Chapter 7 Exponential Functions

Section 7.1 Characteristics of Exponential Functions

Section 7.1 Page 342 Question 1

a) The function $y = x^3$ is a polynomial function, not an exponential function.

b) The function $y = 6^x$ is an exponential function. The base is greater than 0 and the independent variable is the exponent.

c) The function $y = x^{\frac{1}{2}}$ is a square root function, not an exponential function.

d) The function $y = 0.75^x$ is an exponential function. The base is greater than 0 and the independent variable is the exponent.

Section 7.1 Page 342 Question 2

a) For $x = 5$,

$f(x) = 4^x$	$g(x) = \left(\frac{1}{4}\right)^x$	$h(x) = 2^x$
$f(5) = 4^5$	$g(5) = \left(\frac{1}{4}\right)^5$	$h(5) = 2^5$
$f(5) = 1024$	$g(5) = \frac{1}{1024}$	$h(5) = 32$

The function $f(x)$ has the greatest value when $x = 5$.

b) For $x = –5$,

$f(x) = 4^x$	$g(x) = \left(\frac{1}{4}\right)^x$	$h(x) = 2^x$
$f(-5) = 4^{-5}$	$g(-5) = \left(\frac{1}{4}\right)^{-5}$	$h(-5) = 2^{-5}$
$f(-5) = \frac{1}{1024}$	$g(-5) = 1024$	$h(-5) = \frac{1}{32}$

The function $g(x)$ has the greatest value when $x = –5$.

c) Any base raised to the exponent 0 is 1.

$f(x) = 4^x$	$g(x) = \left(\frac{1}{4}\right)^x$	$h(x) = 2^x$
$f(0) = 4^0$	$g(0) = \left(\frac{1}{4}\right)^0$	$h(0) = 2^0$

$f(0) = 1$ $\quad$ $g(0) = 1$ $\quad$ $h(0) = 1$

Section 7.1 Page 342 Question 3

a) For $y = 5^x$, $c > 1$ so the graph is increasing. The graph will pass through the point (1, 5). Graph **B**.

b) For $y = \left(\frac{1}{4}\right)^x$, $c < 1$ so the graph is decreasing. The graph will pass through the point (1, 0.25). Graph **C**.

c) For $y = \left(\frac{2}{3}\right)^x$, $c < 1$ so the graph is decreasing. The graph will pass through the approximate point (1, 0.67). Graph **A**.

Section 7.1 Page 343 Question 4

a) There is a pattern in the ordered pairs.

x	*y*
0	1
1	3
2	9

As the value of x increases by 1 unit, the value of y increases by a factor of 3. Therefore, for this function, $c = 3$.
Use the point (1, 3) to check the function $y = 3^x$:

Left Side	Right Side
y	3^x
$= 3$	$= 3^1$
	$= 3$

The function equation for the graph is $y = 3^x$.

b) There is a pattern in the ordered pairs.

x	*y*
–2	25
–1	5
0	1

As the value of x increases by 1 unit, the value of y decreases by a factor of $\frac{1}{5}$. Therefore, for this function, $c = \frac{1}{5}$.

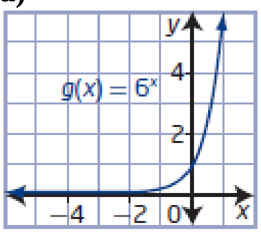
Use the point (–1, 5) to check the function $y = \left(\frac{1}{5}\right)^x$:

Left Side	Right Side
y	$\left(\frac{1}{5}\right)^x$
$= 5$	$= \left(\frac{1}{5}\right)^{-1}$
	$= 5$

The function equation for the graph is $y = \left(\frac{1}{5}\right)^x$.

