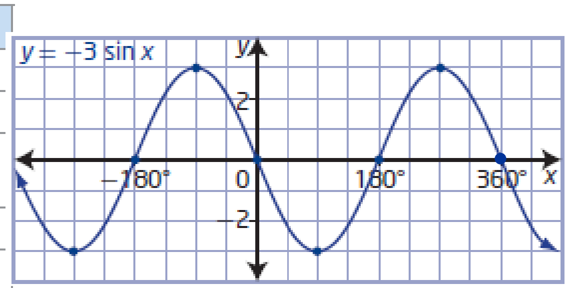
a) For $y = 2 \cos x$ in the interval [–360°, 360°]:

Property	Points on the Graph of $y = 2 \cos x$
maximum	(–360°, 2), (0°, 2), (360°, 2)
minimum	(–180°, –2), (180°, –2)
x-intercepts	(–270°, 0), (–90°, 0), (90°, 0), (270°, 0)
y-intercept	(0, 2)
period	360°
range	$\{y \mid -2 \le y \le 2, y \in R\}$



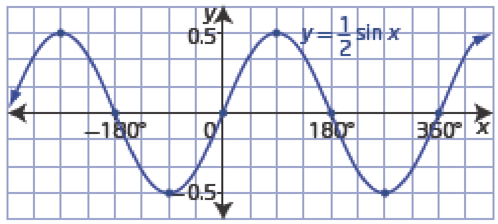
b) For $y = –3 \sin x$ in the interval [–360°, 360°]:

Property	Points on the Graph of $y = -3 \sin x$
maximum	(–90°, 3), (270°, 3)
minimum	(–270°, –3), (90°, –3)
x-intercepts	(–360°, 0), (–180°, 0), (0°, 0), (180°, 0), (360°, 0)
y-intercept	(0, 0)
period	360°
range	$\{y \mid -3 \le y \le 3, y \in R\}$



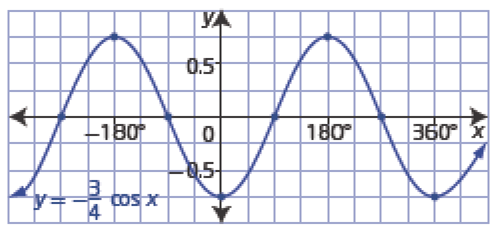
c) For $y = \frac{1}{2}\sin x$ in the interval [–360°, 360°]:

Property	Points on the Graph of $y = \frac{1}{2}\sin x$
maximum	(–270°, 0.5), (90°, 0.5)
minimum	(–90°, –0.5), (270°, –0.5)
x-intercepts	(–360°, 0), (–180°, 0), (0°, 0), (180°, 0), (360°, 0)
y-intercept	(0, 0)
period	360°
range	$\{y \mid -0.5 \le y \le 0.5, y \in R\}$



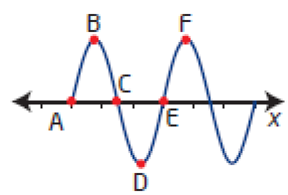
d) For $y = -\frac{3}{4}\cos x$ in the interval [–360°, 360°]:

Property	Points on the Graph of $y = -\frac{3}{4}\cos x$
maximum	(–180°, 0.75), (180°, 0.75)
minimum	(–360°, –0.75), (0°, –0.75), (360°, –0.75)
x-intercepts	(–270°, 0), (–90°, 0), (90°, 0), (270°, 0)
y-intercept	(0, –0.75)
period	360°
range	$\{y \mid -0.75 \le y \le 0.75, y \in R\}$



Section 5.1 Page 234 Question 12

a) Given $y = 3 \sin 2x$ and point A has coordinates (0, 0), find the coordinates of points B, C, D, and E.
Since $a = 3$, the amplitude is $|3|$, or 3. Since $b = 2$, the period is $\frac{2\pi}{|2|}$, or π.

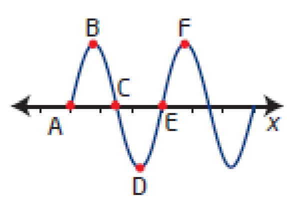


Point B, a maximum, occurs at one-quarter of the period: $\left(\frac{\pi}{4}, 3\right)$.

Points C and E, *x*-intercepts, occur at half the period and at the end of the period: $\left(\frac{\pi}{2}, 0\right)$ and $(\pi, 0)$.

Point D, a minimum, occurs at three-quarters of the period: $\left(\frac{3\pi}{4}, -3\right)$.

b) Given $y = 2 \cos x$ and point B has coordinates (0, 2), find the coordinates of points C, D, E, and F.
Since $a = 2$, the amplitude is $|2|$, or 2. Since $b = 1$, the period is 2π.



Points C and E, x-intercepts, occur at one-quarter and three-quarters of the period: $\left(\frac{\pi}{2}, 0\right)$ and $\left(\frac{3\pi}{2}, 0\right)$.
Point D, a minimum, occurs at half of the period: $(\pi, -2)$.
Point F, a maximum, occurs at the end of the period: $(2\pi, 2)$.

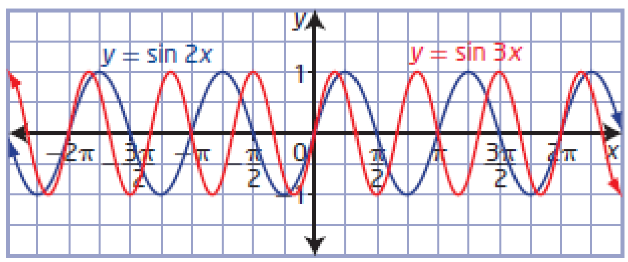
c) Given $y = \sin \frac{1}{2}x$ and point A has coordinates $(-4\pi, 0)$, find the coordinates of points B, C, D, and E.

Since $a = 1$, the amplitude is $|1|$, or 1. Since $b = \frac{1}{2}$, the period is $\frac{2\pi}{\left|\frac{1}{2}\right|}$, or 4π.

Point B, a maximum, occurs at one-quarter of the period: $(-3\pi, 1)$.
Points C and E, x-intercepts, occur at half the period and at the end of the period: $(-2\pi, 0)$ and $(0, 0)$.
Point D, a minimum, occurs at three-quarters of the period: $(-\pi, -1)$.

Section 5.1 Page 234 Question 13

The graphs of $f(x) = \sin 2x$ and $f(x) = \sin 3x$ the same amplitude of 1 but different periods, π and $\frac{2\pi}{3}$, respectively. So, the maximum, minimum, y-intercepts, domain, and range are the same for both graphs. However, the x-intercepts are different.



Section 5.1 Page 234–235 Question 14

a) Use Amplitude $= \frac{\text{maximum value} - \text{minimum value}}{2}$.

The amplitude of the sine wave is $\frac{5-(-5)}{2}$, or 5, and the period is $\frac{4\pi}{3}$.

b) The amplitude of the sine wave is $\frac{4-(-4)}{2}$, or 4, and the period is $\frac{2\pi}{3}$.

Section 5.1 Page 235 Question 15

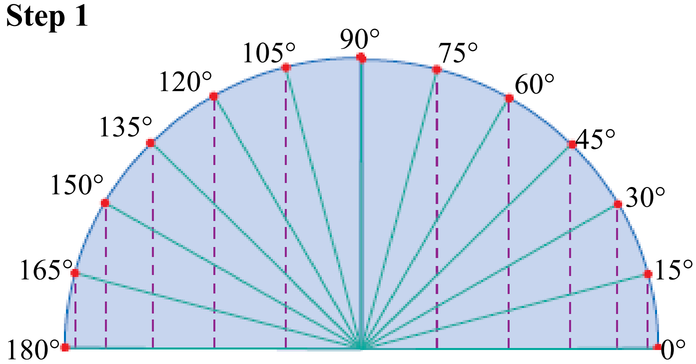
a) The amplitude of the graph is $\frac{120-80}{2}$, or 20 mmHg, and the period is 0.8 s.

b) The pulse rate for this person is $\frac{60}{0.8}$, or 75 beats per minute.

Section 5.1 Page 235 Question 16

Examples:

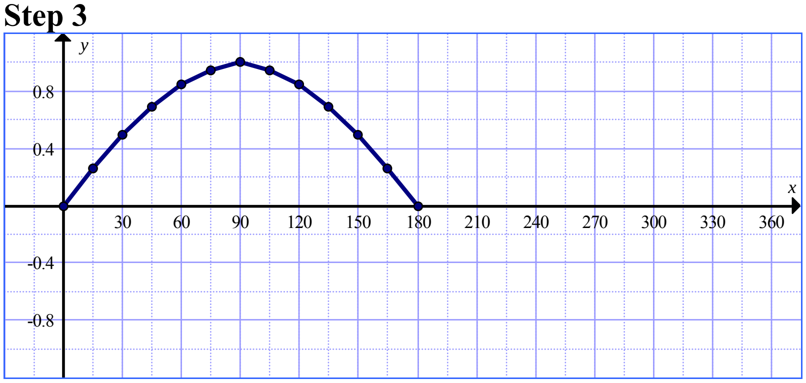
Step 1



Step 2

Angle, x	Opposite (cm)	Hypotenuse (cm)	$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$
0°	0.0	3.9	sin 0° = 0
15°	1.0	3.9	sin 15° ≈ 0.26
30°	1.9	3.9	sin 30° ≈ 0.49
45°	2.7	3.9	sin 45° ≈ 0.69
60°	3.3	3.9	sin 60° ≈ 0.85
75°	3.7	3.9	sin 75° ≈ 0.95
90°	3.9	3.9	sin 90° = 1
105°	3.7	3.9	sin 105° ≈ 0.95
120°	3.3	3.9	sin 120° ≈ 0.85
135°	2.7	3.9	sin 135° ≈ 0.69
150°	1.9	3.9	sin 150° ≈ 0.49
165°	1.0	3.9	sin 165° ≈ 0.26
180°	0.0	3.9	sin 180° = 0

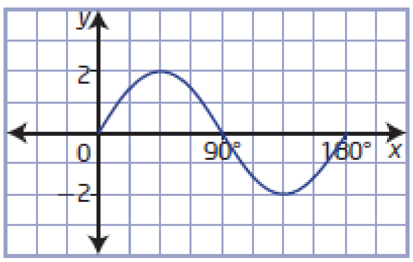
Step 3



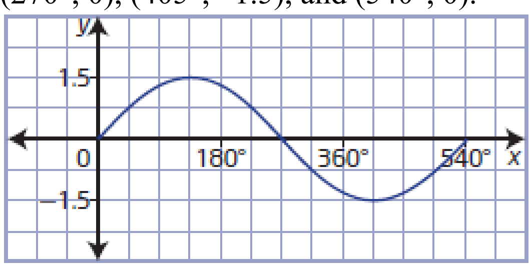
Step 4

Section 5.1 Page 236 Question 17

a) For a sinusoidal curve with amplitude 2, period 180°, and passing through the point (0, 0), the five key points that divide the period into quarters are (0, 0), (45°, 2), (90°, 0), (270°, –2), and (180°, 0).

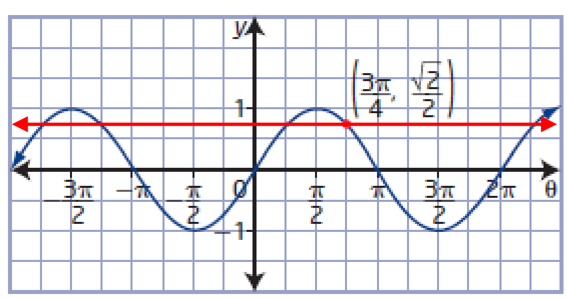


b) For a sinusoidal curve with amplitude 1.5, period 540°, and passing through the point (0, 0), the five key points that divide the period into quarters are (0, 0), (135°, 1.5), (270°, 0), (405°, –1.5), and (540°, 0).



Section 5.1 Page 236 Question 18

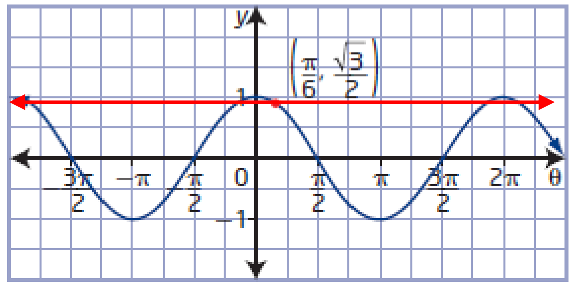
a) Add the horizontal line $y = \frac{\sqrt{2}}{2}$ to the grid. The points of intersection with the graph of $y = \sin \theta$ will have the same y-coordinate as the point given.



$\left(-\frac{7\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(-\frac{5\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$, and $\left(\frac{9\pi}{4}, \frac{\sqrt{2}}{2}\right)$

b) Add the horizontal line $y = \frac{\sqrt{3}}{2}$ to the grid. The points of intersection with the graph of $y = \cos\theta$ will have the same y-coordinate as the point given.

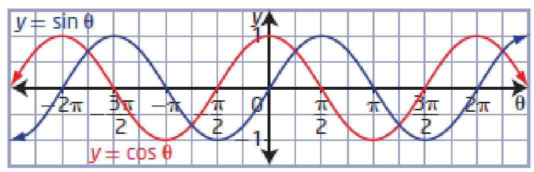
$\left(-\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(-6, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, and $\left(\frac{13\pi}{6}, \frac{\sqrt{3}}{2}\right)$



Section 5.1 Page 236 Question 19

a) The graphs have the same maximum values, minimum values, period, domain, and range.

b) The graphs have different x- and y-intercepts.



c) Example: The graph of $y = \cos\theta$ will be the same as the graph of $y = \sin\theta$, if it is translated horizontally $\frac{\pi}{2}$ units to the right.

Section 5.1 Page 236 Question 20

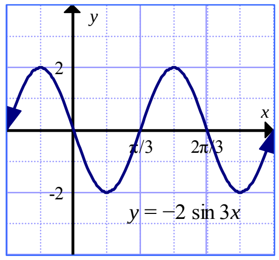
The function $y = f(x)$ has a period of 6. For $y = f\left(\frac{1}{2}x\right)$, $b = \frac{1}{2}$. So, the period for $y = f\left(\frac{1}{2}x\right)$ is $\frac{6}{\left|\frac{1}{2}\right|}$, or 12.

Section 5.1 Page 236 Question 21

One method for determining the period is to graph the function and observe the length of a cycle from the graph. Another method is to use the formula.

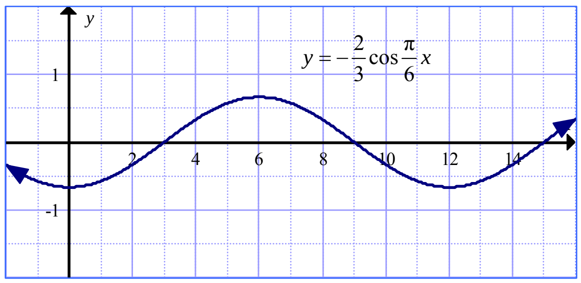
a) For the function $y = -2\sin 3x$, $b = 3$.

$$\text{Period} = \frac{2\pi}{|b|} = \frac{2\pi}{|3|} = \frac{2\pi}{3}$$



b) For the function $y = -\frac{2}{3}\cos\frac{\pi}{6}x$, $b = \frac{\pi}{6}$.

$$\text{Period} = \frac{2\pi}{|b|} = \frac{2\pi}{\left|\frac{\pi}{6}\right|} = 12$$



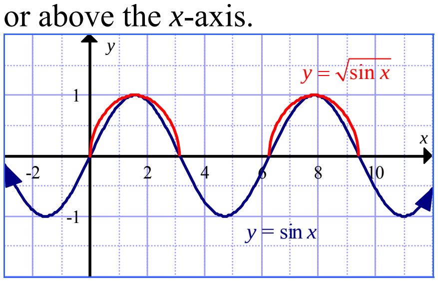
Section 5.1 Page 236 Question 22

If sin θ = 0.3 and the period of y = sin θ is 2π, then sin (θ + 2π) = sin (θ + 4π) = 0.3.

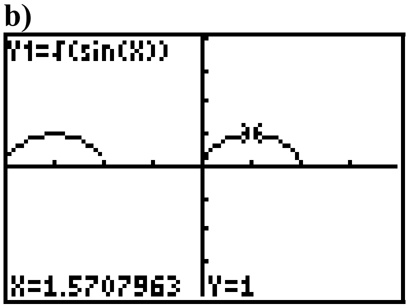
sin θ + sin (θ + 2π) + sin (θ + 4π)
= 0.3 + 0.3 + 0.3
= 0.9

Section 5.1 Page 236 Question 23

a) The graph of $y = \sqrt{\sin x}$ will consists the portions of the graph of $y = \sin x$ that lie on or above the x-axis.

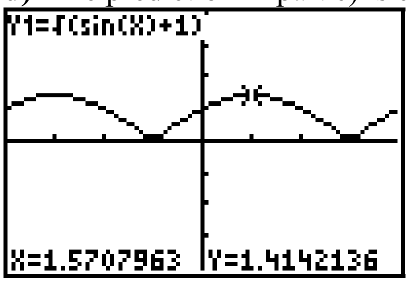


b)



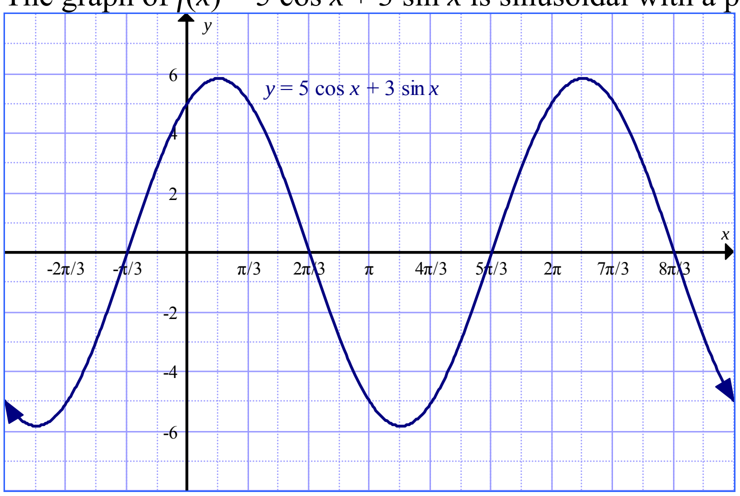
c) Since the range of $y = \sin x$ is $\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$, the range of $y = \sin x + 1$ will be $\{y \mid 0 \le y \le 2, y \in \mathrm{R}\}$. In other words, the entire graph of $y = \sin x + 1$ will lie on or above the x-axis. So, the graph of $y = \sqrt{\sin x + 1}$ will exist for all values of x, as compared to the graph of $y = \sqrt{\sin x}$.

d) The prediction in part c) is correct.



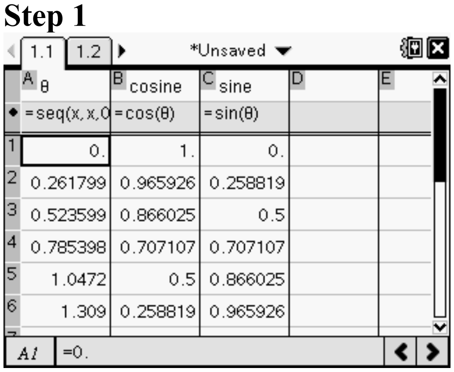
Section 5.1 Page 236 Question 24

The graph of $f(x) = 5 \cos x + 3 \sin x$ is sinusoidal with a period of 2π.



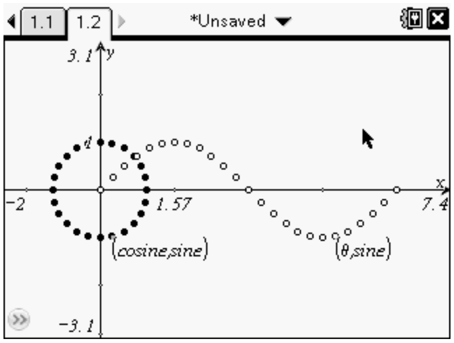
Section 5.1 Page 236–37 Question C1

Step 1

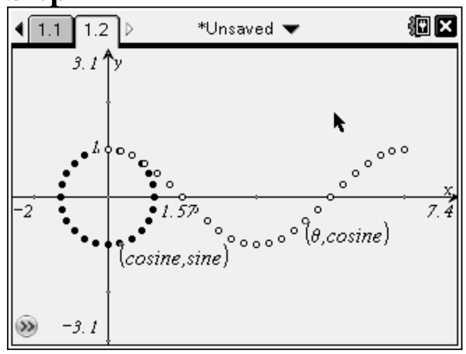


	A θ	B cosine	C sine	D	E
◆	=seq(x,x,0	=cos(θ)	=sin(θ)		
1	0.	1.	0.		
2	0.261799	0.965926	0.258819		
3	0.523599	0.866025	0.5		
4	0.785398	0.707107	0.707107		
5	1.0472	0.5	0.866025		
6	1.309	0.258819	0.965926		

Steps 2–3



Step 4



Step 5 **a)** The *x*-coordinate of each point on the unit circle represents cos θ. The *y*-coordinate of each point on the unit circle represents the sin θ.

b) The *y*-coordinates of the points on the sine graph are the same as the *y*-coordinates of the points on the unit circle. The *y*-coordinates of the points on the cosine graph are the same as the *x*-coordinates of the points on the unit circle.

Section 5.1 Page 237 Question C2

The value of $(\cos \theta)^2 + (\sin \theta)^2$ is 1.
The points on the unit circle have the coordinates (cos θ, sin θ). So, the sum of the squares of the legs of each right triangle is equal to the radius of the unit circle, which is always 1.