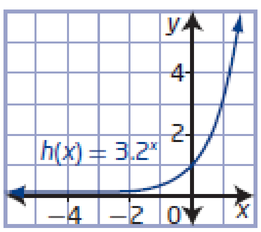
Section 7.1 Page 343 Question 5

a)



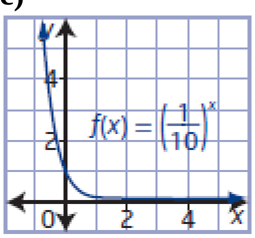
The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 0, y \in \mathrm{R}\}$.
The y-intercept is 1.
The function is increasing.
The equation of the horizontal asymptote is $y = 0$.

b)



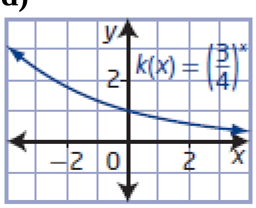
The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 0, y \in \mathrm{R}\}$.
The y-intercept is 1.
The function is increasing.
The equation of the horizontal asymptote is $y = 0$.

c)



The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 0, y \in \mathrm{R}\}$.
The y-intercept is 1.
The function is decreasing.
The equation of the horizontal asymptote is $y = 0$.

d)



The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 0, y \in \mathrm{R}\}$.
The y-intercept is 1.
The function is decreasing.
The equation of the horizontal asymptote is $y = 0$.

Section 7.1 Page 343 Question 6

a) The bacteria in a Petri dish doubling their number every hour represents growth. So, $c > 1$.

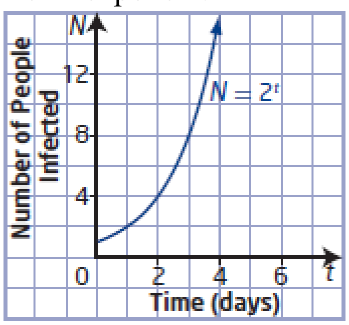
b) The half-life of the radioactive isotope actinium-225 represents decay. So, $c < 1$.

c) The amount of light passing through water decreases with depth represents decay. So, $c < 1$.

d) The population of an insect colony tripling every hour represents growth. So, $c > 1$.

Section 7.1 Page 343 Question 7

a) The function $N = 2^t$ is exponential since the base is greater than zero and the variable t is an exponent.



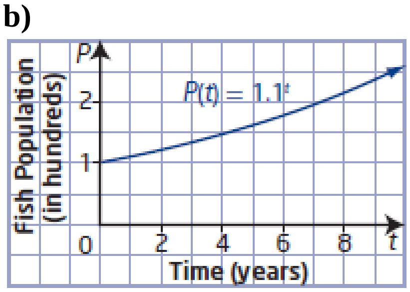
b)

i) For $t = 0$, $N = 2^0$ $= 1$ At the start, 1 person has the virus.	**ii)** For $t = 1$, $N = 2^1$ $= 2$ After 1 day, 2 people have the virus.	**iii)** For $t = 4$, $N = 2^4$ $= 16$ After 4 days, 16 people have the virus.	**iv)** For $t = 10$, $N = 2^{10}$ $= 1024$ After 10 days, 1024 people have the virus.

Section 7.1 Page 343 Question 8

a) If the population increases by 10% each year, the population becomes 110% of the previous year's population. So, the growth rate is 110%, or 1.1 written as a decimal.

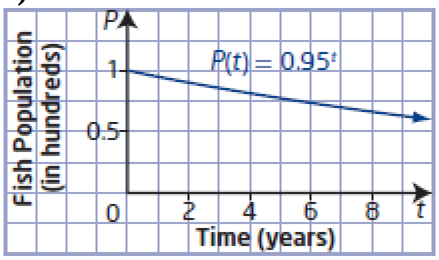
b)



The domain is $\{t \mid t \geq 0, t \in \mathrm{R}\}$, and the range is $\{P \mid P \geq 100, P \in \mathrm{R}\}$.

c) If the population decreases by 5% each year, the population becomes 95% of the previous year's population. So, the growth rate is 95%, or 0.95 written as a decimal.

d)

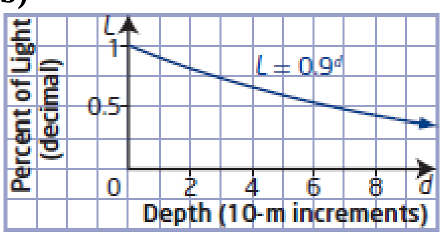


The domain is $\{t \mid t \geq 0, t \in \mathrm{R}\}$, and the range is $\{P \mid 0 < P \leq 100, P \in \mathrm{R}\}$.