Section 5.1 Page 237 Question C3

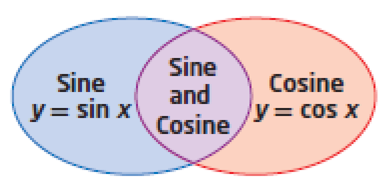
Given: The graph of $y = f(x)$ is sinusoidal with a period of 40° passing through the point (4, 0).
a) The value of $f(0)$ cannot be determined because the amplitude is not given.

b) The value of $f(4)$ is 0, as given.

c) The value of $f(84)$ is 0. Since the period is 40°, the graph will return to the same value every 40°. So, the *y*-coordinate at $x = 4$, $x = 44$, and $x = 84$ will be 0.

Section 5.1 Page 237 Question C4

a) The domain of both $y = \sin x$ and $y = \cos x$ is $\{x \mid x \in \mathrm{R}\}$.



b) The range of both $y = \sin x$ and $y = \cos x$ is $\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$.

c) The period of both $y = \sin x$ and $y = \cos x$ is 2π.

d) The amplitude of both $y = \sin x$ and $y = \cos x$ is 1.

e) The graph of $y = \sin x$ has x-intercepts of $n(180°)$, $n \in I$.

f) The graph of $y = \cos x$ has x-intercepts of $90° + n(180°)$, $n \in I$.

g) The graph of $y = \cos x$ has y-intercept 1.

h) The graph of $y = \sin x$ has y-intercept 0.

i) The graph of $y = \cos x$ passes through the point (0, 1).

j) The graph of $y = \sin x$ passes through the point (0, 0).

k) The graph of $y = \cos x$ has a maximum value at (360°, 1).

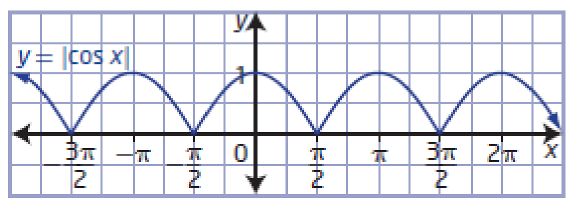
l) The graph of $y = \sin x$ has a maximum value at (90°, 1).

m) This is the graph of $y = \sin x$.

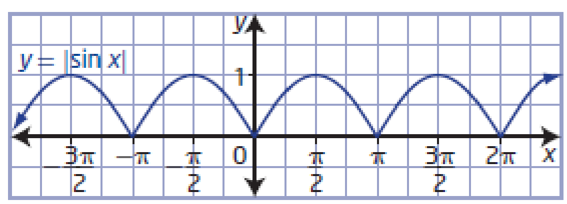
n) This is the graph of $y = \cos x$.

Section 5.1 Page 237 Question C5

a) The graph of $y = |\cos x|$ can be obtained from the graph of $y = \cos x$ by reflecting all parts of the graph below the x-axis in the x-axis.



b) The graph of $y = |\sin x|$ can be obtained from the graph of $y = \sin x$ by reflecting all parts of the graph below the x-axis in the x-axis.

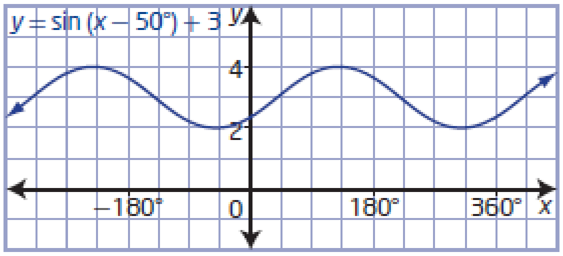


Section 5.2 Transformations of Sinusoidal Functions

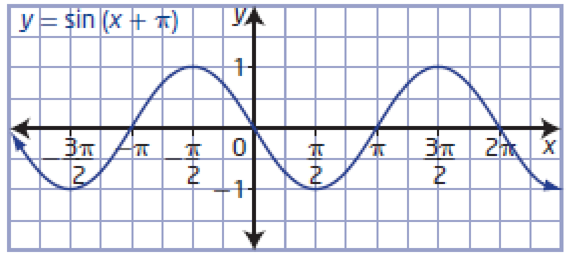
Section 5.2 Page 250 Question 1

Compare each function to the form $y = a \sin b(x - c) + d$. The phase shift is determined by parameter c, while the vertical displacement is determined by parameter d.

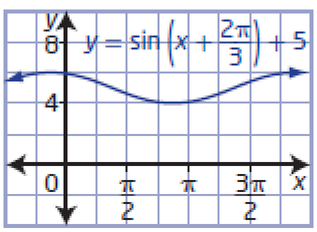
a) For $y = \sin(x - 50°) + 3$, $a = 1$, $b = 1$, $c = 50°$, and $d = 3$. So, the phase shift is 50° to the right and the vertical displacement is 3 units up. To sketch the graph, also use amplitude 1 and period 360°, since there is no vertical or horizontal stretch.



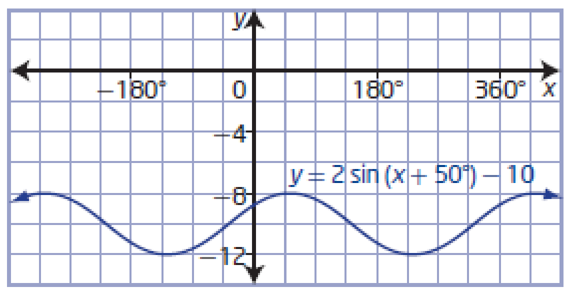
b) For $y = \sin(x + \pi)$, $a = 1$, $b = 1$, $c = -\pi$, and $d = 0$. So, the phase shift is π units to the left and there is no vertical displacement. To sketch the graph, also use amplitude 1 and period 2π, since there is no vertical or horizontal stretch.



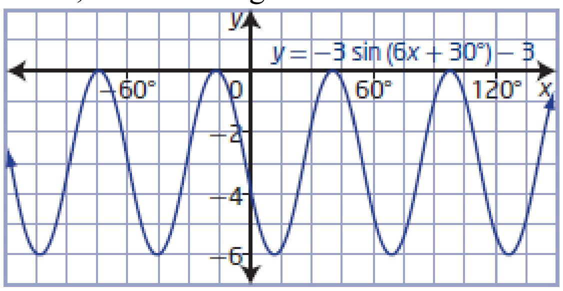
c) For $y = \sin\left(x + \frac{2\pi}{3}\right) + 5$, $a = 1$, $b = 1$, $c = -\frac{2\pi}{3}$, and $d = 5$. So, the phase shift is $\frac{2\pi}{3}$ units to the left and the vertical displacement is 5 units up. To sketch the graph, also use amplitude 1 and period 2π, since there is no vertical or horizontal stretch.



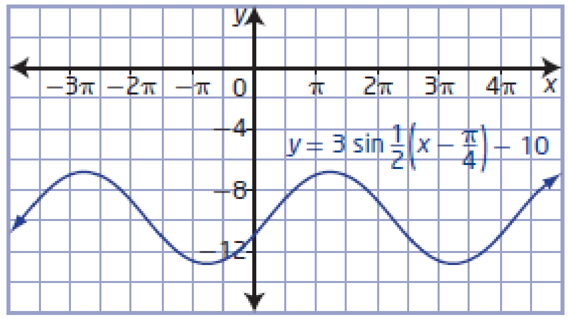
d) For $y = 2\sin(x + 50°) - 10$, $a = 2$, $b = 1$, $c = -50°$, and $d = -10$. So, the phase shift is 50° to the left and the vertical displacement is 10 units down. To sketch the graph, also use amplitude 2 and period 360°, since there is a vertical stretch but no horizontal stretch.



e) First rewrite $y = -3 \sin (6x + 30°) - 3$ as $y = -3 \sin 6(x + 5°) - 3$. Then, $a = -3$, $b = 6$, $c = -5°$, and $d = -3$. So, the phase shift is 5° to the left and the vertical displacement is 3 units down. To sketch the graph, also use amplitude 3 and period 60°, since there is a vertical stretch and a horizontal stretch. Reflect the graph in the x-axis, since a is negative.



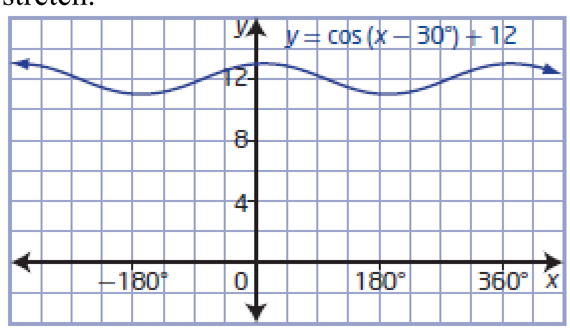
f) For $y = 3\sin\frac{1}{2}\left(x - \frac{\pi}{4}\right) - 10$, $a = 3$, $b = \frac{1}{2}$, $c = \frac{\pi}{4}$, and $d = -10$. So, the phase shift is $\frac{\pi}{4}$ units to the right and the vertical displacement is 10 units down. To sketch the graph, also use amplitude 3 and period 4π, since there is a vertical stretch and a horizontal stretch.



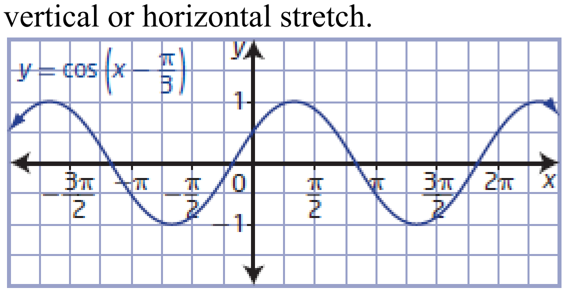
Section 5.2 Page 250 Question 2

Compare each function to the form $y = a \cos b(x - c) + d$. The phase shift is determined by parameter c, while the vertical displacement is determined by parameter d.

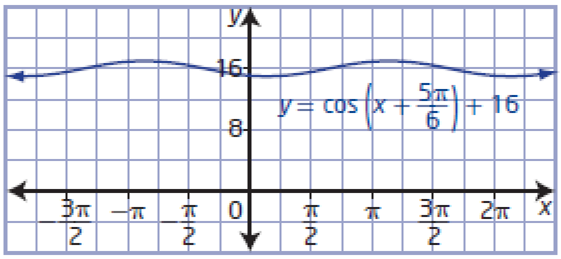
a) For $y = \cos (x - 30°) + 12$, $a = 1$, $b = 1$, $c = 30°$, and $d = 12$. So, the phase shift is 30° to the right and the vertical displacement is 12 units up. To sketch the graph, also use amplitude 1 and period 360°, since there is no vertical or horizontal stretch.



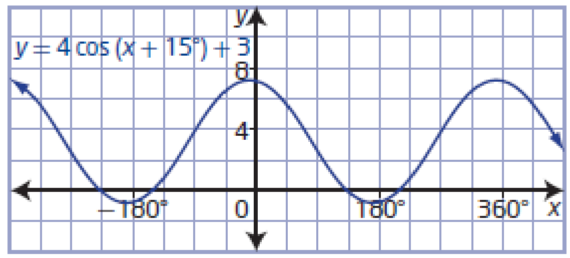
b) For $y = \cos\left(x - \frac{\pi}{3}\right)$, $a = 1$, $b = 1$, $c = \frac{\pi}{3}$, and $d = 0$. So, the phase shift is $\frac{\pi}{3}$ units to the right and there is no vertical displacement. To sketch the graph, also use amplitude 1 and period 2π, since there is no vertical or horizontal stretch.



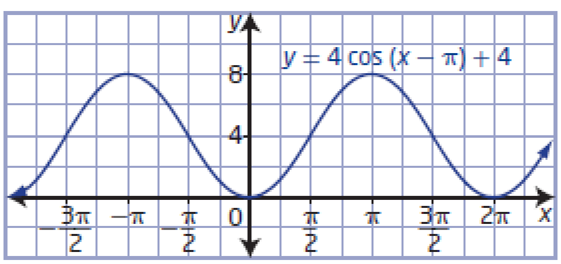
c) For $y = \cos\left(x + \frac{5\pi}{6}\right) + 16$, $a = 1$, $b = 1$, $c = -\frac{5\pi}{6}$, and $d = 16$. So, the phase shift is $\frac{5\pi}{6}$ units to the left and the vertical displacement is 16 units up. To sketch the graph, also use amplitude 1 and period 2π, since there is no vertical or horizontal stretch.



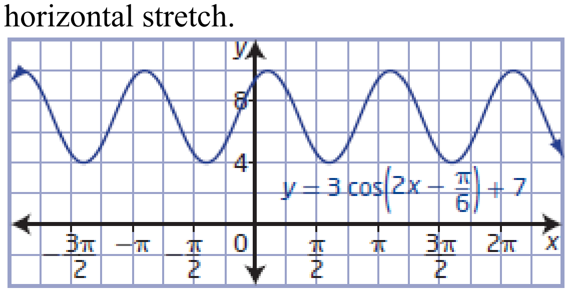
d) For $y = 4 \cos (x + 15°) + 3$, $a = 4$, $b = 1$, $c = -15°$, and $d = 3$. So, the phase shift is 15° to the left and the vertical displacement is 3 units up. To sketch the graph, also use amplitude 2 and period 360°, since there is a vertical stretch but no horizontal stretch.



e) For $y = 4 \cos (x - \pi) + 4$, $a = 4$, $b = 1$, $c = \pi$, and $d = 4$. So, the phase shift is π to the right and the vertical displacement is 4 units up. To sketch the graph, also use amplitude 4 and period 2π, since there is a vertical stretch but no horizontal stretch.



f) First rewrite $y = 3\cos\left(2x - \frac{\pi}{6}\right) + 7$ as $y = 3\cos 2\left(x - \frac{\pi}{12}\right) + 7$. Then, $a = 3$, $b = 2$, $c = \frac{\pi}{12}$, and $d = 7$. So, the phase shift is $\frac{\pi}{12}$ units to the right and the vertical displacement is 7 units up. To sketch the graph, also use amplitude 3 and period π, since there is a vertical stretch and a horizontal stretch.



Section 5.2 Page 250 Question 3

Use the amplitude, *a*, and vertical displacement, *d*, to determine the maximum and minimum values.

a) i) For $y = 3\cos\left(x - \frac{\pi}{2}\right) + 5$, $a = 3$, $b = 1$, $c = \frac{\pi}{2}$, and $d = 5$.

maximum: $d + |a| = 5 + |3|$
$= 8$

minimum: $d - |a| = 5 - |3|$
$= 2$

So, the range of the function is $\{y \mid 2 \le y \le 8, y \in \mathrm{R}\}$.

ii) For $y = -2 \sin (x + \pi) - 3$, $a = -2$, $b = 1$, $c = \pi$, and $d = -3$.

maximum: $d + |a| = -3 + |-2|$
$= -1$

minimum: $d - |a| = -3 - |-2|$
$= -5$

So, the range of the function is $\{y \mid -5 \le y \le -1, y \in \mathrm{R}\}$.

iii) For $y = 1.5 \sin x + 4$, $a = 1.5$, $b = 1$, $c = 0$, and $d = 4$.

maximum: $d + |a| = 4 + |1.5|$
$= 5.5$

minimum: $d - |a| = 4 - |1.5|$
$= 2.5$

So, the range of the function is $\{y \mid 2.5 \le y \le 5.5, y \in \mathrm{R}\}$.

iv) For $y = \frac{2}{3}\cos(x + 50°) + \frac{3}{4}$, $a = \frac{2}{3}$, $b = 1$, $c = 50°$, and $d = \frac{3}{4}$.

maximum: $d + |a| = \frac{3}{4} + \left|\frac{2}{3}\right|$
$= \frac{17}{12}$

minimum: $d - |a| = \frac{3}{4} - \left|\frac{2}{3}\right|$
$= \frac{1}{12}$

So, the range of the function is $\left\{y \mid \frac{1}{12} \le y \le \frac{17}{12}, y \in \mathrm{R}\right\}$.

b) To determine the range of a function of the form $y = a \cos b(x - c) + d$ or $y = a \sin b(x - c) + d$, add and subtract the amplitude to/from the vertical displacement. This gives the maximum and minimum values of the function, and thus the range.

Section 5.2 Page 250 Question 4

a) For $y = -2 \cos 2(x + 4) - 1$, $a = -2$, $b = 2$, $c = -4$, and $d = -1$. So, the amplitude is $|-2|$, or 2, the period is $\frac{2\pi}{|2|}$ or π, the phase shift is 4 units to the left, and the vertical displacement is 1 unit down. Choice **D**.

b) For $y = 2 \sin 2(x - 4) - 1$, $a = 2$, $b = 2$, $c = 4$, and $d = -1$. So, the amplitude is $|2|$, or 2, the period is $\frac{2\pi}{|2|}$ or π, the phase shift is 4 units to the right, and the vertical displacement is 1 unit down. Choice **C**.

c) First, rewrite $y = 2 \sin (2x - 4) - 1$ as $y = 2 \sin 2(x - 2) - 1$. Then, $a = 2$, $b = 2$, $c = 2$, and $d = -1$. So, the amplitude is $|2|$, or 2, the period is $\frac{2\pi}{|2|}$ or π, the phase shift is 2 units to the right, and the vertical displacement is 1 unit down. Choice **B**.

d) First, rewrite $y = 3 \sin (3x - 9) - 1$ as $y = 3 \sin 3(x - 3) - 1$. Then, $a = 3$, $b = 3$, $c = 3$, and $d = -1$. So, the amplitude is $|3|$, or 3, the period is $\frac{2\pi}{|3|}$ or $\frac{2\pi}{3}$, the phase shift is 3 units to the right, and the vertical displacement is 1 unit down. Choice **A**.

e) First, rewrite $y = 3 \sin (3x + \pi) - 1$ as $y = 3\sin 3\left(x + \frac{\pi}{3}\right) - 1$. Then, $a = 3$, $b = 3$, $c = -\frac{\pi}{3}$, and $d = -1$. So, the amplitude is $|3|$, or 3, the period is $\frac{2\pi}{|3|}$ or $\frac{2\pi}{3}$, the phase shift is $\frac{\pi}{3}$ units to the left, and the vertical displacement is 1 unit down. Choice **E**.

Section 5.2 Page 250 Question 5

a) For $y = \sin\left(x - \frac{\pi}{4}\right)$, $a = 1$, $b = 1$, $c = \frac{\pi}{4}$, and $d = 0$. So, the graph will have amplitude 1, period 2π, phase shift of $\frac{\pi}{4}$ units to the right, and no vertical displacement. Choice **D**.

b) For $y = \sin\left(x + \frac{\pi}{4}\right)$, $a = 1$, $b = 1$, $c = -\frac{\pi}{4}$, and $d = 0$. So, the graph will have amplitude 1, period 2π, phase shift of $\frac{\pi}{4}$ units to the left, and no vertical displacement. Choice **B**.

c) For $y = \sin x - 1$, $a = 1$, $b = 1$, $c = 0$, and $d = -1$. So, the graph will have amplitude 1, period 2π, no phase shift, and a vertical displacement of 1 unit down. Choice **C**.

d) For $y = \sin x + 1$, $a = 1$, $b = 1$, $c = 0$, and $d = 1$. So, the graph will have amplitude 1, period 2π, no phase shift, and a vertical displacement of 1 unit up. Choice **A**.

Section 5.2 Page 251 Question 6

a) For amplitude 4, period π, phase shift $\frac{\pi}{2}$ to the right, and vertical displacement 6 units down, $a = 4$, $b = 2$, $c = \frac{\pi}{2}$, and $d = -6$. Then, the equation of the sine function in the form $y = a \sin b(x - c) + d$ is $y = 4\sin 2\left(x - \frac{\pi}{2}\right) - 6$.

b) For amplitude 0.5, period 4π, phase shift $\frac{\pi}{6}$ to the left, and vertical displacement 1 unit up, $a = 0.5$, $b = \frac{1}{2}$, $c = -\frac{\pi}{6}$, and $d = 1$. Then, the equation of the sine function in the form $y = a \sin b(x - c) + d$ is $y = 0.5\sin\frac{1}{2}\left(x + \frac{\pi}{6}\right) + 1$.

c) For amplitude $\frac{3}{4}$, period 720°, no phase shift, and vertical displacement 5 units down, $a = \frac{3}{4}$, $b = \frac{1}{2}$, $c = 0$, and $d = -5$. Then, the equation of the sine function in the form $y = a \sin b(x - c) + d$ is $y = \frac{3}{4}\sin\frac{1}{2}x - 5$.

Section 5.2 Page 251 Question 7

a) For a vertical stretch by a factor of 3 about the *x*-axis, horizontal stretch by a factor of 2 about the *y*-axis, and translated 2 units to the left and 3 units up, $a = 3$, $b = \frac{1}{2}$, $c = -2$, and $d = 3$. Then, the equation of the cosine function in the form $y = a \cos b(x - c) + d$ is $y = 3\cos\frac{1}{2}(x + 2) + 3$.

b) For a vertical stretch by a factor of $\frac{1}{2}$ about the *x*-axis, horizontal stretch by a factor of $\frac{1}{4}$ about the *y*-axis, and translated 3 units to the right and 5 units down, $a = \frac{1}{2}$, $b = 4$, $c = 3$, and $d = -5$. Then, the equation of the cosine function in the form $y = a \cos b(x - c) + d$ is $y = \frac{1}{2}\cos 4(x - 3) - 5$.

c) For a vertical stretch by a factor of $\frac{3}{2}$ about the *x*-axis, horizontal stretch by a factor of 3 about the *y*-axis, reflected in the *x*-axis, and translated $\frac{\pi}{4}$ units to the right and 1 unit down, $a = -\frac{3}{2}$, $b = \frac{1}{3}$, $c = \frac{\pi}{4}$, and $d = -1$. Then, the equation of the cosine function in the form $y = a \cos b(x - c) + d$ is $y = -\frac{3}{2}\cos\frac{1}{3}\left(x - \frac{\pi}{4}\right) - 1$.

Section 5.2 Page 251 Question 8

The colours of the wavelengths, in order from greatest to least period are red, orange, yellow, green, blue, indigo, and violet.

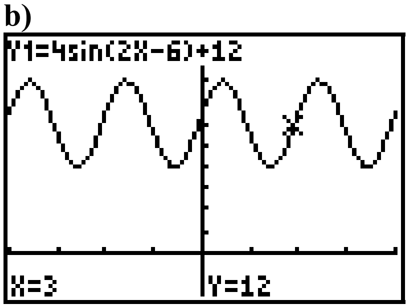
Section 5.2 Page 251 Question 9

For the piston to move faster, the period of the sine curve must be reduced. So, in the equation $y = a \sin b(x - c) + d$, parameter *b* would be affected.

Section 5.2 Page 252 Question 10

a) Rewrite $f(x) = 4 \sin (2x - 6) + 12$ as $f(x) = 4 \sin 2(x - 3) + 12$. So, the phase shift, *c*, is 3 units to the right. Stewart is correct.

b)



Section 5.2 Page 252 Question 11

If the range of the original function is $\{y \mid -3 \le y \le 3, y \in \mathrm{R}\}$, then $a = 3$.

a) For $d = 2$:

maximum: $d + |a| = 2 + |3| = 5$

minimum: $d - |a| = 2 - |3| = -1$

So, the range of the function becomes $\{y \mid -1 \le y \le 5, y \in \mathrm{R}\}$.

b) For $d = -3$:

maximum: $d + |a| = -3 + |3| = 0$

minimum: $d - |a| = -3 - |3| = -6$

So, the range of the function becomes $\{y \mid -6 \le y \le 0, y \in \mathrm{R}\}$.

c) For $d = -10$:

maximum: $d + |a| = -10 + |3|$
$= -7$

minimum: $d - |a| = -10 - |3|$
$= -13$

So, the range of the function becomes $\{y \mid -13 \le y \le -7, y \in \mathrm{R}\}$.

d) For $d = 8$:

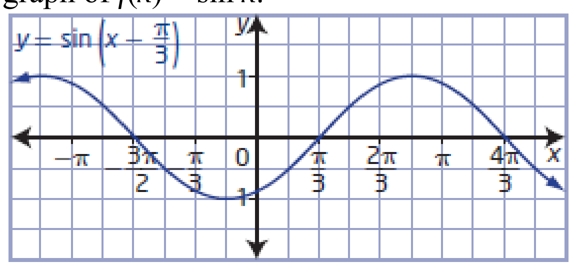
maximum: $d + |a| = 8 + |3|$
$= 11$

minimum: $d - |a| = 8 - |3|$
$= 5$

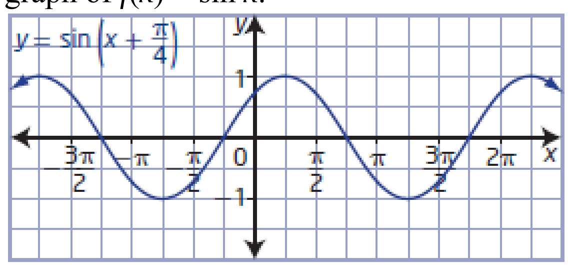
So, the range of the function becomes $\{y \mid 5 \le y \le 11, y \in \mathrm{R}\}$.

Section 5.2 Page 252 Question 12

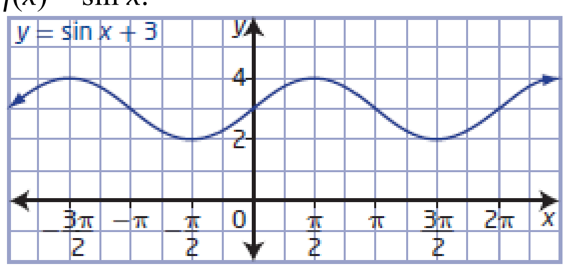
a) For the graph of $f\left(x - \frac{\pi}{3}\right)$, apply a translation of $\frac{\pi}{3}$ units to the right to the graph of $f(x) = \sin x$.



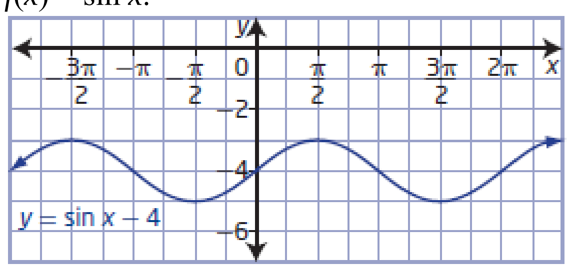
b) For the graph of $f\left(x + \frac{\pi}{4}\right)$, apply a translation of $\frac{\pi}{4}$ units to the left to the graph of $f(x) = \sin x$.



c) For the graph of $f(x) + 3$, apply a translation of 3 units up to the graph of $f(x) = \sin x$.



c) For the graph of $f(x) - 4$, apply a translation of 4 units down to the graph of $f(x) = \sin x$.



Section 5.2 Page 252 Question 13

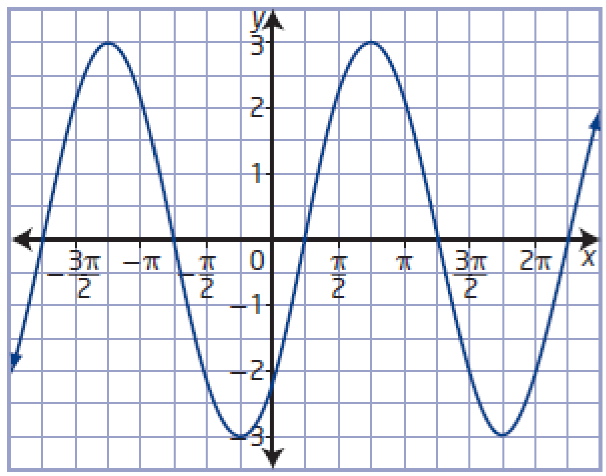
Given the range of a trigonometric function in the form $y = a \sin b(x - c) + d$ is $\{y \mid -13 \le y \le 5, y \in \mathrm{R}\}$, then the amplitude is

$$\text{amplitude} = \frac{\text{maximum value} - \text{minimum value}}{2}$$
$$a = \frac{5 - (-13)}{2}$$
$$a = 9$$

The mid-line is halfway between the maximum and minimum values.

$$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$
$$= \frac{5 + (-13)}{2}$$
$$= -4$$

Section 5.2 Page 252 Question 14



a) i) The amplitude is $\frac{3 - (-3)}{2}$, or 3.

ii) The period is $\frac{9\pi}{4} - \frac{\pi}{4}$, or 2π.

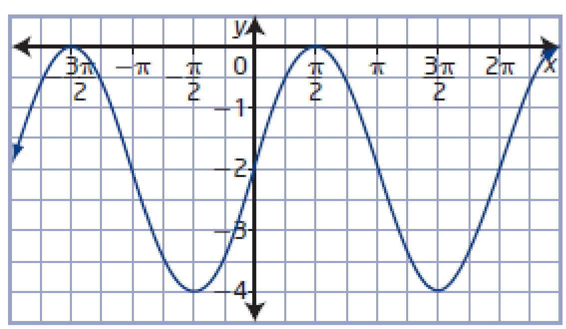
iii) The start of the first cycle of the sine curve is to the right of the y-axis. So, the phase shift is $\frac{\pi}{4}$ units to the right.

iv) There is no vertical displacement.

v) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid -3 \le y \le 3, y \in \mathrm{R}\}$.

vi) Over the interval $0 \le x \le 2\pi$, the maximum value of y is 3, which occurs at $x = \frac{3\pi}{4}$.

vii) Over the interval $0 \le x \le 2\pi$, the minimum value of y is –3, which occurs at $x = \frac{7\pi}{4}$.



b) i) The amplitude is $\frac{0 - (-4)}{2}$, or 2.

ii) The period is $2\pi - 0$, or 2π.

iii) The start of the first cycle of the cosine curve is to the right of the y-axis. So, the phase shift is $\frac{\pi}{2}$ units to the right.

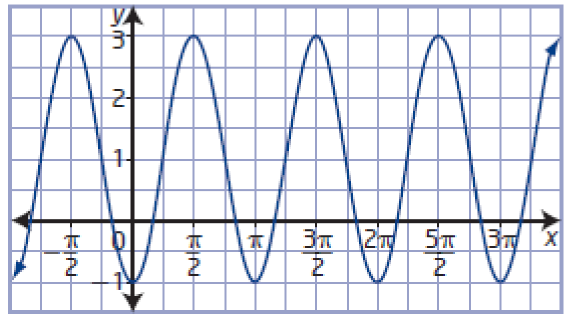
iv) The vertical displacement is 2 units down.

v) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid -4 \le y \le 0, y \in \mathrm{R}\}$.

vi) Over the interval $0 \le x \le 2\pi$, the maximum value of y is 3, which occurs at $x = \frac{\pi}{2}$.

vii) Over the interval $0 \le x \le 2\pi$, the minimum value of y is –3, which occurs at $x = \frac{3\pi}{2}$.

c) i) The amplitude is $\frac{3-(-1)}{2}$, or 2.



ii) The period is $\frac{5\pi}{4} - \frac{\pi}{4}$, or π.

iii) The start of the first cycle of the sine curve is to the right of the y-axis. So, the phase shift is $\frac{\pi}{4}$ units to the right.

iv) The vertical displacement is 1 unit up.

v) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid -1 \le y \le 3, y \in \mathrm{R}\}$.

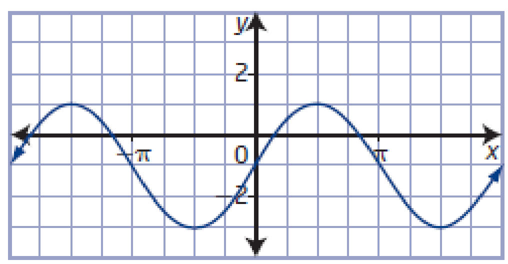
vi) Over the interval $0 \le x \le 2\pi$, the maximum value of y is 3, which occurs at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

vii) Over the interval $0 \le x \le 2\pi$, the minimum value of y is –1, which occurs at $x = 0$, $x = \pi$, and $x = 2\pi$.

Section 5.2 Page 253 Question 15

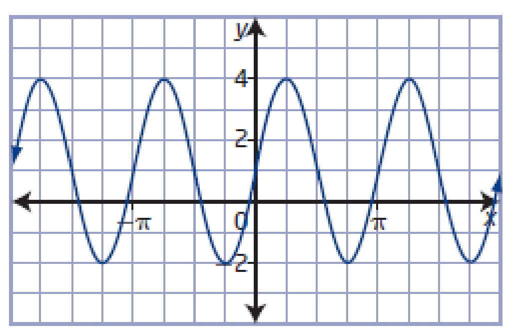
a) The amplitude, a, is $\frac{1-(-3)}{2}$, or 2.

The period is $2\pi - 0$, or 2π. So, $b = 1$.
The start of the first cycle of the sine curve is on the y-axis. So, $c = 0$.
The vertical displacement is 1 unit down.
Locating the mid-line gives $d = -1$.
So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 2 \sin x - 1$.

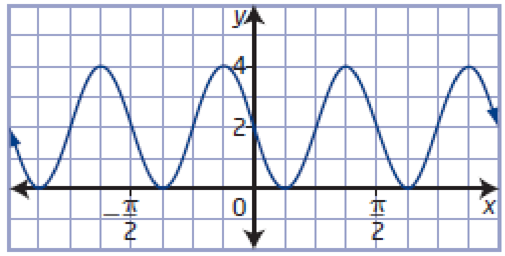


b) The amplitude, a, is $\frac{4-(-2)}{2}$, or 3.

The period is $\pi - 0$, or π. So, $b = 2$.
The start of the first cycle of the sine curve is on the y-axis. So, $c = 0$.
The vertical displacement is 1 unit up. Locating the mid-line gives $d = 1$.
So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 3 \sin 2x + 1$.



c) The amplitude, a, is $\frac{4-0}{2}$, or 2.

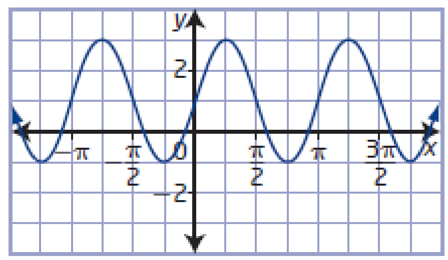


The period is $\frac{\pi}{2} - 0$, or $\frac{\pi}{2}$. So, $b = 4$.
The start of the first cycle of the sine curve is to the right of the y-axis. So, $c = \frac{\pi}{4}$.
The vertical displacement is 2 units up. Locating the mid-line gives $d = 2$.

So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 2\sin 4\left(x - \frac{\pi}{4}\right) + 2$.

Section 5.2 Page 253 Question 16

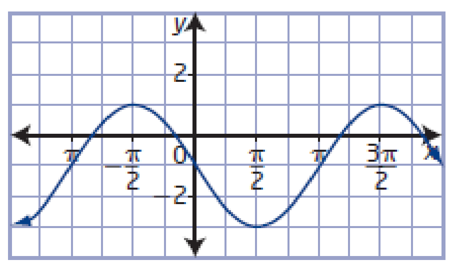
a) The amplitude, a, is $\frac{3-(-1)}{2}$, or 2.



The period is $\pi - 0$, or π. So, $b = 2$.
The start of the first cycle of the cosine curve is to the right of the y-axis. So, $c = \frac{\pi}{4}$.
The vertical displacement is 1 unit up. Locating the mid-line gives $d = 1$.

So, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = 2\cos 2\left(x - \frac{\pi}{4}\right) + 1$.

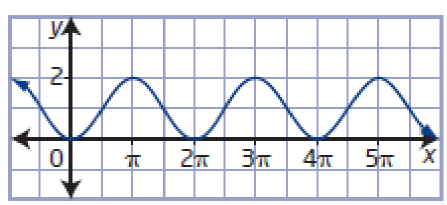
b) The amplitude, a, is $\frac{1-(-3)}{2}$, or 2.



The period is $\frac{3\pi}{2} - \left(-\frac{\pi}{2}\right)$, or 2π. So, $b = 1$.
The start of the first cycle of the cosine curve is to the left of the y-axis. So, $c = -\frac{\pi}{2}$.
The vertical displacement is 1 unit down. Locating the mid-line gives $d = -1$.

So, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = 2\cos\left(x + \frac{\pi}{2}\right) - 1$.

c) The amplitude, a, is $\frac{2-0}{2}$, or 1.

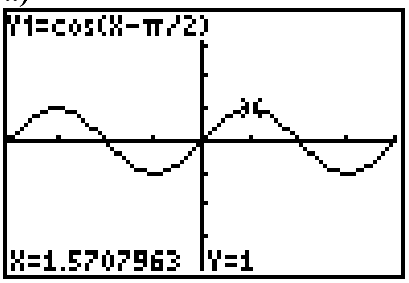


The period is $3\pi - \pi$, or 2π. So, $b = 1$.
The start of the first cycle of the cosine curve is to the right of the y-axis. So, $c = \pi$.

The vertical displacement is 1 unit up. Locating the mid-line gives $d = 1$.
So, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = \cos (x - \pi) + 1$.

Section 5.2 Page 253 Question 17

a)



b) An equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = \sin x$.

c) The graph of the cosine function shifted $\frac{\pi}{2}$ units to the right is equivalent to the graph of the sine function.

Section 5.2 Page 253 Question 18

The graph of the sine function shifted $\frac{\pi}{2}$ units to the left is equivalent to the graph of the cosine function: $g(x) = \sin\left(x + \frac{\pi}{2}\right)$.

Section 5.2 Page 253 Question 19

a) i) For one cycle $30° \le x \le 390°$ of a cosine function in the form $y = 3 \cos b(x - c)$, the phase shift is 30° to the right, the period is 360°, and the x-intercepts are 120° and 300°.
ii) The minimum is located at (210°, –3) and the maximum is located at (30°, 3) and (390°, 3).

b) i) For one cycle $\frac{\pi}{4} \le x \le \frac{5\pi}{4}$ of a cosine function in the form $y = 3 \cos b(x - c)$, the phase shift is $\frac{\pi}{4}$ to the right, the period is π, and the x-intercepts are $\frac{\pi}{2}$ and π.

ii) The minimum is located at $\left(\frac{3\pi}{4}, -3\right)$ and the maximum is located at $\left(\frac{\pi}{4}, 3\right)$ and $\left(\frac{5\pi}{4}, 3\right)$.

Section 5.2 Page 253 Question 20

The amplitude, *a*, is $\frac{5100-5000}{2}$, or 50.

The period is 2.75 – 1.75, or 1 mi, which is equivalent to 5280 ft. So, $b = \frac{\pi}{2640}$.

The start of the first cycle of the cosine curve is 1.75 mi, or 9240 ft, to the right of the *y*-axis. So, *c* = 9240.

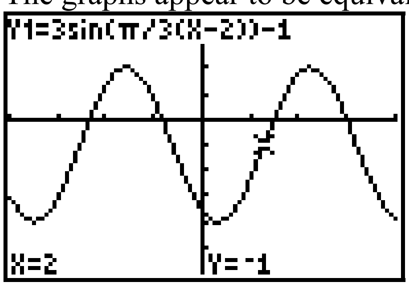
The vertical displacement is $\frac{5100+5000}{2}$, or 5050 units up. So, *d* = 5050.

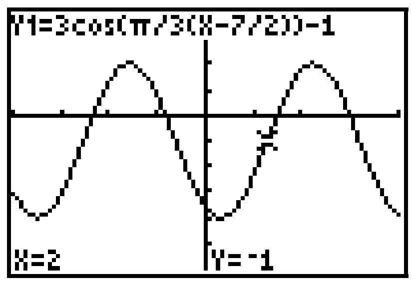
So, an equation in the form *y* = *a* cos *b*(*x* – *c*) + *d* for the pattern is

$y = 50\cos\frac{\pi}{2640}(x-9240)+5050$.

Section 5.2 Page 254 Question 21

The graphs appear to be equivalent.





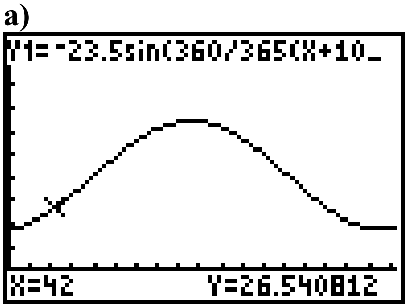
Section 5.2 Page 254 Question 22

An outside noise sound wave with amplitude 4 and period $\frac{\pi}{2}$ has equation *y* = 4 sin 4*x*.

To cancel this, the headphones must produce a sound wave that is shifted by π units: *y* = 4 sin 4(*x* + π).

Section 5.2 Page 254 Question 23

a)

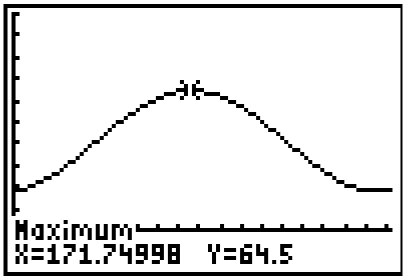


b) February 12 is day 42. Use the graph from part a) or the formula.

$$A = -23.5\sin\frac{360}{365}(x+102)+41$$
$$= -23.5\sin\frac{360}{365}(42+102)+41$$
$$= -23.5\sin\frac{51\,840}{365}+41$$
$$\approx 26.5$$

The sun's angle of inclination on February 12 is approximately 26.5°.

c) Find the maximum value of the graph. The angle of inclination is the greatest in Estevan on approximately day 171 or June 21.

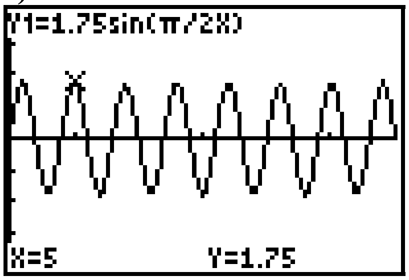


Section 5.2 Page 254 Question 24

a) The period of $r = 1.75 \sin \frac{\pi}{2} t$ is $\frac{2\pi}{\left|\frac{\pi}{2}\right|}$, or 4. So, the time for one full respiratory cycle is 4 s.

b) The number of cycles per minute is $\frac{60}{4}$, or 15.

c)



d) Substitute $t = 30$.

$$r = 1.75 \sin \frac{\pi}{2} t$$
$$= 1.75 \sin \frac{\pi}{2}(30)$$
$$= 1.75 \sin 15\pi$$
$$= 0$$

The rate of air flow at a time of 30 s is 0 L/s. In the context of the respiratory cycle, this corresponds to when the lungs are either completely full or completely empty.

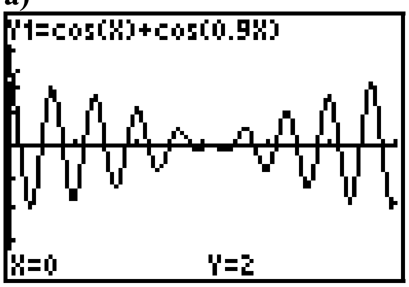
e) Substitute $t = 7.5$.

$$\begin{aligned} r &= 1.75 \sin \frac{\pi}{2} t \\ &= 1.75 \sin \frac{\pi}{2}(7.5) \\ &= 1.75 \sin 3.75\pi \\ &\approx -1.237 \end{aligned}$$

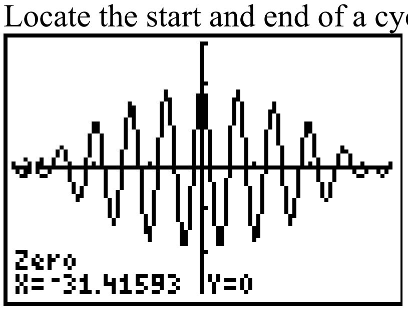
The rate of air flow at a time of 7.5 s is approximately –1.237 L/s. In the context of the respiratory cycle, this corresponds to a portion of the cycle when air is flowing out of the lungs.