Section 7.1 Page 344 Question 9

a) The exponential function that relates the amount, L, as a percent expressed as a decimal, of light available to the depth, d, in 10-m increments, is $L = 0.9^d$.

b)



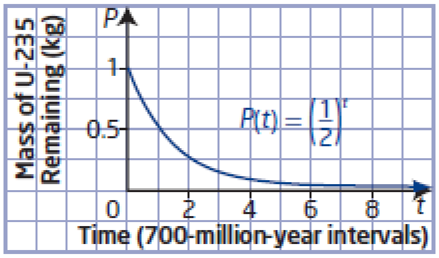
c) The domain is $\{d \mid d \geq 0, d \in \mathrm{R}\}$, and the range is $\{L \mid 0 < L \leq 1, L \in \mathrm{R}\}$.

d) 25 m is the same as 2.5 10-m increments.
For 2.5,
$L = 0.9^d$
$= 0.9^{2.5}$
$= 0.7684\ldots$
The percent of light that will reach Petra if she dives to a depth of 25 m is approximately 76.8%.

Section 7.1 Page 344 Question 10

a) Let P represent the percent, as a decimal, of U-235 remaining. Let t represent time, in 700-million-year intervals. Then, the exponential function that represents the radioactive decay of 1 kg of U-235 is $P(t) = \left(\frac{1}{2}\right)^t$.

b)

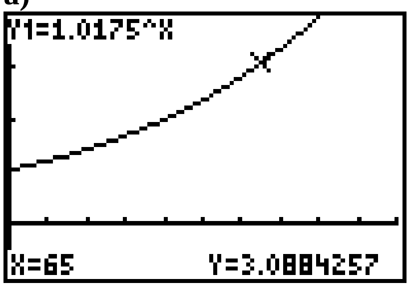


c) From the graph, it will take 3 700-million-year intervals, or 2 100 000 000 years, for 1 kg of U-235 to decay to 0.125 kg.

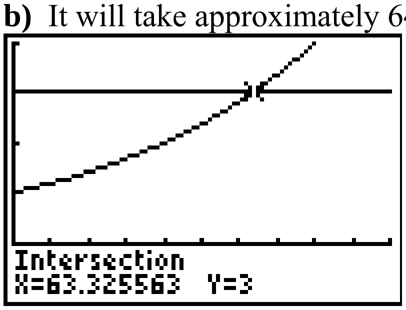
d) The sample in part c) will never decay to 0 kg, since $P = 0$ is the horizontal asymptote.

Section 7.1 Page 344 Question 11

a)

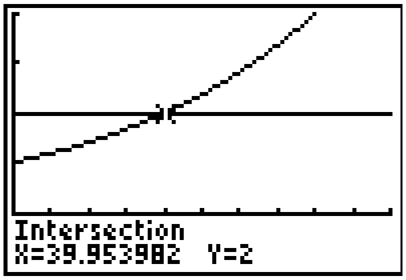


b) It will take approximately 64 years for the deposit to triple in value.



c) The amount of time it takes for a deposit to triple does not depend on the value of the initial deposit. Since each $1 amount invested triples, it does not matter what the initial investment was.

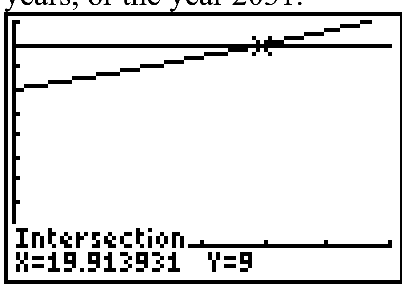
d) From the graph, the approximate doubling time for this investment is 40 years.



From the rule of 72, the approximate doubling time for this investment is $\frac{72}{1.75}$, or 42 years.

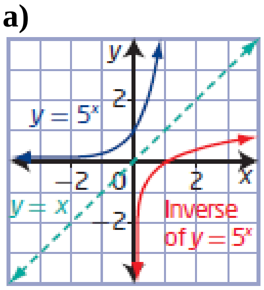
Section 7.1 Page 344 Question 12

Let P represent the world population, in billions. Let t represent time, in years, since 2011. Then, the exponential function that represents world population over time is $P(t) = 7(1.0127)^t$. The population of the world will reach 9 billion in approximately 20 years, or the year 2031.



Section 7.1 Page 344 Question 13

a)



b) The points (x, y) on the graph $y = 5^x$ become the points (y, x) on the graph of the inverse of the function. Thus, the domains and ranges are interchanged. Also, the horizontal asymptote of the graph $y = 5^x$ becomes a vertical asymptote of the graph of the inverse of the function.

c) The equation of the inverse of the function is $x = 5^y$.

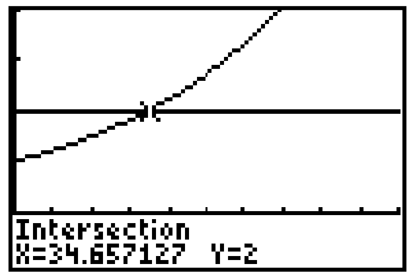
Section 7.1 Page 345 Question 14

a) The function $D = 2^{-\varphi}$ can be written as $D = \left(\frac{1}{2}\right)^{\varphi}$. The coarser the material, the greater the diameter. Therefore, a negative value of φ represents a greater value of D.

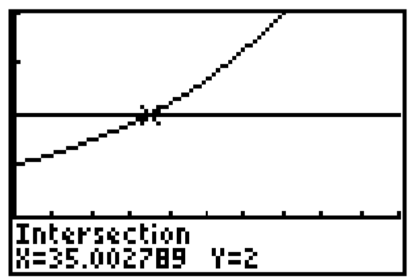
b) The diameter of fine sand is $\left(\frac{1}{2}\right)^{3}$, or 0.125 mm. The diameter of coarse gravel is $\left(\frac{1}{2}\right)^{-5}$, or 32 mm. Thus, fine sand is $\frac{1}{256}$ the diameter of coarse gravel.

Section 7.1 Page 345 Question 15

a) Graph $A(t) = (2.7183)^{0.02t}$ and $A(t) = 2$ and determine the point of intersection. The approximate doubling period is 34.7 years.



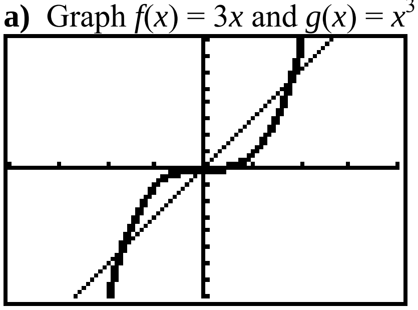
b) Graph $A = (1.02)^t$ and $A = 2$ and determine the point of intersection. The approximate doubling period is 35 years.

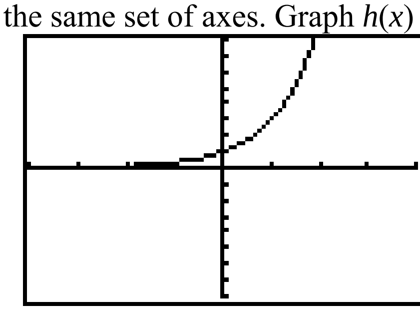


c) The results are similar, but the continuous compounding function gives a shorter doubling period by approximately 0.3 years.

Section 7.1 Page 345 Question C1

a) Graph $f(x) = 3x$ and $g(x) = x^3$ on the same set of axes. Graph $h(x) = 3^x$ separately.





b)

Feature	$f(x) = 3x$	$g(x) = x^3$	$h(x) = 3^x$
domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
range	$\{y \mid y \in R\}$	$\{y \mid y \in R\}$	$\{y \mid y > 0, y \in R\}$
intercepts	x-intercept 0, y-intercept 0	x-intercept 0, y-intercept 0	no x-intercept, y-intercept 1
equations of asymptotes	none	none	$y = 0$

c) Example: All three functions have the same domain, and each of their graphs has a y-intercept. The functions $f(x)$ and $g(x)$ have all key features in common.

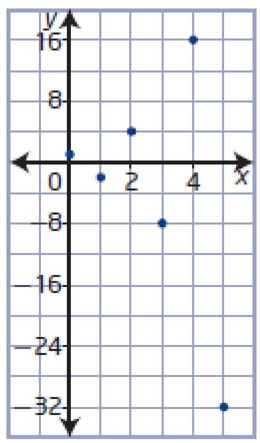
d) Example: The function $h(x)$ is the only function with an asymptote, which restricts its range and results in no x-intercept.