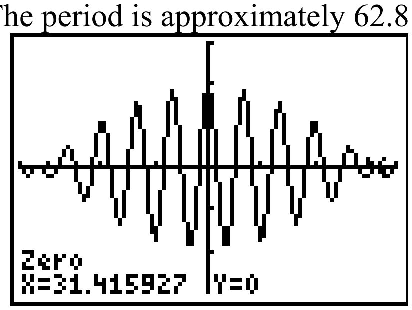
Section 5.2 Page 254 Question 25

a)



b) From the graph, the maximum value is 2 and the minimum value is approximately –2. So, the amplitude is 2.
Locate the start and end of a cycle. The period is approximately 62.8 or 20π.





Section 5.2 Page 254 Question 26

a) From question 17, $\sin x = \cos (x - 90°)$ or $\sin x = \cos \left(x - \frac{\pi}{2}\right)$.

i) Then, $4 \sin (x - 30°) = 4 \cos (x - \mathbf{120°})$.

ii) Then, $2 \sin \left(x - \frac{\pi}{4}\right) = \cos \left(x - \frac{\mathbf{3\pi}}{\mathbf{4}}\right)$.

iii) Then, $-3\cos\left(x-\frac{\pi}{2}\right) = 3\sin(x-\boldsymbol{\pi})$.

iv) First rewrite, cos (–2*x* + 6π) as cos (–2(*x* – 3π)). Use the fact that $\cos(-x) = \sin\left(x+\frac{\pi}{2}\right)$. Then, $\cos(-2x) = \sin 2\left(x+\frac{\pi}{4}\right)$.

$$\begin{aligned}\cos(-2(x-3\pi)) &= \sin 2\left(x+\frac{\pi}{4}-3\pi\right)\\ &= \sin 2\left(x-\frac{11\pi}{4}\right)\\ &= \sin 2\left(x-\boldsymbol{\frac{\pi}{4}}\right)\end{aligned}$$

b) See explanation for part a) iv) above.

Section 5.2 Page 255 Question 27

Examples:

a) Given: sine function with amplitude 3, maximum $\left(-\frac{\pi}{2}, 5\right)$, and nearest maximum to the right at $\left(\frac{3\pi}{2}, 5\right)$.

$a = 3$

The period is $\frac{3\pi}{2}-\left(-\frac{\pi}{2}\right)$, or 2π. So, $b = 1$.

The maximum for a sine curve appears at one-quarter of the cycle, or $x = \frac{\pi}{2}$. So, $c = -\pi$.

$$\begin{aligned}\text{maximum} &= d + |a|\\ 5 &= d + |3|\\ d &= 2\end{aligned}$$

An equation for the sine function is $y = 3\sin(x + \pi) + 2$.

Alternatively, the maximum for a sine curve appears at three-quarters of the cycle to the left of the *y*-axis, or $x = -\frac{3\pi}{2}$. So, $c = \pi$. Then, an equation for the sine function is $y = 3\sin(x - \pi) + 2$.

b) Given: sine function with amplitude 3, minimum $\left(\frac{\pi}{4}, -2\right)$, and nearest maximum to the right at $\left(\frac{3\pi}{4}, 4\right)$.

$a = 3$

The distance between the minimum and the maximum of a sine function is one-half the period. The period is $2\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)$, or π. So, $b = 2$.

The minimum for a sine curve appears at three-quarters of the cycle, or $x = \frac{3\pi}{4}$.

So, $c = -\frac{\pi}{2}$.

minimum = $d - |a|$
$-2 = d - |3|$
$d = 1$

An equation for the sine function is $y = 3\sin 2\left(x+\frac{\pi}{2}\right)+1$.

Alternatively, the minimum for a sine curve appears at one-quarter of the cycle to the left of the y-axis, or $x = -\frac{\pi}{4}$. So, $c = \frac{\pi}{2}$. Then, an equation for the sine function is $y = 3\sin 2\left(x-\frac{\pi}{2}\right)+1$.

c) Given: sine function with minimum (–π, 3) and nearest maximum to the right at (0, 7).

The amplitude is $\frac{7-3}{2}$, or 2.

The distance between the minimum and the maximum of a sine function is one-half the period. The period is 2(0 – (–π)), or 2π. So, $b = 1$.

The maximum for a sine curve appears at one-quarter of the cycle, or $x = \frac{\pi}{2}$. So, $c = -\frac{\pi}{2}$.

minimum = $d - |a|$
$3 = d - |2|$
$d = 5$

An equation for the sine function is $y = 2\sin\left(x+\frac{\pi}{2}\right)+5$.

Alternatively, the maximum for a sine curve appears at three-quarters of the cycle to the left of the y-axis, or $x = -\frac{3\pi}{2}$. So, $c = \frac{3\pi}{2}$. Then, an equation for the sine function is $y = 2\sin\left(x-\frac{3\pi}{2}\right)+5$.

d) Given: sine function with minimum (90°, −6) and nearest maximum to the right at (150°, 4).

The amplitude is $\frac{4-(-6)}{2}$, or 5.

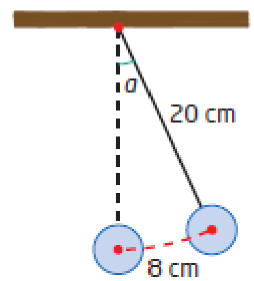
The distance between the minimum and the maximum of a sine function is one-half the period. The period is 2(150° – 90°), or 120°. So, $b = 3$.

The maximum for a sine curve appears at one-quarter of the cycle, or $x = 30°$. So, $c = 120°$.

$$\text{minimum} = d - |a|$$
$$-6 = d - |5|$$
$$d = -1$$

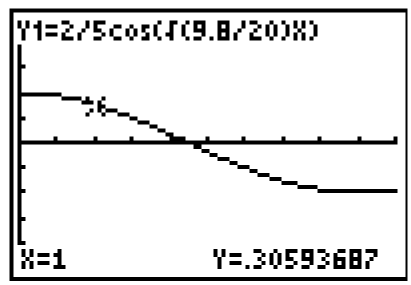
An equation for the sine function is $y = 5 \sin 3(x - 120°) - 1$.
Alternatively, since the period is 120°, the first maximum to the right of the y-axis occurs at $x = 30°$. So, $c = 0$. Then, an equation for the sine function is $y = 5 \sin 3x - 1$.

Section 5.2 Page 255 Question 28



a) Given: $P = a \cos bt$, where a represents the maximum distance from its equilibrium and b is the horizontal stretch factor.
Use the formula for the length of an arc to determine a.
arc length = (central angle)(radius)

$$8 = a(20)$$
$$a = \frac{8}{20}$$
$$a = \frac{2}{5}$$



The period is given by $2\pi\sqrt{\frac{20}{9.8}}$.

So, $b = \frac{2\pi}{2\pi\sqrt{\frac{20}{9.8}}}$ or $\sqrt{\frac{9.8}{20}}$.

So, graph $P = \frac{2}{5}\cos\sqrt{\frac{9.8}{20}}t$ for $0 \le t \le 5$.

b) Substitute $t = 6$.

$$P = \frac{2}{5}\cos\sqrt{\frac{9.8}{20}}t$$
$$= \frac{2}{5}\cos\sqrt{\frac{9.8}{20}}(6)$$
$$= -0.196...$$

The position of the pendulum is approximately –0.20 radians, or 3.9 cm along the arc to the left of the vertical.

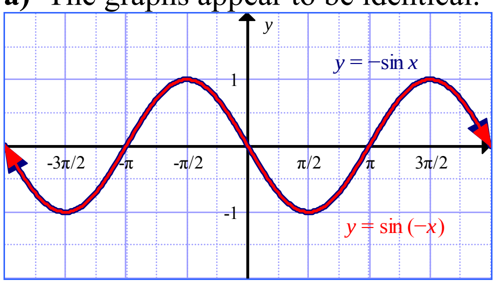
Section 5.2 Page 255 Question C1

For the graph of a sinusoidal function of the form $y = a \sin b(x - c) + d$,
- the parameter a changes the amplitude to $|a|$
- the parameter b changes the period to $\frac{360°}{|b|}$ or $\frac{2\pi}{|b|}$
- the parameter c changes the phase shift
- the parameter d changes the vertical displacement

These parameters can be related to function transformations from Chapter 1:
- vertical stretch by a factor of $|a|$ and reflected in the x-axis if $a < 0$
- horizontal stretch by a factor of $\frac{1}{|b|}$ and reflected in the y-axis if $b < 0$
- horizontal translation of c units, to right if $c > 0$ and to left if $c < 0$
- vertical translation of d units, up if $d > 0$ and down if $d < 0$

Section 5.2 Page 255 Question C2

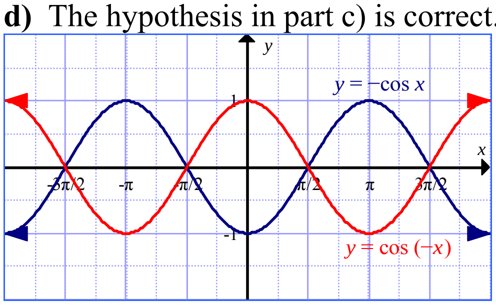
a) The graphs appear to be identical.



b) The graph of $y = –\sin x$ is a reflection in the x-axis of the graph of $y = \sin x$. The graph of $y = \sin (–x)$ is a reflection in the y-axis of the graph of $y = \sin x$. These result in the same graph.

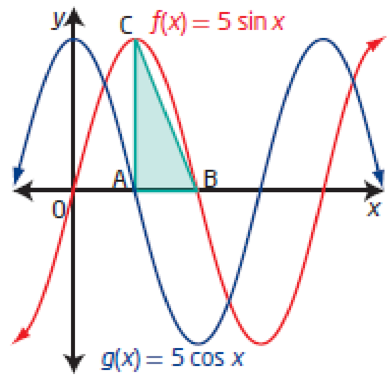
c) The graph of $y = –\cos x$ is a reflection in the x-axis of the graph of $y = \cos x$. The graph of $y = \cos (–x)$ is a reflection in the y-axis of the graph of $y = \cos x$. The two graphs will be reflections of each in the x-axis because the graph $y = \cos (–x)$ is the same as the graph $y = \cos x$.

d) The hypothesis in part c) is correct.



Section 5.2 Page 255 Question C3

Determine the height and base of the triangle.
The height, CA, is the amplitude of $f(x) = 5 \sin x$, or 5.
The base, AB, is the distance between the x-intercepts of the two graphs, $\pi - \frac{\pi}{2}$, or $\frac{\pi}{2}$.



Now, find the area of the triangle.

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}\left(\frac{\pi}{2}\right)(5)$$
$$= \frac{5\pi}{4}$$

The area of △ABC is $\frac{5\pi}{4}$ units^2.

Section 5.2 Page 255 Question C4

Given: equation of a sine function $y = a \sin b(x - c) + d$, where $a > 0$ and $b > 0$.

a) For the period to be greater than 2π, $0 < b < 1$. The other parameters do not affect the period.

b) For the amplitude to be greater than 1 unit, $a > 1$. The other parameters do not affect the amplitude.

c) For the graph to pass through the origin, $c = 0$ and $d = 0$.

d) For the graph to have no x-intercepts, the entire graph must lie above or below the x-axis. Either $d - a > 0$ or $d + a < 0$, respectively. The other parameters do not affect the x-intercepts.

e) For the graph to have a y-intercept at a, $c = -\frac{\pi}{2}$ and $d = 0$.

f) For the length of one cycle to be 120°, $b = 3$. The other parameters do not affect the period.

Section 5.3 The Tangent Function

Section 5.3 Page 262 Question 1

a) $\tan \theta = \frac{1}{1}$ in quadrant I

$\tan \theta = 1$
$\theta = \tan^{-1} 1$
$\theta = 45°$

b) $\tan \theta = \frac{-1.7}{1}$ in quadrant II

$\tan \theta = -1.7$
$\theta = \tan^{-1} (-1.7)$
$\theta \approx 120.5°$

c) $\tan \theta = \frac{-1.7}{1}$ in quadrant IV

$\tan \theta = -1.7$
$\theta = \tan^{-1} (-1.7)$
$\theta \approx 300.5°$

d) $\tan \theta = \frac{1}{1}$ in quadrant III

$\tan \theta = 1$
$\theta = \tan^{-1} 1$
$\theta = 225°$

Section 5.3 Page 263 Question 2

a) $\tan \frac{\pi}{2}$ is undefined

b) $\tan \frac{3\pi}{4} = -1$

c) $\tan \left(-\frac{7\pi}{4}\right) = 1$

d) $\tan 0 = 0$

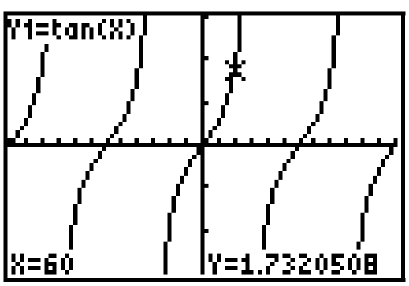
e) $\tan \pi = 0$

f) $\tan \frac{5\pi}{4} = 1$

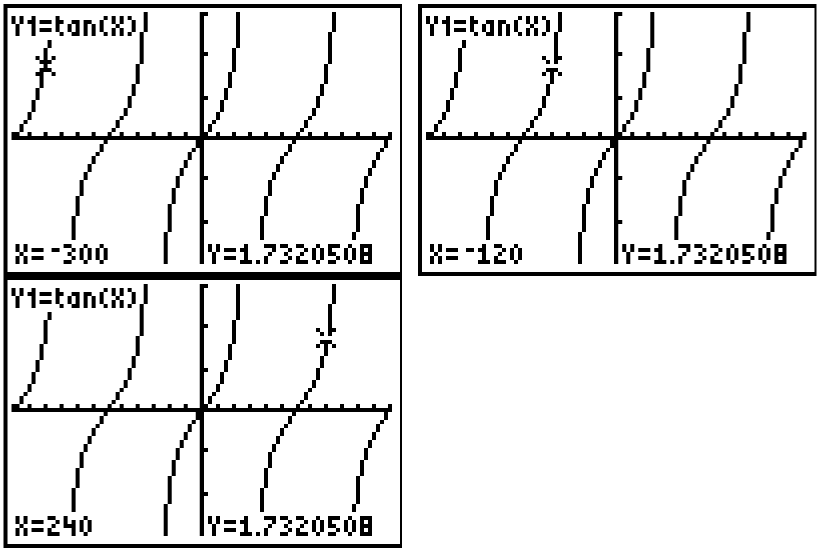
Section 5.3 Page 263 Question 3

No, $y = \tan x$ does not have an amplitude because it has no maximum or minimum values.

Section 5.3 Page 263 Question 4

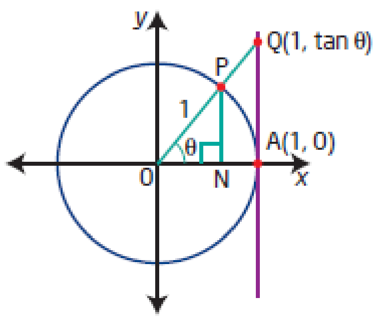


In the domain [–360°, 360°], I predict the other values of x with the same value as tan 60° will be –300°, –120°, and 240°.



Section 5.3 Page 263 Question 5

Given: △PON and △QOA are similar triangles.



Any point on the unit circle has coordinates (cos θ, sin θ).
Then, from properties of similar triangles

$$\frac{QA}{PN} = \frac{AO}{NO}$$

$$\frac{\tan\theta}{\sin\theta} = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Section 5.3 Page 263 Question 6

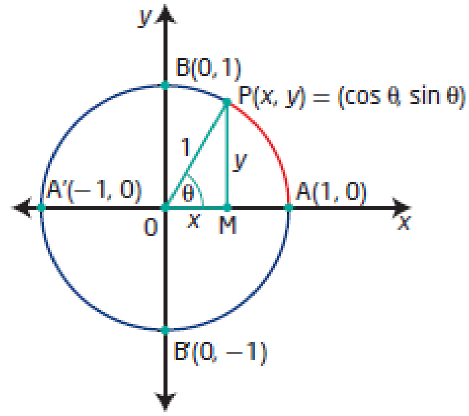
a) For a terminal arm of angle θ that intersects the unit circle at P(x, y), slope = $\frac{y}{x}$.

b) Any point on the unit circle also has coordinates (cos θ, sin θ). So, the slope of the terminal arm can be expressed as slope = tan θ.

c) In terms of sine and cosine, slope = $\frac{\sin\theta}{\cos\theta}$.

d) You can determine tan θ when the coordinates of point P are known by tan θ = $\frac{y}{x}$.

Section 5.3 Page 263 Question 7



a) In △POM, $\tan \theta = \dfrac{y}{x}$.

b) Using cos θ and sin θ, $\tan \theta = \dfrac{\sin \theta}{\cos \theta}$.

c) From parts a) and b), $y = \sin \theta$ and $x = \cos \theta$.

Section 5.3 Page 264 Question 8

a)

θ	tan θ
89.5°	114.59
89.9°	572.96
89.999°	57 295.78
89.999 999°	57 295 779.51

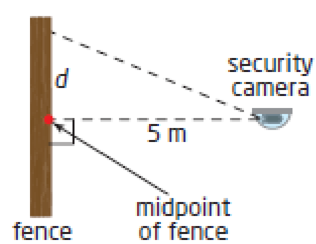
b) As θ approaches 90°, tan θ approaches infinity.

c) I predict a similar result as θ approaches 90° from the other direction.

θ	tan θ
90.5°	−114.59
90.01°	−5729.58
90.001°	−57 295.78
90.000 001°	−57 295 779.51

As θ approaches 90° from the other direction, tan θ approaches negative infinity.

Section 5.3 Page 264 Question 9



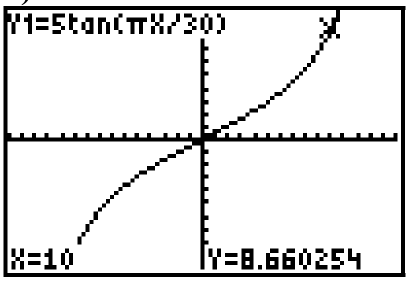
a) From the diagram, $\tan \theta = \dfrac{d}{5}$.

The period is 60 s. So, $\theta = \dfrac{2\pi}{60}t$ or $\dfrac{\pi}{30}t$. Then,

$$\tan \frac{\pi}{30}t = \frac{d}{5}$$

$$d = 5 \tan \frac{\pi}{30}t$$

b)



c) Substitute $t = 10$.

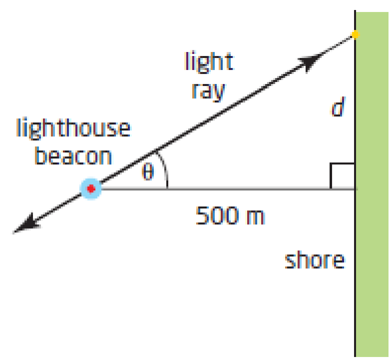
$$d = 5 \tan \frac{\pi}{30} t$$
$$= 5 \tan \frac{\pi}{30}(10)$$
$$= 5 \tan \frac{\pi}{3}$$
$$= 5\sqrt{3}$$
$$\approx 8.7$$

The distance from the midpoint of the fence at $t = 10$ s is 8.7 m, to the nearest tenth of a metre.

d) When $t = 15$, $d = 5 \tan \frac{\pi}{2}$, which is undefined. The camera is pointing along a line parallel to the wall and is turning away from the wall.

Section 5.3 Page 264 Question 10

a) From the diagram, $\tan \theta = \frac{d}{500}$.

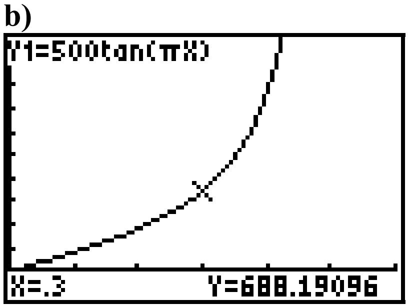


The period is 2 min. So, $\theta = \frac{2\pi}{2} t$ or πt. Then,

$$\tan \pi t = \frac{d}{500}$$
$$d = 500 \tan \pi t$$

The equation that expresses the distance, d, in metres, as a function of time, t, in minutes is $d = 500 \tan \pi t$.

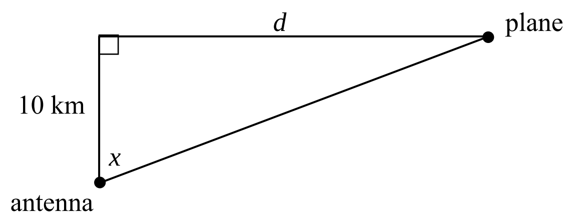
b)

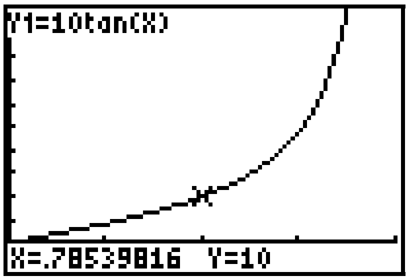


c) The asymptote in the graph at $\theta = 90°$, or $t = \frac{90}{\pi}$ represents the moment when the ray of light shines along a line that is parallel to the shore.

Section 5.3 Page 265 Question 11

$$\tan x = \frac{d}{10}$$
$$d = 10 \tan x$$



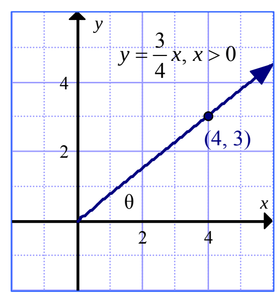


Section 5.3 Page 265 Question 12

a) Since Andrea measures the length of the shadow of a pole and calculates the slope of the string from the top of the pole to the ground, I would expect the graph representing her data to show a tangent function.

b) When the Sun is directly overhead and no shadow results, the slope of the string is undefined. This is represented by a vertical asymptote in the graph.

Section 5.3 Page 265 Question 13



a) Choose the integral point (4, 3).

b) From the diagram, $\tan \theta = \frac{3}{4}$.

c) The slope of the line is $\tan \theta$.

Section 5.3 Page 265 Question 14

a) Substitute $x = 0.5$.

Left Side	Right Side
$\tan x$	$x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315}$
$= \tan 0.5$	$= 0.5 + \frac{0.5^3}{3} + \frac{2(0.5)^5}{15} + \frac{17(0.5)^7}{315}$
$= 0.546\,302...$	$= 0.546\,254...$
≈ 0.5463	≈ 0.5463

Left Side = Right Side

b) Substitute $x = 0.5$.

Left Side

$\sin x$
$= \sin 0.5$
$= 0.479\ 425...$
≈ 0.4794

Right Side

$x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$
$= 0.5 - \frac{0.5^3}{6} + \frac{0.5^5}{120} - \frac{0.5^7}{5040}$
$= 0.479\ 525...$
≈ 0.4795

Left Side ≈ Right Side

c) Substitute $x = 0.5$.

Left Side

$\cos x$
$= \cos 0.5$
$= 0.877\ 582...$
≈ 0.8776

Right Side

$1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$
$= 1 - \frac{0.5^2}{2} + \frac{0.5^4}{24} - \frac{0.5^6}{720}$
$= 0.877\ 582...$
≈ 0.8776

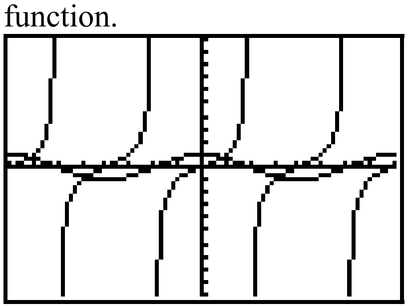
Left Side = Right Side

Section 5.3 Page 265 Question C1

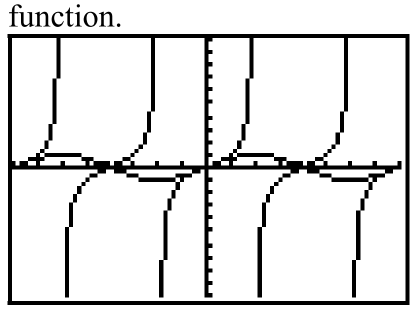
While the domain of both sin x and cos x is $\{x \mid x \in \mathrm{R}\}$, the domain of tan x is $\left\{x \mid x \neq \frac{\pi}{2} + n\pi, x \in \mathrm{R}, n \in \mathrm{I}\right\}$. The domains differ because tan x can be expressed as $\frac{\sin x}{\cos x}$. In this form, you can see that the domain is restricted to non-zero values of cos x.

Section 5.3 Page 265 Question C2

a) The graph of the tangent function has asymptotes at the same locations as the x-intercepts of the graph of the cosine function.



b) The graph of the tangent function has x-intercepts at the same locations as the x-intercepts of the graph of the sine function.



Section 5.3 Page 265 Question C3

Example: A circular or periodic function repeats its values over a specific period. In the case of $y = \tan x$, the period is π. So, the equation $\tan (x + \pi) = \tan x$ is true for all values of x in the domain of $\tan x$.

Section 5.4 Equations and Graphs of Trigonometric Functions

Section 5.4 Page 275 Question 1

a) From the graph, the solutions to the equation $\sin x = 0$ in the interval $0 \le x \le 2\pi$ are $x = 0$, π, and 2π.

b) The general solution for $\sin x = 0$ is $x = n\pi$, $n \in I$.

c) $\sin 3x = 0$

$\sin^{-1} 0 = 3x$

$3x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots$

For the interval $0 \le x \le 2\pi$,

$$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$$

Section 5.4 Page 275 Question 2

Examples:

a) From the graph, $x \approx 1.3$ and $x \approx 4.6$.

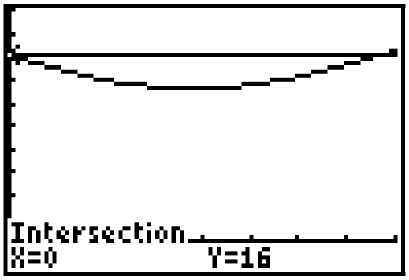
b) From the graph, $x \approx -3, -2, 0.1, 1.1, 3.2, 4.2, 6.3$, and 7.3.

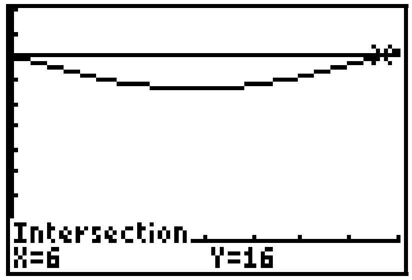
Section 5.4 Page 275 Question 3

From the graph, the solutions to the equation $4 \cos (2(x - 60°)) + 6 = 3$ are $x \approx -50°, -10°, 130°, 170°, 310°$, and $350°$.

Section 5.4 Page 275 Question 4

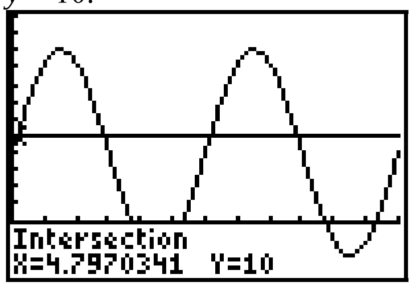
a) Determine the points of intersection for the graphs of $y = -2.8 \sin\left(\frac{\pi}{6}(x - 12)\right) + 16$ and $y = 16$.

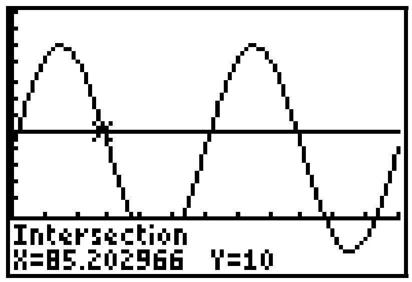


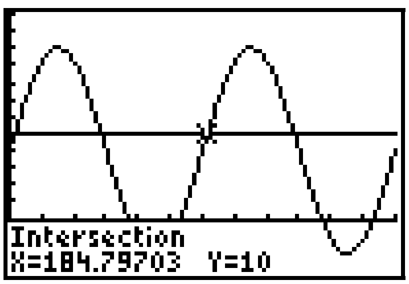


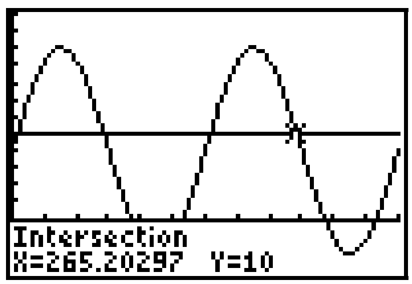
In the interval $0 \le x \le 2\pi$, the solutions are $x = 0$ and $x = 6$.

b) Determine the points of intersection for the graphs of $y = 12 \cos (2(x - 45°)) + 8$ and $y = 10$.



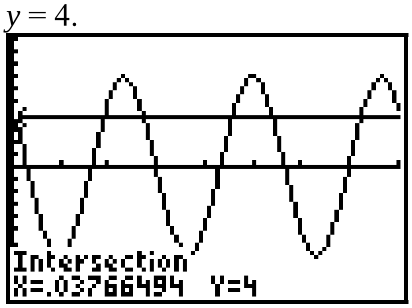


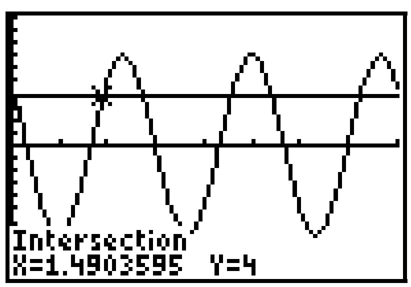


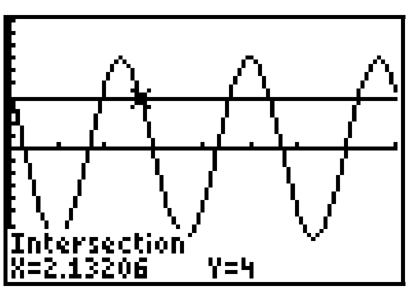


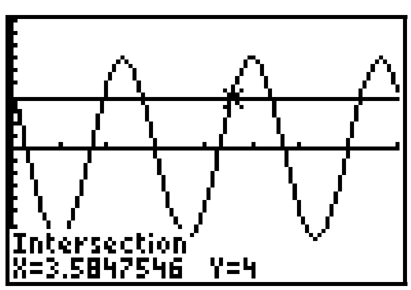
In the interval $0 \le x \le 360°$, the solutions are $x \approx 4.8°$, 85.2°, 184.8°, and 265.2°.

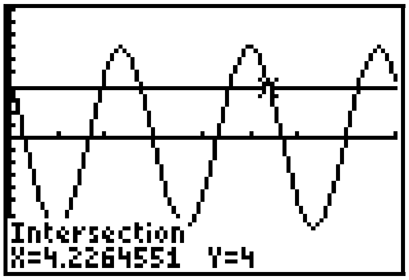
c) Determine the points of intersection for the graphs of $y = 7 \cos (3x - 18)$ and $y = 4$.

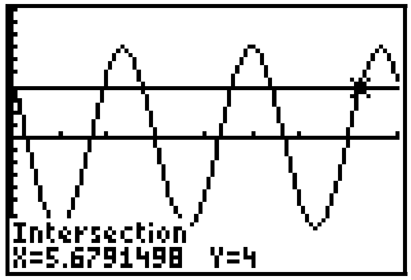






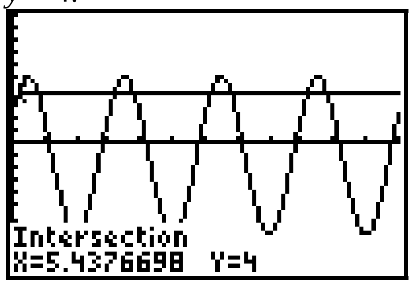


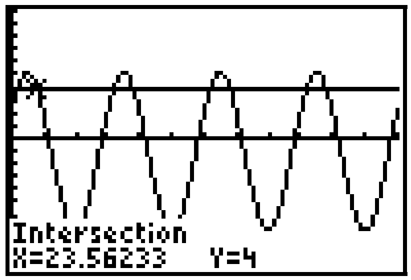


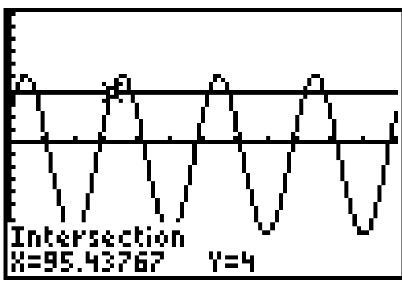


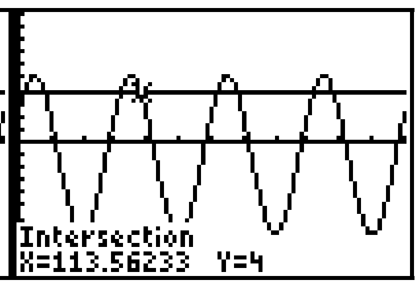
In the interval $0 \le x \le 2\pi$, the solutions are $x \approx 0.04$, 1.49, 2.13, 3.58, 4.23, and 5.68.

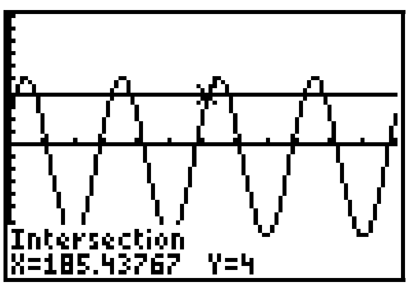
d) Determine the points of intersection for the graphs of $y = 6.2 \sin (4(x + 8°)) - 1$ and $y = 4$.

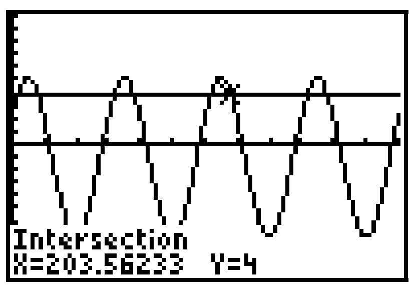


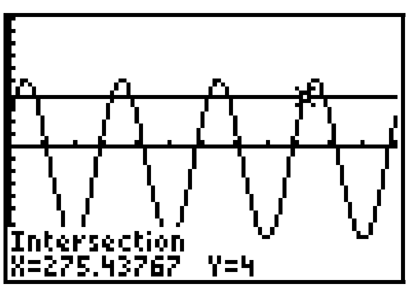


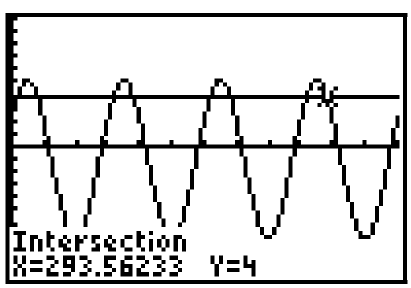








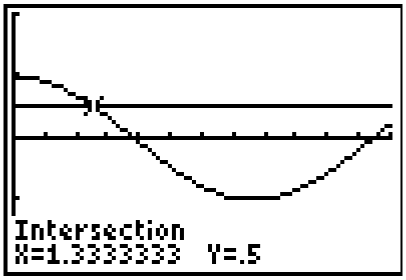




In the interval $0 \le x \le 360°$, the solutions are $x \approx 5.44°$, 23.56°, 95.44°, 113.56°, 185.44°, 203.56°, 275.44°, and 293.56°.

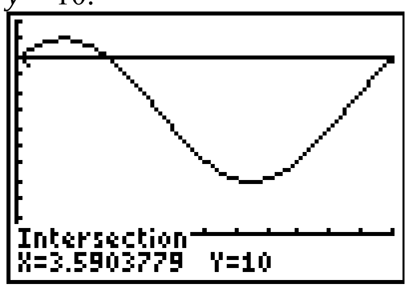
Section 5.4 Page 275 Question 5

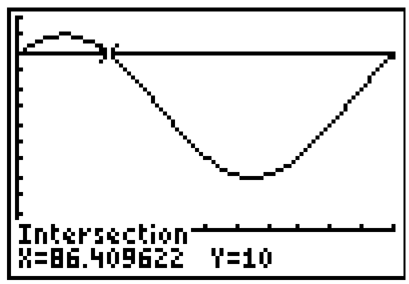
a) Determine the points of intersection for the graphs of $y = \sin\left(\frac{\pi}{4}(x-6)\right)$ and $y = 0.5$.



In the interval $0 \le x \le 2\pi$, the solution is $x = \frac{4}{3}$ or approximately 1.33.

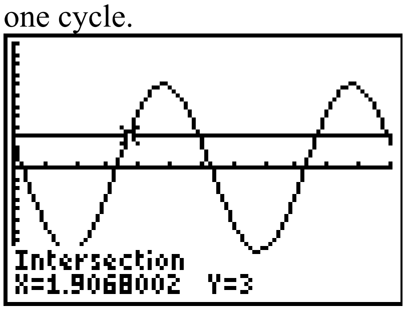
b) Determine the points of intersection for the graphs of $y = 4 \cos (x - 45°) + 7$ and $y = 10$.

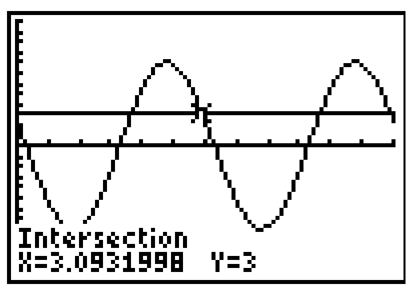




In the interval $0 \le x \le 360°$, the solutions are $x \approx 3.59°$ and 86.41°.

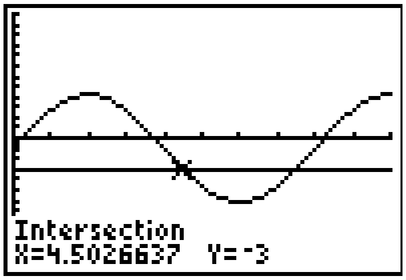
c) Determine the points of intersection for the graphs of $y = 8 \cos (2x - 5)$ and $y = 3$ in one cycle.

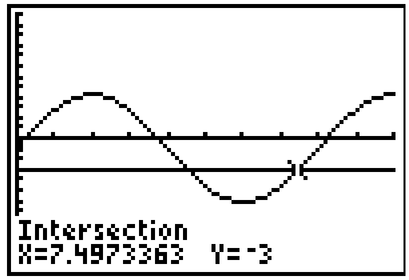




The general solution is $x \approx 1.91 + \pi n$ and $x \approx 3.09 + \pi n$, where $n \in I$.

d) Determine the points of intersection for the graphs of $y = 5.2 \sin (45(x + 8°)) - 1$ and $y = -3$ in one cycle.





The general solution is $x \approx 4.5° + (8°)n$ and $x \approx 7.5° + (8°)n$, where $n \in I$.

Section 5.4 Page 276 Question 6

a) For the population of a lakeside town with large numbers of seasonal residents that is modelled by the function $P(t) = 6000 \sin (t - 8) + 8000$, a possible domain is domain $\{t \mid t \geq 0, t \in R\}$ and a possible range is $\{P \mid 2000 \leq P \leq 14\,000, P \in N\}$.

b) For the height of the tide on a given day that is modelled using the function $h(t) = 6 \sin (t - 5) + 7$, a possible domain is $\{t \mid t \geq 0, t \in R\}$ and a possible range is $\{h \mid 1 \leq h \leq 13, h \in R\}$.

c) For the height above the ground of a rider on a Ferris wheel that can be modelled by $h(t) = 6 \sin 3(t - 30) + 12$, a possible domain is $\{t \mid t \geq 0, t \in R\}$ and a possible range is $\{h \mid 6 \leq h \leq 18, h \in R\}$.

d) For the average daily temperature that may be modelled by the function $h(t) = 9 \cos \frac{2\pi}{365}(t - 200) + 14$, a possible domain is $\{t \mid t \geq 0, t \in R\}$ and a possible range is $\{h \mid 5 \leq h \leq 23, h \in R\}$.

Section 5.4 Page 276 Question 7

If the fly's wings flap 200 times in one second, the period of the musical note is $\frac{1}{200}$ s.

Section 5.4 Page 276 Question 8

a) Given: sine function with first maximum at (30°, 24) and first minimum to the right of the maximum at (80°, 6)

period: $2(80° - 30°) = 100°$ amplitude: $\frac{24-6}{2} = 9$ sinusoidal axis: $y = 6 + 9 = 15$

b) Given: cosine function with first maximum at (0, 4) and first minimum to the right of the maximum at $\left(\frac{2\pi}{3}, -16\right)$

period: $2\left(\frac{2\pi}{3} - 0\right) = \frac{4\pi}{3}$ amplitude: $\frac{4-(-16)}{2} = 10$
sinusoidal axis: $y = –16 + 10 = –6$

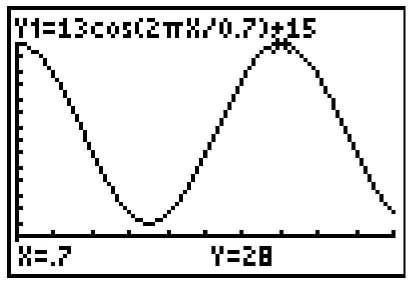
c) Given: electron oscillates back and forth 50 times per second, the maximum occurs at +10, and the minimum occurs at –10
period: $\frac{1}{50}$ s amplitude: $\frac{10-(-10)}{2} = 10$ sinusoidal axis: $y = –10 + 10 = 0$

Section 5.4 Page 276 Question 9

Given: $h(t) = 13 \cos\left(\frac{2\pi}{0.7}t\right) + 15$

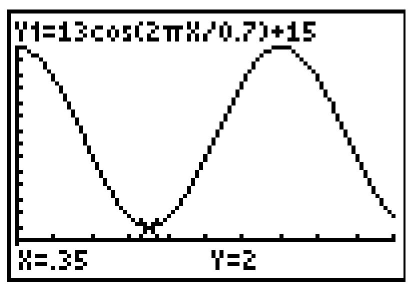
a) The maximum height of the point is 15 + 13, or 28 m.

b) The maximum height is reached at 0 min, 0.7 min, 1.4 min ….

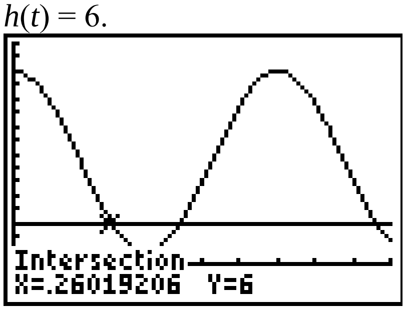


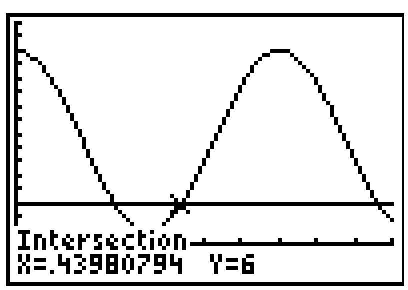
c) The minimum height of the point is 15 – 13, or 2 m.

d) The minimum height is reached at 0.35 min, 1.05 min, 1.75 min ….



e) Determine the points of intersection for the graphs of $h(t) = 13 \cos\left(\frac{2\pi}{0.7}t\right) + 15$ and $h(t) = 6$.





Within one cycle, the point is less than 6 m above the ground for approximately 0.44 – 0.26, or 0.18 min.

f) Determine the height at 1 h 12 min, or 72 min.

$$h(t) = 13\cos\left(\frac{2\pi}{0.7}t\right) + 15$$

$$h(72) = 13\cos\left(\frac{2\pi}{0.7}(72)\right) + 15$$

$h(72) = 23.105\ldots$

The height of the point if the wheel is allowed to turn for 1 h 12 min is approximately 23.1 m.

Section 5.4 Page 276 Question 10

Substitute $t = 17.5$.

$$h(t) = 59 + 24\sin 125t$$

$$h(17.5) = 59 + 24\sin 125(17.5)$$

$h(17.5) = 78.54\ldots$

The height of the mark, to the nearest tenth of a centimetre, when $t = 17.5$ s, is 78.5 cm.

Section 5.4 Page 276 Question 11

For a sinusoidal function that oscillates between –155 V and +155 V and makes 60 complete cycles each second, $a = 155$, $b = 120\pi$, $c = 0$, and $d = 0$. An equation for the voltage as a function of time, t is $V = 155\sin 120\pi t$.

Section 5.4 Page 277 Question 12

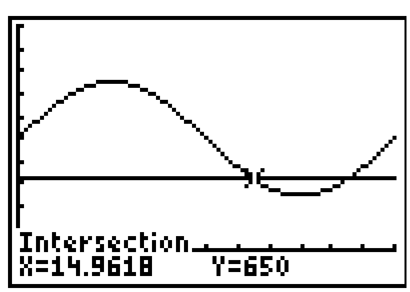
a) The period of the satellite is $\frac{2\pi}{28\pi}$, or $\frac{1}{14}$ days.

b) It will take the satellite $\frac{1}{14}(24)(60)$, or approximately 102.9 min to orbit Earth.

c) The satellite will make $\frac{60(24)}{102.9}$, or approximately 14 orbits per day.

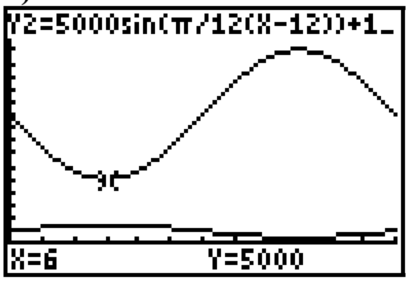
Section 5.4 Page 277 Question 13

a) Determine the point of intersection of the graphs of $F(t) = 500\sin\frac{\pi}{12}t + 1000$ and $F(t) = 650$.



It takes approximately 15 months for the fox population to drop to 650.

b)



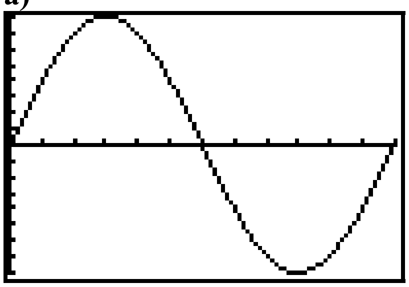
c)

	Arctic Fox	Lemming
Maximum Population	1500	15 000
Month	6	18
Minimum Population	500	5000
Month	18	6

d) Example: The maximum for the predator occurs at a minimum for the prey and vice versa. The predators population depends on the prey, so every time the lemming population changes the arctic fox population changes in accordance.

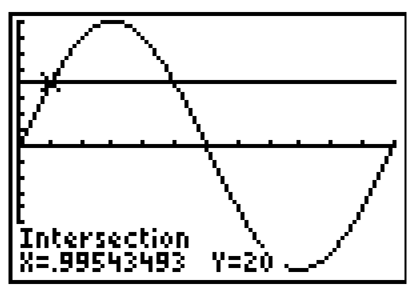
Section 5.4 Page 277 Question 14

a)



b) Substitute $t = 2.034$.
$h = 40 \sin 0.526t$
$= 40 \sin 0.526(2.034)$
$= 35.085\ldots.$
The guest will have swayed approximately 35.1 cm after 2.034 s.

c) Determine the point of intersection of the graphs $h = 40 \sin 0.526t$ and $h = 20$. It will take approximately 1 s for the guest to sway 20 cm.



Section 5.4 Page 278 Question 15

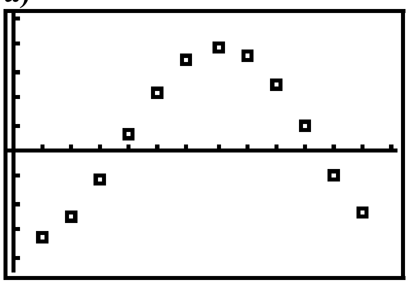
a) From the graph, the maximum height of the Sun is approximately 7.5 Sun widths and the minimum height of the Sun is approximately 1 Sun width.

b) The period is 24 h.

c) Since the amplitude is 3.25, $a = -3.25$. Since the period is 24, $b = \frac{\pi}{12}$. The sinusoidal axis is 1+ 3.25, or $d = 4.25$. Then, the sinusoidal equation that models the midnight Sun is $y = -3.25 \sin \frac{\pi}{12}x + 4.25$.

Section 5.4 Page 278 Question 16

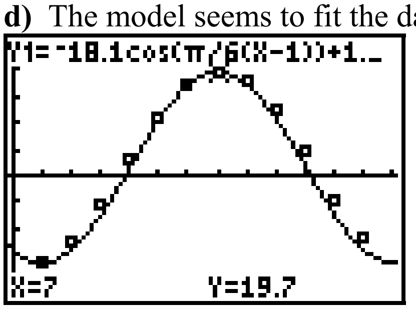
a)



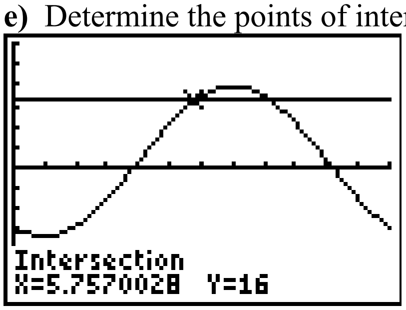
b) The temperature that is halfway between the maximum and minimum average monthly temperatures is $\frac{19.7 + (-16.5)}{2}$, or 1.6 °C.

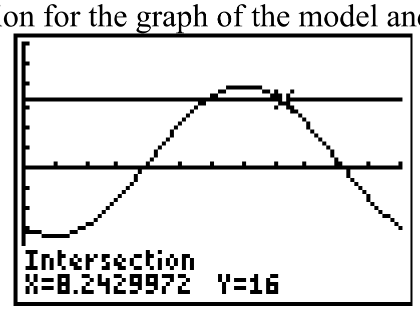
c) The best fit curve is of the form $y = -\cos x$. Since the amplitude is $\frac{19.7 - (-16.5)}{2} = 18.1$, $a = 18.1$. Since the period is 12 months, $b = \frac{\pi}{6}$. Since the minimum occurs in January, $c = 1$. From part b), $d = 1.6$. Then, a sinusoidal function to model the temperature for Winnipeg is $T = -18.1 \cos \frac{\pi}{6}(M - 1) + 1.6$, where T is the average monthly temperature, in degrees Celsius, and M is the time, in months.

d) The model seems to fit the data relatively well.



e) Determine the points of intersection for the graph of the model and $T = 16$.





The average monthly temperature in Winnipeg is greater than or equal to 16 °C for approximately 8.24 – 5.78, or about 2.5 months.

Section 5.4 Page 278 Question 17

a) The best fit curve is of the form $y = -\cos x$. Use the turn on and off temperatures to determine $a = \frac{43-34}{2}$, or 4.5. The period is 60 min, so $b = \frac{\pi}{30}$. The sinusoidal axis is $d = \frac{43+34}{2}$, or 38.5. Then, the equation that expresses temperature as a function of time is $T = -4.5 \cos \frac{\pi}{15}t + 38.5$.

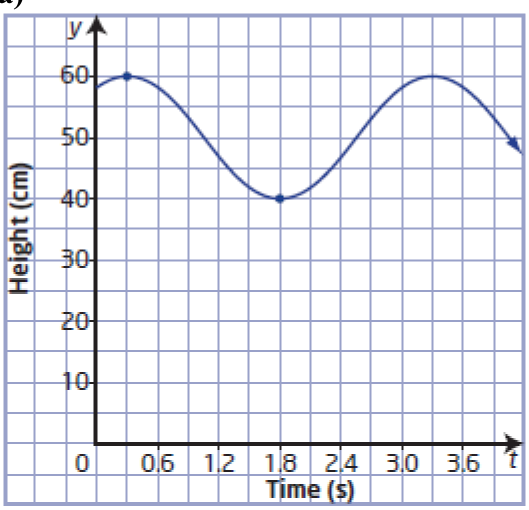
b) Substitute $t = 10$.

$$\begin{aligned} T &= -4.5 \cos \frac{\pi}{30}t + 38.5 \\ &= -4.5 \cos \frac{\pi}{30}(10) + 38.5 \\ &= -4.5 \cos \frac{\pi}{3} + 38.5 \\ &= 36.25 \end{aligned}$$

The temperature 10 min after the heater turns on is 36.25 °C.

Section 5.4 Page 279 Question 18

a)



b) Example: From the graph, $a = 10$, $b = \frac{2\pi}{3}$, $c = 0.3$, and $d = 50$. Then, the cosine equation for the distance, d, in centimetres, from the floor as a function of time, t, in seconds, is $d = 10 \cos \frac{2\pi}{3}(t - 0.3) + 50$.

c) Substitute $t = 17.2$.

$d = 10 \cos \frac{2\pi}{3}(t - 0.3) + 50$

$= 10 \cos \frac{2\pi}{3}(17.2 - 0.3) + 50$

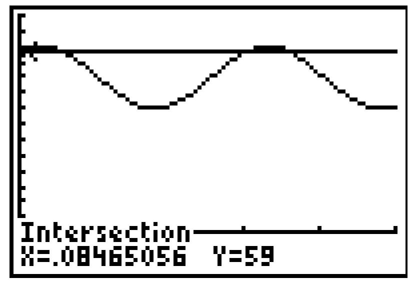
$= 43.308\ldots$

The distance from the floor when the stopwatch reads 17.2 s is approximately 43.3 cm.

d) Determine the first point of intersection of the graphs of

$d = 10 \cos \frac{2\pi}{3}(t - 0.3) + 50$ and $d = 59$.

The first time when the mass is 59 cm above the floor is approximately 0.085 s.



Section 5.4 Page 279 Question 19

a) The best fit curve is of the form $y = -\cos x$. Since the radius is 10 m, $a = 10$. The period is 60 s, so $b = \frac{\pi}{30}$. Since the minimum height is 2 m, $d = 2 + 10$, or 12. Then, an equation to model the path of a passenger on the Ferris wheel, where the height, h, in metres, is a function of time, t, in seconds is $h = -10 \cos \frac{\pi}{30}t + 12$.

b) 2.3 min = 138 s

Substitute $t = 138$.

$h = -10 \cos \frac{\pi}{30}t + 12$

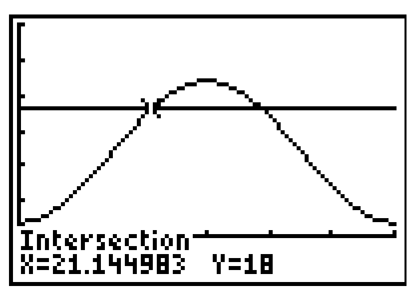
$= -10 \cos \frac{\pi}{30}(138) + 12$

$= 15.090\ldots$

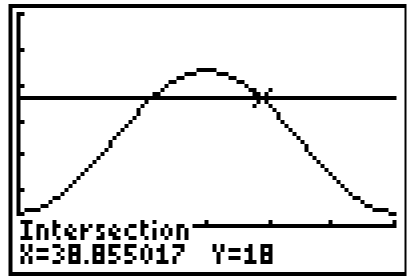
Emily's height above the ground, to the nearest tenth of a metre, when the wheel has been in motion for 2.3 min is 15.1 m.

c) Determine the first point of intersection of the graphs of $h = -10 \cos \frac{\pi}{30}t + 12$ and $h = 18$.

The amount of time that passes before a rider reaches a height of 18 m for the first time is approximately 21.1 s.



Another time when the rider is at that same height within the first cycle is approximately 38.9 s.



Section 5.4 Page 279 Question 20

a) Use a cosine function. From the maximum and minimum heights, $a = \frac{22-8}{2}$, or 7.

The period is 5 s, so $b = \frac{2\pi}{5}$. The minimum is reached at 2 s, but the minimum of a cosine curve typically occurs at the halfway point in the period, or 2.5, then $c = -0.5$. Since the minimum height is 8 m, $d = 8 + 7$, or 15. Then, an equation to model the rotation of the tip of the blade, where the height, h, in metres, is a function of time, t, in seconds is $h = 7\cos\frac{2\pi}{5}(t + 0.5) + 15$.

An equivalent sine function is $h = 7\sin\frac{2\pi}{5}(t + 1.75) + 15$.

b) Substitute $t = 4$.

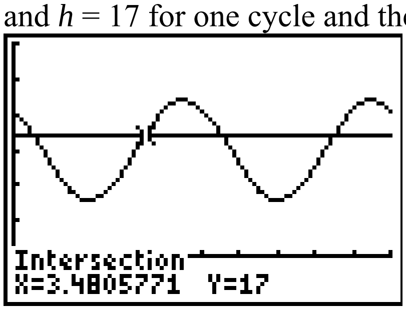
$h = 7\cos\frac{2\pi}{5}(t + 0.5) + 15$

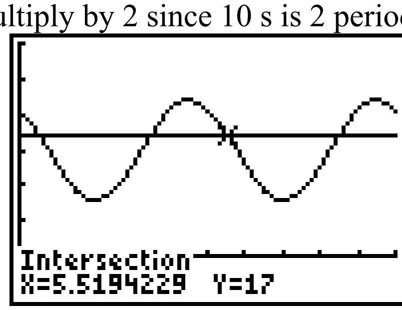
$\quad = 7\cos\frac{2\pi}{5}(4 + 0.5) + 15$

$\quad = 20.663\ldots$

The height of the tip of the blade after 4 s is approximately 20.7 m.

c) Determine the points of intersection of the graphs of $h = 7\cos\frac{2\pi}{5}(t + 0.5) + 15$ and $h = 17$ for one cycle and then multiply by 2 since 10 s is 2 periods.





$2(5.5194 - 3.4806) = 4.0776$

The tip of the blade above a height of 17 m in the first 10 s is approximately 4.08 s.

Section 5.4 Page 280 Question 21

a) The best fit curve is of the form $y = -\cos x$. From the maximum and minimum temperatures, $a = \frac{23.6 - 4.2}{2}$, or 9.7. The period is 366 days, so $b = \frac{\pi}{183}$. The minimum is reached on day 26, but the minimum of a reflected cosine curve typically occurs at the start point in the period, so $c = 26$. Since the minimum temperature is 4.2 °C, $d = 4.2 + 9.7$, or 13.9. Then, an equation to model the temperature, T, in degrees Celsius, is a function of time, t, in days, is

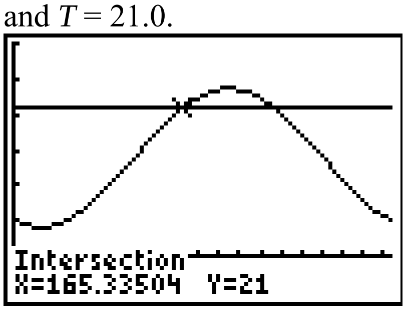
$T = -9.7 \cos \frac{\pi}{183}(t - 26) + 13.9$.

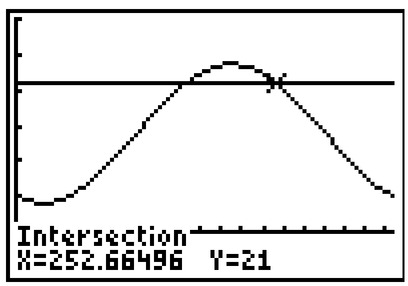
b) Substitute $t = 147$.

$$T = -9.7 \cos \frac{\pi}{183}(t - 26) + 13.9$$
$$= -9.7 \cos \frac{\pi}{183}(147 - 26) + 13.9$$
$$= 18.605\ldots$$

The temperature for day 147, May 26, is approximately 18.6 °C.

c) Determine the points of intersection for the graphs of $T = -9.7 \cos \frac{\pi}{183}(t - 26) + 13.9$ and $T = 21.0$.



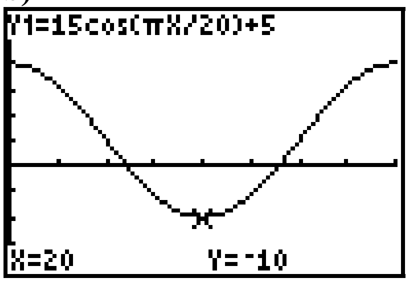


A maximum temperature of 21.0 °C or higher is expected for approximately 253 – 165, or 88 days.

Section 5.4 Page 280 Question 22

a) Use a cosine function. From the maximum and minimum cash flow percents, $a = \frac{20 - (-10)}{2}$, or 15. The period is 40 years, so $b = \frac{\pi}{20}$. Since the minimum percent is -10, $d = -10 + 15$, or 5. Then, an equation to model the cash flow, C, in percent of total assets, is a function of time, t, in years, is $C = 15 \cos \frac{\pi}{20} t + 5$.

b)



c) For 2008, substitute $t = 88$.

$C = 15 \cos \frac{\pi}{20} t + 5$

$\quad = 15 \cos \frac{\pi}{20}(88) + 5$

$\quad = 9.635\ldots$

The cash flow for the company in 2008 is approximately 9.6% of total assets.

d) Example: No. I would not invest with this company, because too great a portion of the cycle has losses.

Section 5.4 Page 280 Question 23

a) Use a sine function. Since the distance of the turns from the midline is 1.2 m, $a = 1.2$. For 10 turns in 20 s, the period is 4 s (2 turns per cycle) and $b = \frac{\pi}{2}$. Then, the function that models the path of the skier is $y = 1.2 \sin \frac{\pi}{2} t$, where t represents the time, in seconds, and y represents the distance for one turn, in metres, from the midline.

b) If the skier made only eight turns in 20 s, then the period would change to 5 s and equation becomes $y = 1.2 \sin \frac{2\pi}{5} t$.

Section 5.4 Page 280 Question C1

Examples:

a) Use a sine function as a model when the curve or data begins at or near the intersection of the vertical axis and the sinusoidal axis.

b) Use a cosine function as a model when the curve or data has a maximum or minimum near or at the vertical axis.

Section 5.4 Page 280 Question C2

Examples:

a) $y = a \sin b(x - c) + d$

The parameter b has the greatest influence on the graph of the function. It changes the period of the function. Parameters c and d change the location of the curve, but not the shape. Parameter a changes the maximum and minimum values.

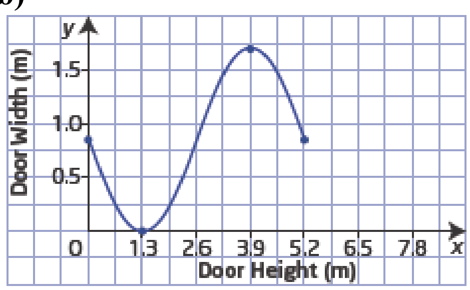
b) $y = a \cos b(x - c) + d$

The parameter b has the greatest influence on the graph of the function. It changes the period of the function. Parameters c and d change the location of the curve, but not the shape. Parameter a changes the maximum and minimum values.

Section 5.4 Page 281 Question C3

a) The best fit curve is of the form $y = -\sin x$. Since the amplitude is half the width of the door, $a = 0.85$. Since the period is the height of the door, $b = \frac{2\pi}{5.2}$. Then, a sinusoidal function that could model the shape of the open door is $y = -0.85 \sin \frac{2\pi}{5.2}x + 0.85$, where x represents the height of the door, in metres, and y represents the width of the door, in metres.

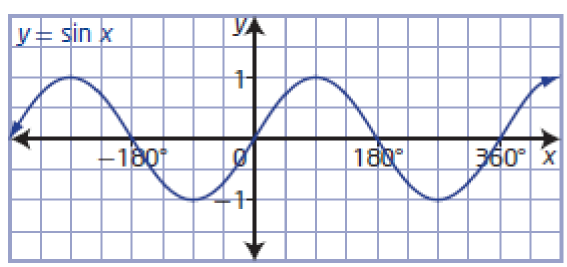
b)



Chapter 5 Review

Chapter 5 Review Page 282 Question 1

a) The x-intercepts in the interval $-360° \le x \le 360°$ are $-360°$, $-180°$, $0°$, $180°$, and $360°$.



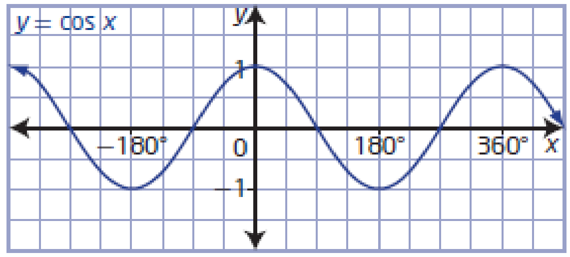
b) The y-intercept is 0.

c) The domain is $\{x \mid x \in R\}$. The range is $\{y \mid -1 \le y \le 1, y \in R\}$. The period is 360°.

d) The greatest value of $y = \sin x$ is 1.

Chapter 5 Review Page 282 Question 2

a) The x-intercepts in the interval $-360° \le x \le 360°$ are $-270°$, $-90°$, $0°$, $90°$, and $270°$.



b) The y-intercept is 1.

c) The domain is $\{x \mid x \in R\}$. The range is $\{y \mid -1 \le y \le 1, y \in R\}$. The period is 360°.

d) The greatest value of $y = \cos x$ is 1.

Chapter 5 Review Page 282 Question 3

a) For $y = \sin x$, the amplitude is 1 and the period is 2π. This matches graph **A**.

b) For $y = \sin 2x$, the amplitude is 1 and the period is π. This matches graph **D**.

c) For $y = -\sin x$, the amplitude is 1, the period is 2π, and the graph is reflected in the x-axis. This matches graph **B**.

d) For $y = \frac{1}{2}\sin x$, the amplitude is $\frac{1}{2}$ and the period is 2π. This matches graph **C**.

Chapter 5 Review Page 282 Question 4

a) For $y = -3 \sin 2x$, $a = -3$ and $b = 2$. So, the amplitude is $|-3| = 3$ and the period is $\frac{2\pi}{2} = \pi$ or $\frac{360°}{2} = 180°$.

b) For $y = 4 \cos 0.5x$, $a = 4$ and $b = 0.5$. So, the amplitude is $|4| = 4$ and the period is $\frac{2\pi}{0.5} = 4\pi$ or $\frac{360°}{0.5} = 720°$.

c) For $y = \frac{1}{3}\sin \frac{5}{6}x$, $a = \frac{1}{3}$ and $b = \frac{5}{6}$. So, the amplitude is $\left|\frac{1}{3}\right| = \frac{1}{3}$ and the period is

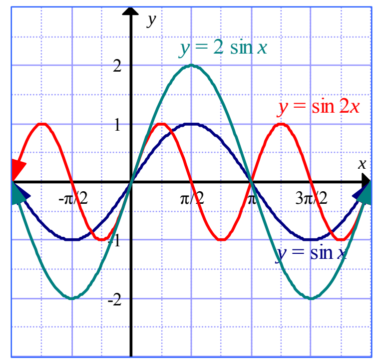
$$\frac{2\pi}{\frac{5}{6}} = \frac{12\pi}{5} \text{ or } \frac{360°}{\frac{5}{6}} = 432°.$$

d) For $y = -5 \cos \frac{3}{2}x$, $a = -5$ and $b = \frac{3}{2}$. So, the amplitude is $|-5| = 5$ and the period is

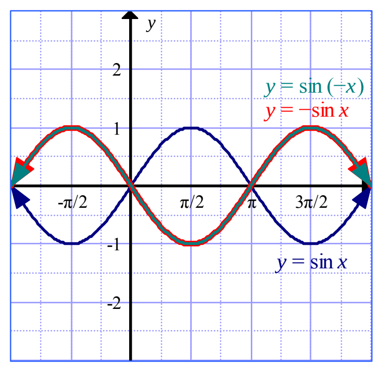
$\frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$ or $\frac{360°}{\frac{3}{2}} = 240°$.

Chapter 5 Review Page 282 Question 5

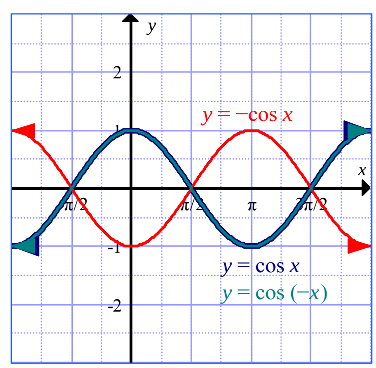
a) The graph of $y = \sin x$ has an amplitude of 1 and a period of 2π. The graph of $y = \sin 2x$ has the same amplitude as the graph of $y = \sin x$, but a period of π. The graph of $y = 2 \sin x$ has an amplitude of 2 and the same period as the graph of $y = \sin x$.



b) The graph of $y = \sin x$ has an amplitude of 1 and a period of 2π. The graph of $y = -\sin x$ has the same amplitude and period as the graph of $y = \sin x$, but is reflected in the x-axis. The graph of $y = \sin(-x)$ has the same amplitude and period as the graph of $y = \sin x$, but is reflected in the y-axis.



c) The graph of $y = \cos x$ has an amplitude of 1 and a period of 2π. The graph of $y = -\cos x$ has the same amplitude and period as the graph of $y = \cos x$, but is reflected in the x-axis. The graph of $y = \cos(-x)$ has the same amplitude and period as the graph of $y = \cos x$, but is reflected in the y-axis.



Chapter 5 Review Page 282 Question 6

a) For a cosine function with an amplitude of 3 and a period of π, $a = 3$ and $b = 2$. Then, the equation of the function in the form $y = a \cos bx$ is $y = 3 \cos 2x$.

b) For a cosine function with an amplitude of 4, $a = 4$.

For a period of 150°, $b = \frac{360°}{150°}$.

$$= \frac{12}{5}$$

Then, the equation of the function in the form $y = a \cos bx$ is $y = 4 \cos \frac{12}{5}x$.

c) For a cosine function with an amplitude of $\frac{1}{2}$ and a period of 720°, $a = \frac{1}{2}$ and $b = \frac{1}{2}$.

Then, the equation of the function in the form $y = a \cos bx$ is $y = \frac{1}{2} \cos \frac{1}{2}x$.

d) For a cosine function with an amplitude of $\frac{3}{4}$, $a = \frac{3}{4}$.

For a period of $\frac{\pi}{6}$, $b = \frac{2\pi}{\frac{\pi}{6}}$.

$$= 12$$

Then, the equation of the function in the form $y = a \cos bx$ is $y = \frac{3}{4} \cos 12x$.

Chapter 5 Review Page 283 Question 7

a) For a sine function with an amplitude of 8 and a period of 180°, $a = 8$ and $b = 2$. Then, the equation of the function in the form $y = a \sin bx$ is $y = 8 \sin 2x$.

b) For a sine function with an amplitude of 0.4 and a period of 60°, $a = 0.4$ and $b = 6$. Then, the equation of the function in the form $y = a \sin bx$ is $y = 0.4 \sin 6x$.

c) For a sine function with an amplitude of $\frac{3}{2}$ and a period of 4π, $a = \frac{3}{2}$ and $b = \frac{1}{2}$.

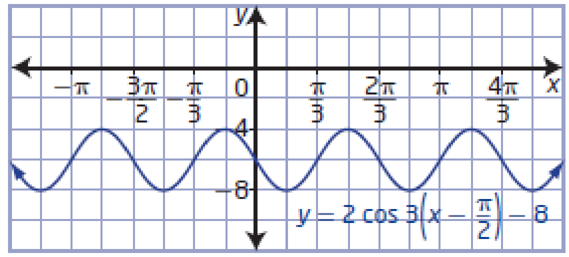
Then, the equation of the function in the form $y = a \sin bx$ is $y = \frac{3}{2} \sin \frac{1}{2}x$.

d) For a sine function with an amplitude of 2 and a period of $\frac{2\pi}{3}$, $a = 2$ and $b = 3$. Then, the equation of the function in the form $y = a \sin bx$ is $y = 2 \cos 3x$.

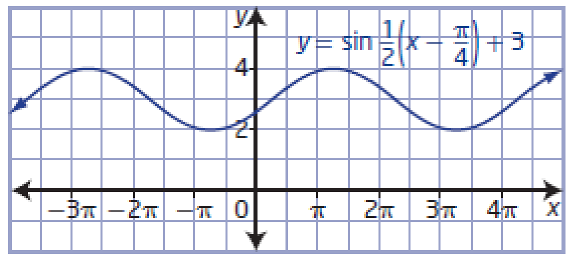
Chapter 5 Review Page 283 Question 8

a) For $y = 2\cos 3\left(x - \frac{\pi}{2}\right) - 8$,

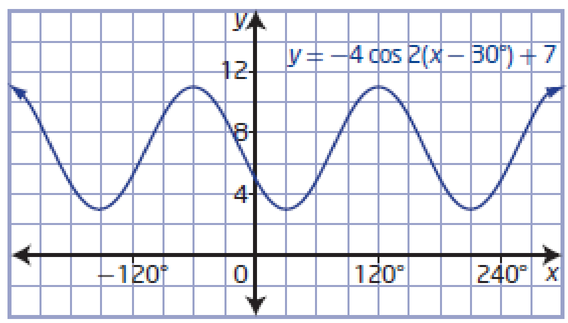
$a = 2$, $b = 3$, $c = \frac{\pi}{2}$, and $d = -8$. The amplitude is $|2| = 2$, the period is $\frac{2\pi}{3}$, the phase shift is $\frac{\pi}{2}$ units to the right, and the vertical displacement is 8 units down.



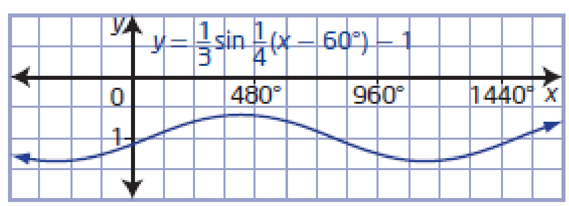
b) For $y = \sin \frac{1}{2}\left(x - \frac{\pi}{4}\right) + 3$, $a = 1$, $b = \frac{1}{2}$, $c = \frac{\pi}{4}$, and $d = 3$. The amplitude is $|1| = 1$, the period is $\frac{2\pi}{\frac{1}{2}} = 4\pi$, the phase shift is $\frac{\pi}{4}$ units to the right, and the vertical displacement is 3 units up.



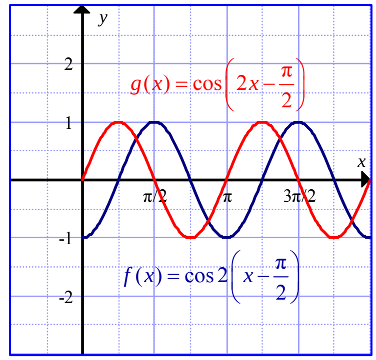
c) For $y = -4\cos 2(x - 30°) + 7$, $a = -4$, $b = 2$, $c = 30°$, and $d = 7$. The amplitude is $|-4| = 4$, the period is $\frac{360°}{2} = 180°$, the phase shift is 30° to the right, and the vertical displacement is 7 units up.



d) For $y = \frac{1}{3}\sin \frac{1}{4}(x - 60°) - 1$, $a = \frac{1}{3}$, $b = \frac{1}{4}$, $c = 60°$, and $d = -1$. The amplitude is $\left|\frac{1}{3}\right| = \frac{1}{3}$, the period is $\frac{360°}{\frac{1}{4}} = 1440°$, the phase shift is 60° to the right, and the vertical displacement is 1 unit down.



Chapter 5 Review Page 283 Question 9



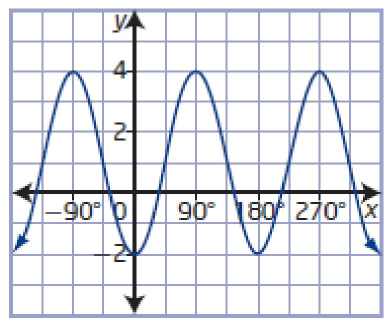
a) The period of both functions is π.

b) The phase shift for $f(x)$ is $\frac{\pi}{2}$ units to the right, while the phase shift for $g(x)$ is $\frac{\pi}{4}$ units to the right.

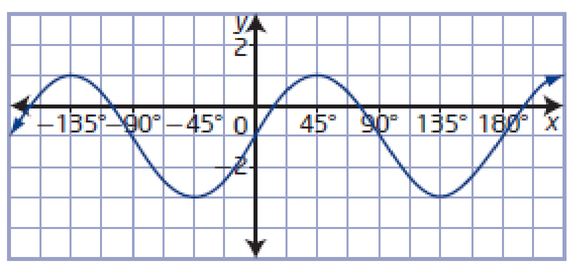
c) The phase shift of the function $y = b(x - \pi)$ is π units to the right.

d) The phase shift of the function $y = (bx - \pi)$ is $\frac{\pi}{b}$ units to the right.

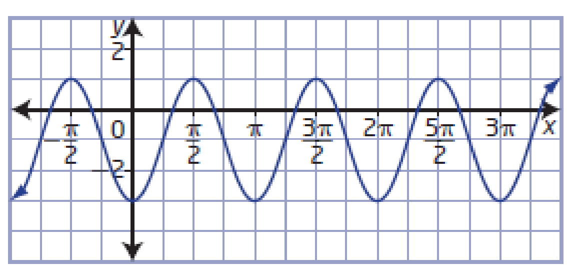
Chapter 5 Review Page 283 Question 10



a) The amplitude, a, is $\frac{4-(-2)}{2}$, or 3.
The period is 180° – 0°, or 180°. So, $b = 2$.
The start of the first cycle of the sine curve to the right of the y-axis is at 45°. So, $c = 45°$.
The vertical displacement is 1 unit up. Locating the mid-line gives $d = 1$.
So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 3 \sin 2(x - 45°) + 1$. Similarly, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = -3 \cos 2x + 1$.



b) The amplitude, a, is $\frac{1-(-3)}{2}$, or 2.
The period is 180° – 0°, or 180°. So, $b = 2$.
The start of the first cycle of the sine curve is at the y-axis. So, $c = 0°$.
The vertical displacement is 1 unit down. Locating the mid-line gives $d = -1$.
So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 2 \sin 2x - 1$. Similarly, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = 2 \cos 2(x - 45°) - 1$.



c) The amplitude, a, is $\frac{1-(-3)}{2}$, or 2.
The period is π – 0, or π. So, $b = 2$.

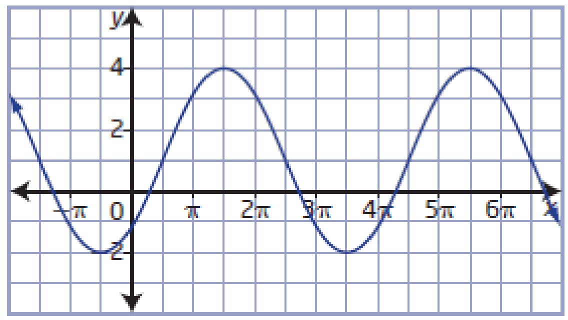
The start of the first cycle of the sine curve is to the right of the *y*-axis. So, $c = \frac{\pi}{4}$.

The vertical displacement is 1 unit down. Locating the mid-line gives $d = -1$.

So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 2\sin 2\left(x - \frac{\pi}{4}\right) - 1$.

Similarly, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = -2 \cos 2x - 1$.

d) The amplitude, *a*, is $\frac{4-(-2)}{2}$, or 3.



The period is $4\pi - 0$, or 4π. So, $b = 0.5$.
The start of the first cycle of the sine curve is to the right of the *y*-axis. So, $c = \frac{\pi}{2}$.

The vertical displacement is 1 unit up.
Locating the mid-line gives $d = 1$.
So, an equation in the form $y = a \sin b(x - c) + d$ for the graph is $y = 3\sin 0.5\left(x - \frac{\pi}{2}\right) + 1$. Similarly, an equation in the form $y = a \cos b(x - c) + d$ for the graph is $y = 3\cos 0.5\left(x - \frac{3\pi}{2}\right) + 1$.

Chapter 5 Review Page 283 Question 11

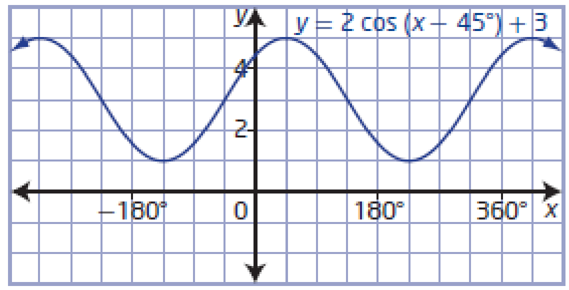
a) For amplitude 4, period π, phase shift $\frac{\pi}{3}$ to the right, and vertical displacement 5 units down, $a = 4$, $b = 2$, $c = \frac{\pi}{3}$, and $d = -5$. Then, the equation of the sine function in the form $y = a \sin b(x - c) + d$ is $y = 4\sin 2\left(x - \frac{\pi}{3}\right) - 5$.

b) For amplitude 0.5, period 4π, phase shift $\frac{\pi}{6}$ to the left, and vertical displacement 1 unit up, $a = 0.5$, $b = \frac{1}{2}$, $c = -\frac{\pi}{6}$, and $d = 1$. Then, the equation of the cosine function in the form $y = a \cos b(x - c) + d$ is $y = 0.5\cos\frac{1}{2}\left(x + \frac{\pi}{6}\right) + 1$.

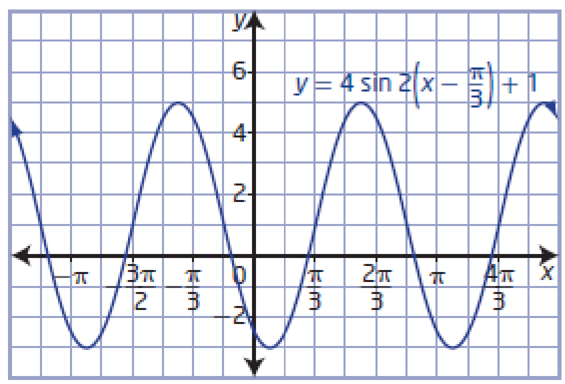
c) For amplitude $\frac{2}{3}$, period 540°, no phase shift, and vertical displacement 5 units down, $a = \frac{2}{3}$, $b = \frac{2}{3}$, $c = 0$, and $d = -5$. Then, the equation of the sine function in the form $y = a \sin b(x - c) + d$ is $y = \frac{2}{3}\sin\frac{2}{3}x - 5$.

Chapter 5 Review Page 284 Question 12

a) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range $\{y \mid 1 \le y \le 5, y \in \mathrm{R}\}$. The maximum value is 5 and the minimum value is 1. There are no x-intercepts. The y-intercept is $\sqrt{2}+3$, or approximately 4.41.



b) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range $\{y \mid -3 \le y \le 5, y \in \mathrm{R}\}$. The maximum value is 5 and the minimum value is –3. The x-intercepts are approximately $0.9 + n\pi$ and $2.7 + n\pi$, $n \in \mathrm{I}$.
The y-intercept is $-2\sqrt{3}+1$, or approximately –2.5.



Chapter 5 Review Page 284 Question 13

a) For $y = 3 \sin 2\left(x - \frac{\pi}{3}\right) + 6$, $a = 3$, $b = 2$, $c = \frac{\pi}{3}$, and $d = 6$. Compared to the graph of $y = \sin x$, the graph of $y = 3 \sin 2\left(x - \frac{\pi}{3}\right) + 6$ is stretched horizontally by a factor of $\frac{1}{2}$ about the y-axis, stretched vertically by a factor of 3 about the x-axis, and translated $\frac{\pi}{3}$ units to the right and 6 units up.

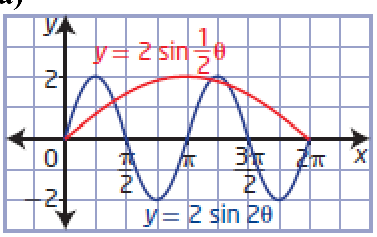
b) For $y = -2 \cos \frac{1}{2}\left(x + \frac{\pi}{4}\right) - 3$, $a = -2$, $b = \frac{1}{2}$, $c = -\frac{\pi}{4}$, and $d = -3$. Compared to the graph of $y = \cos x$, the graph of $y = -2 \cos \frac{1}{2}\left(x + \frac{\pi}{4}\right) - 3$ is stretched horizontally by a factor of 2 about the y-axis, stretched vertically by a factor of 2 about the x-axis, reflected in the x-axis, and translated $\frac{\pi}{4}$ units to the left and 3 units down.

c) For $y = \frac{3}{4} \cos 2(x - 30°) + 10$, $a = \frac{3}{4}$, $b = 2$, $c = 30°$, and $d = 10$. Compared to the graph of $y = \cos x$, the graph of $y = \frac{3}{4} \cos 2(x - 30°) + 10$ is stretched horizontally by a factor of $\frac{1}{2}$ about the y-axis, stretched vertically by a factor of $\frac{3}{4}$ about the x-axis, and translated 30° to the right and 10 units up.

d) For $y = -\sin 2(x + 45°) - 8$, $a = -1$, $b = 2$, $c = -45°$, and $d = -8$. Compared to the graph of $y = \sin x$, the graph of $y = -\sin 2(x + 45°) - 8$ is stretched horizontally by a factor of $\frac{1}{2}$ about the y-axis, reflected in the x-axis, and translated 45° to the left and 8 units down.

Chapter 5 Review Page 284 Question 14

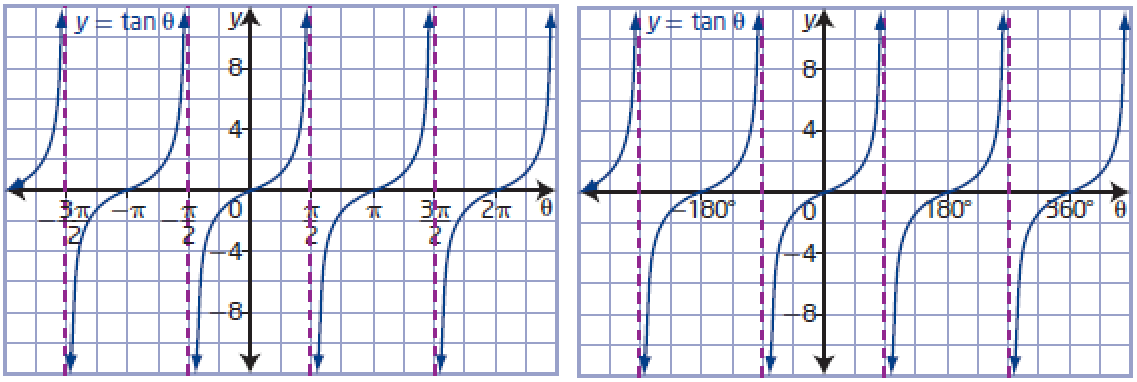
a)



b) Compared to the graph of $y = \sin \theta$, the graph of $y = 2 \sin 2\theta$ is vertically stretched by a factor of 2 about the x-axis and half the period. Compared to the graph of $y = \sin \theta$, the graph of $y = 2 \sin \frac{1}{2}\theta$ is vertically stretched by a factor of 2 about the x-axis and double the period.

Chapter 5 Review Page 284 Question 15

a)



b) i) For the interval $-2\pi \le \theta \le 2\pi$, the domain is $\left\{x \mid x \neq -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, x \in \mathrm{R}\right\}$. For the interval $-360° \le \theta \le 360°$, the domain is $\{x \mid x \neq -270°, -90°, 90°, 270°, x \in \mathrm{R}\}$.

ii) The range is $\{y \mid y \in \mathrm{R}\}$.

iii) The *y*-intercept is 0.

iv) For the interval $-2\pi \le \theta \le 2\pi$, the *x*-intercepts are -2π, $-\pi$, 0, π, and 2π. For the interval $-360° \le \theta \le 360°$, the *x*-intercepts are –360°, –180°, 0°, 180°, and 360°.

v) For the interval $-2\pi \le \theta \le 2\pi$, the equations of the asymptotes are $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$. For the interval $-360° \le \theta \le 360°$, the equations of the asymptotes are $x = -270°, -90°, 90°, 270°$.

Chapter 5 Review Page 284 Question 16

a) The exact coordinates of point Q are (1, tan θ) or $\left(1, \frac{1}{\sqrt{3}}\right)$.

b) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

c) As θ approaches 90°, tan θ approaches infinity.

d) When θ = 90°, cos 90° = 0 and thus tan 90° is undefined.

Chapter 5 Review Page 284 Question 17

a) Since $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, the asymptotes of the graph of $y = \tan\theta$ occur when $\cos\theta = 0$.

b) Since $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, the x-intercepts of the graph of $y = \tan\theta$ occur when $\sin\theta = 0$.

Chapter 5 Review Page 284 Question 18

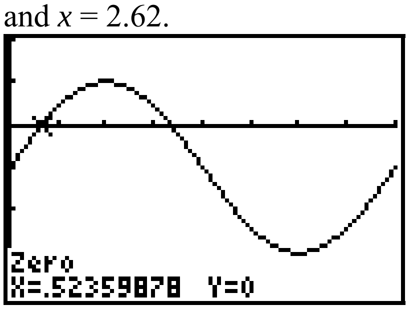
When the Sun is directly overhead, the shadow has no length which means $\tan\theta$ is undefined. This relates to the asymptote of the graph of $y = \tan\theta$ at $\theta = 90°$.

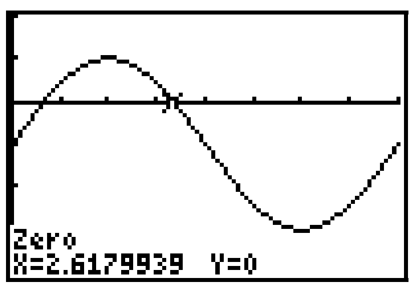
Chapter 5 Review Page 284 Question 19

A vertical asymptote is an imaginary line that the graph approaches. If a trigonometric function is represented by a quotient, such as the tangent function, asymptotes occur at values for which the function is undefined; that is, when the function in the denominator is equal to zero.

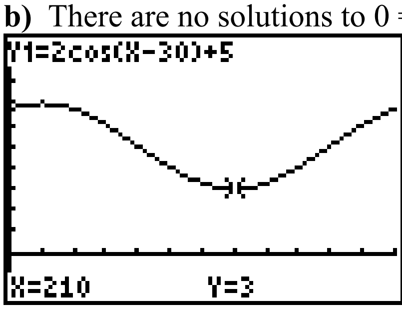
Chapter 5 Review Page 284 Question 20

a) The solutions to $2\sin x - 1 = 1$ in the interval $0 \le x \le 2\pi$ are approximately $x = 0.52$ and $x = 2.62$.

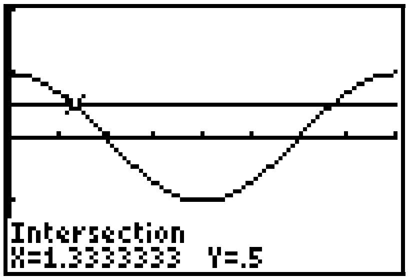


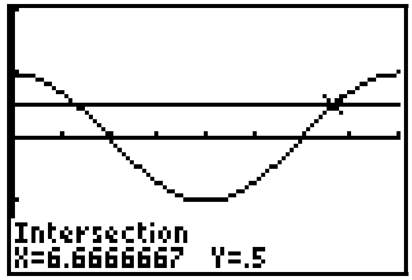


b) There are no solutions to $0 = 2\cos(x - 30°) + 5$ in the interval $0° \le x \le 360°$.



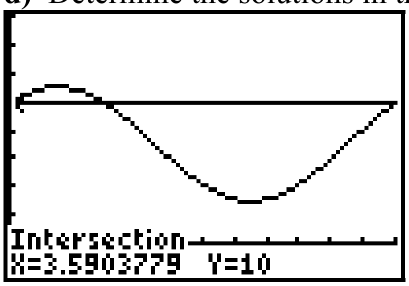
c) Determine the solutions in the first cycle.

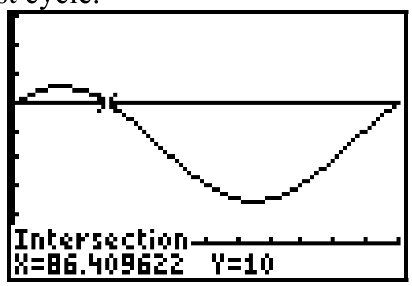




The general solution to $\sin\left(\frac{\pi}{4}(x-6)\right) = 0.5$ is approximately 1.33 + 8*n* and 6.67 + 8*n*, where $n \in$ I.

d) Determine the solutions in the first cycle.

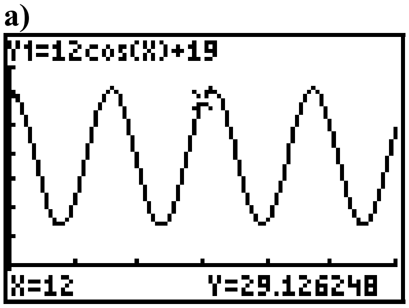




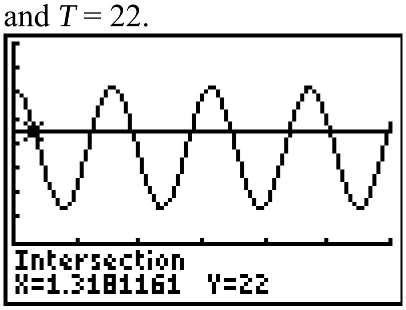
The general solution to 4 cos (x – 45°) + 7 = 10 is approximately 3.59° + (360°)*n* and 86.41° + (360°)*n*, where $n \in$ I.

Chapter 5 Review Page 285 Question 21

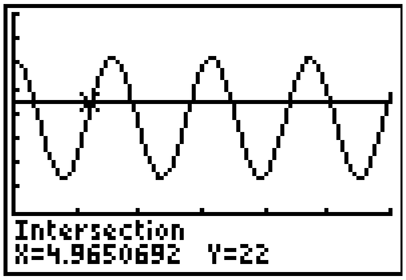
a)

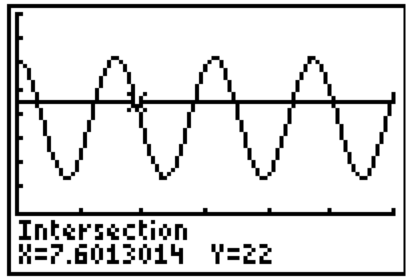


b) Determine several points of intersection between the graphs of $T = 12 \cos t + 19$ and $T = 22$.

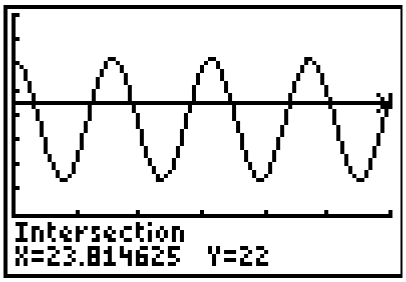


The first point of intersection gives approximately 1.32 h.





Using the approximate second and third points of intersection gives 7.60 – 4.97, or 2.63 h. So, in one day 3(2.63), or 7.89 h.

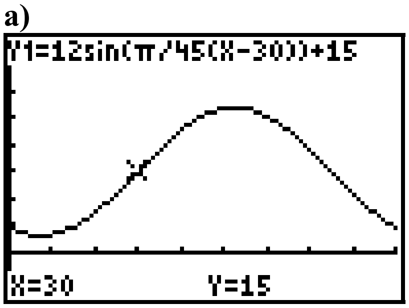


The last point of intersection gives approximately 24 – 23.81, or 0.19 h.
To the nearest tenth of an hour, the amount of time in one day that the air conditioning will operate is 1.32 + 7.89 + 0.19, or 9.4 h.

c) Example: A model for temperature variance is important for maintaining constant temperatures to preserve artifacts.

Chapter 5 Review   Page 285   Question 22

a)



b) The maximum height is 12 + 15, or 27 m. The minimum height is 15 – 12, or 3 m.

c) The time required for the Ferris wheel to complete one revolution is $\dfrac{2\pi}{\frac{\pi}{45}}$, or 90 s.

d) Substitute $t = 45$.

$$h(t) = 12 \sin \frac{\pi}{45}(t - 30) + 15$$

$h(45) = 12 \sin \frac{\pi}{45}(45 - 30) + 15$

$h(45) = 12 \sin \frac{\pi}{3} + 15$

$h(45) = 12\left(\frac{\sqrt{3}}{2}\right) + 15$

$h(45) = 6\sqrt{3} + 15$

$h(45) = 25.392\ldots$

The height of the rider above the ground after 45 s is approximately 25.4 m.

Chapter 5 Review Page 285 Question 23

a) The best fit curve is of the form $y = -\cos x$. Use the hours of daylight to determine $a = \frac{15.7 - 8.3}{2}$, or 3.7. The period is 365 days, so $b = \frac{2\pi}{365}$. The minimum is reached on day 355, but the minimum of a reflected cosine curve typically occurs at the start point in the period, so $c = -10$. The sinusoidal axis is $d = \frac{15.7 + 8.3}{2}$, or 12. Then, the equation that expresses the number of hours of daylight, L, as a function of time, t, in days, is $L = -3.7 \cos \frac{2\pi}{365}(t + 10) + 12$.

Chapter 5 Review Page 285 Question 24

a) The best fit curve is of the form $y = \cos x$. Use the number of sunspots to determine $a = \frac{110 - 10}{2}$, or 50. The period is $\frac{1948 - 1750}{18}$, or 11 years. So, $b = \frac{2\pi}{11}$. The sinusoidal axis is $d = \frac{110 + 10}{2}$, or 60. Then, the equation that expresses the sunspots, S, as a function of time, t, in years, is $S = 50 \cos \frac{2\pi}{11}t + 60$.

For the year 2000, substitute $t = 250$.

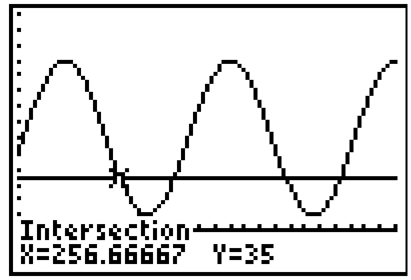
$S = 50 \cos \frac{2\pi}{11}t + 60$

$= 50 \cos \frac{2\pi}{11}(250) + 60$

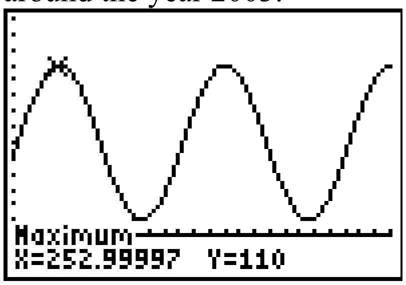
$= 52.884\ldots$

You would expect about 53 sunspots in the year 2000.

b) Determine the point of intersection of the graphs of $S = 50 \cos \frac{2\pi}{11}t + 60$ and $S = 35$. The first year after 2000 in which the number of sunspots will be about 35 is around the year 2007.



c) The first year after 2000 in which the number of sunspots will be a maximum is around the year 2003.



Chapter 5 Practice Test

Chapter 5 Practice Test Page 286 Question 1

For the function $y = 2 \sin x + 1$, $a = 2$ and $d = 1$. So, the range is $\{y \mid -1 \leq y \leq 3, y \in \mathrm{R}\}$: Choice **A**.

Chapter 5 Practice Test Page 286 Question 2

For $f(x) = 3 \sin 2\left(x - \frac{\pi}{3}\right) + 1$, $a = 3$, $b = 2$, $c = \frac{\pi}{3}$, and $d = 1$. The phase shift is $\frac{\pi}{3}$ units to the right. The period is $\frac{2\pi}{2}$ or π. The amplitude is $|3|$, or 3. Choice **D**.

Chapter 5 Practice Test Page 286 Question 3

Since $\sin x = \cos\left(x - \frac{\pi}{2}\right)$, then $\sin\left(x - \frac{\pi}{4}\right) = \cos\left(x - \frac{3\pi}{4}\right)$. Choice **C**.

Chapter 5 Practice Test Page 286 Question 4

Given: cosine function with maximum (3, 14) and nearest minimum to the right at (8, 2).

The amplitude is $\frac{14 - 2}{2}$, or 6.

The distance between the minimum and the maximum of a cosine function is one-half the period. The period is 2(8 – 3), or 10. So, $b = \frac{\pi}{5}$.

The maximum for a cosine curve appears at the start of the cycle, or $x = 3$. So, $c = 3$.

minimum = $d - |a|$

$2 = d - |6|$

$d = 8$

An equation for the cosine function is $y = 6\sin\frac{\pi}{5}(x-3)+8$. Choice **D**.

Chapter 5 Practice Test Page 286 Question 5

From the graph, the amplitude is 2 and the period is π. So, $a = 2$ and $b = 2$. A possible equation for the function is $y = 2 \sin 2\theta$. Choice **B**.

Chapter 5 Practice Test Page 286 Question 6

For a sine function of the form $y = a \sin b(x - c) + d$,

- the values of a and d affect the range of the function
- the values of c and d determine the horizontal and vertical translations, respectively
- the value of b determines the number of cycles within the distance of 2π
- the values of a and b are vertical and horizontal stretches

All of Monique's statements are true. Choice **A**.

Chapter 5 Practice Test Page 286 Question 7

If the bicycle were pedalled at a greater constant speed, the period of the graph would decrease. Choice **C**.

Chapter 5 Practice Test Page 287 Question 8

For the function $f(x) = \sin 2x$, the period is $\frac{2\pi}{2}$ or π. The horizontal distance between two consecutive zeros is half of the period, $\frac{\pi}{2}$.

Chapter 5 Practice Test Page 287 Question 9

For $y = \tan\theta$, the asymptotes occur at $x = \frac{\pi}{2} + n\pi$, $n \in I$, the domain is $\left\{x \mid x \neq \frac{\pi}{2} + n\pi, x \in R, n \in I\right\}$, the range is $\{y \mid y \in R\}$, and the period is π.

Chapter 5 Practice Test Page 287 Question 10

For $f(x) = -4 \sin x$, $a = -4$, $b = 1$, $c = 0$, and $d = 0$.
For $g(x) = -4 \cos \frac{1}{2}x$, $a = -4$, $b = \frac{1}{2}$, $c = 0$, and $d = 0$.
Functions $f(x)$ and $g(x)$ have the same amplitude of 4, the same phase shift of 0, and the same vertical displacement of 0.

Chapter 5 Practice Test Page 287 Question 11

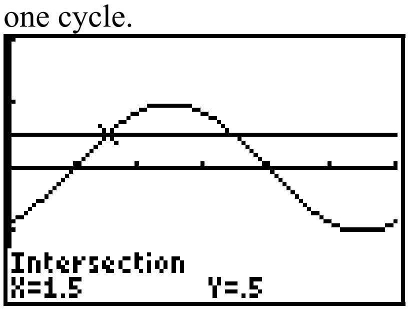
For $V(t) = 120 \sin 2513t$, $a = 120$ and $b = 2513$. So, the amplitude is $|120|$, or 120, and the period is $\frac{2\pi}{2513}$, or approximately 0.0025 s.

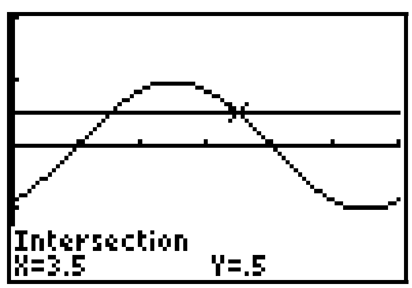
Chapter 5 Practice Test Page 287 Question 12

For $d(t) = -3 \cos \frac{\pi}{6}t + 5$, $a = -3$, $b = \frac{\pi}{6}$, and $d = 5$. So, the amplitude is $|-3|$, or 3, the period is $\frac{2\pi}{\frac{\pi}{6}}$, or 12, the maximum is 5 + 3, or 8, and the minimum is 5 – 3, or 2. The minimum of this cosine function is 2 m; it occurs at the start/end of the cycle, 00:00, 12:00, and 24:00. The maximum of this cosine function is 8 m; it occurs halfway into a cycle, 6:00 and 18:00.

Chapter 5 Practice Test Page 287 Question 13

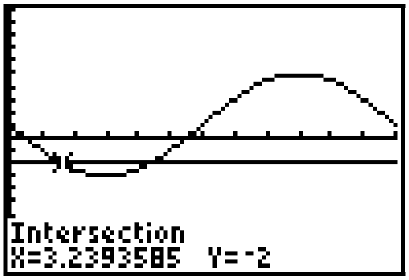
a) Determine the points of intersection for the graphs of $y = \sin \frac{\pi}{3}(x-1)$ and $y = 0.5$ in one cycle.

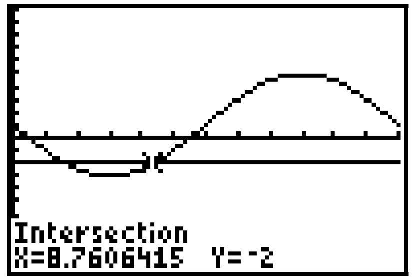




The general solution is $x = 1.5 + 6n$ and $x = 3.5 + 6n$, where $n \in \mathrm{I}$.

b) Determine the points of intersection for the graphs of $y = 4 \cos 15(x + 30°) + 1$ and $y = -2$ in one cycle.





The general solution is $x \approx 3.24° + (24°)n$ and $x \approx 8.76° + (24°)n$, where $n \in I$.

Chapter 5 Practice Test   Page 287   Question 14

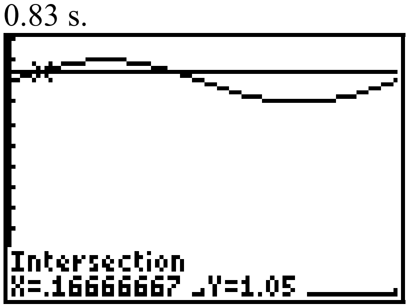
Graph I shows a cosine function with amplitude 2 and period 1 radian. Graph II shows a sine function reflected in the x-axis, amplitude 2, and period 0.5 radians.

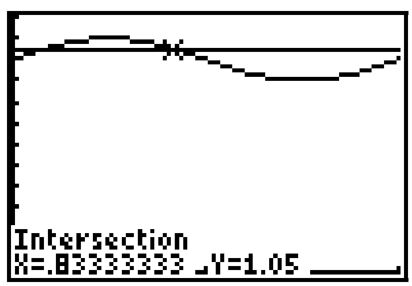
Example: Graph II has half the period of graph I. Graph I represents a cosine curve with no phase shift. Graph II represents a sine curve with no phase shift. Graph I and II have the same amplitude and both graphs have no vertical translations.

Chapter 5 Practice Test   Page 287   Question 15

a) The best fit curve is of the form $y = \sin x$. Use the maximum and rest displacements to determine $a = 0.1$. The period is 2 s, so $b = \frac{2\pi}{2}$, or π. The sinusoidal axis is given as $d = 1$. Then, the equation that models the displacement, d, in metres, as a function of time, t, in seconds, is $d = 0.1 \sin \pi t + 1$.

b) The mass is 1.05 m above the floor in the first cycle at approximately 0.17 s and 0.83 s.





c) Verify $x = 0.17$ and $x = 0.83$.

For $x = 0.17$,
$d = 0.1 \sin \pi t + 1$
$= 0.1 \sin \pi(0.17) + 1$
$= 1.050…$

For $x = 0.83$,
$d = 0.1 \sin \pi t + 1$
$= 0.1 \sin \pi(0.83) + 1$
$= 1.050…$

Chapter 5 Practice Test Page 287 Question 16

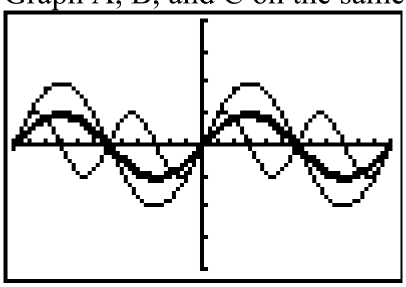
From the graph, the amplitude is 3, the period is π, the sinusoidal axis is $y = –1$.

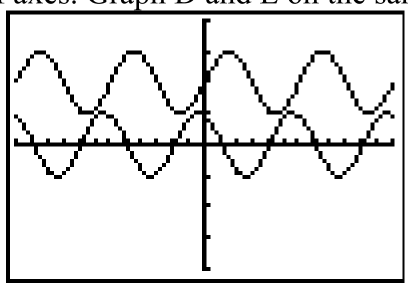
a) For a sine function, the cycle starts at $\frac{\pi}{4}$. So, $a = 3$, $b = 2$, $c = \frac{\pi}{4}$, $d = –1$. Then, the equation in the form $y = a \sin b(x – c) + d$ is $y = 3 \sin 2\left(x - \frac{\pi}{4}\right) - 1$.

b) For a cosine function, the curve is reflected in the x-axis. So, $a = –3$, $b = 2$, $c = 0$, $d = –1$. Then, the equation in the form $y = a \cos b(x – c) + d$ is $y = –3 \cos 2x – 1$.

Chapter 5 Practice Test Page 287 Question 17

Graph A, B, and C on the same set of axes. Graph D and E on the same set of axes.





a) The graphs of $f(x) = \sin x$ (A) and $g(x) = 2 \sin x$ (B) have the same x-intercepts.

b) The graphs of $f(x) = \sin x$ (A) and $g(x) = 2 \sin x$ (B) have the same period. The graphs of $h(x) = \sin 2x$ (C), $k(x) = \sin (2x + 2)$ (D), and $m(x) = \sin 2x + 2$ (E) have the same period.

c) The graph of $g(x) = 2 \sin x$ (B) has a different amplitude than the others.