Section 7.1 Page 345 Question C2

a)

x	$f(x)$
0	1
1	−2
2	4
3	−8
4	16
5	−32

b)



c) No, the points do not form a smooth curve. The locations of the points alternate between above the x-axis and below the x-axis.

d) Using technology to evaluate $f\left(\frac{1}{2}\right)$ and $f\left(\frac{5}{2}\right)$ results in an error: non-real answer.

For $x = \frac{1}{2}$,

$$f(x) = (-2)^x$$
$$f\left(\frac{1}{2}\right) = (-2)^{\frac{1}{2}}$$
$$f\left(\frac{1}{2}\right) = \sqrt{-2}$$

For $x = \frac{5}{2}$,

$$f(x) = (-2)^x$$
$$f\left(\frac{5}{2}\right) = (-2)^{\frac{5}{2}}$$
$$f\left(\frac{5}{2}\right) = \sqrt{(-2)^5}$$

Both values are undefined.

e) Example: Exponential functions are defined to only include positive bases, because only positive bases result in smooth curves.

Section 7.2 Transformations of Exponential Functions

Section 7.2 Page 354 Question 1

Compare each function to the form $y = a(c)^{b(x-h)} + k$.
a) For $y = 2(3)^x$, $a = 2$. This is a vertical stretch by a factor of 2: choice **C**.

b) For $y = 3^{x-2}$, $h = 2$. This is a horizontal translation of 2 units to the right: choice **D**.

c) For $y = 3^x + 4$, $k = 4$. This is a vertical translation of 4 units up: choice **A**.

d) For $y = 3^{\frac{x}{5}}$, $b = \frac{1}{5}$. This is a horizontal stretch by a factor of 5: choice **B**.

Section 7.2 Page 354 Question 2

Compare each function to the form $y = a(c)^{b(x-h)} + k$.
a) For $y = \left(\frac{3}{5}\right)^{x+1}$, $h = -1$. This is a horizontal translation of 1 unit to the left: choice **D**.

b) For $y = -\left(\frac{3}{5}\right)^x$, $a = -1$. This is a reflection in the x-axis: choice **A**.

c) For $y = \left(\frac{3}{5}\right)^{-x}$, $b = -1$. This is a reflection in the y-axis: choice **B**.

d) For $y = \left(\frac{3}{5}\right)^x - 2$, $k = -2$. This is a vertical translation of 2 units down: choice **C**.

Section 7.2 Page 354 Question 3

Compare each function to the form $y = a(c)^{b(x-h)} + k$.
a) For $f(x) = 2(3)^x - 4$,
$a = 2$, vertical stretch by a factor of 2
$b = 1$, no horizontal stretch
$h = 0$, no horizontal translation
$k = -4$, vertical translation of 4 units down

b) For $g(x) = 6^{x-2} + 3$,
$a = 1$, no vertical stretch
$b = 1$, no horizontal stretch
$h = 2$, horizontal translation of 2 units to the right
$k = 3$, vertical translation of 3 units up

c) For $m(x) = -4(3)^{x+5}$,
$a = -4$, vertical stretch by a factor of 4 and a reflection in the x-axis
$b = 1$, no horizontal stretch
$h = -5$, horizontal translation of 5 units to the left
$k = 0$, no vertical translation

d) For $y = \left(\frac{1}{2}\right)^{3(x-1)}$,
$a = 1$, no vertical stretch
$b = 3$, horizontal stretch by a factor of $\frac{1}{3}$
$h = 1$, horizontal translation of 1 unit to the right
$k = 0$, no vertical translation

e) For $n(x) = -\frac{1}{2}(5)^{2(x-4)} + 3$,
$a = -\frac{1}{2}$, vertical stretch by a factor of $\frac{1}{2}$ and a reflection in the x-axis
$b = 2$, horizontal stretch by a factor of $\frac{1}{2}$
$h = 4$, horizontal translation of 4 units to the right
$k = 3$, vertical translation of 3 units up

f) For $y = -\left(\frac{2}{3}\right)^{2x-2}$, or $y = -\left(\frac{2}{3}\right)^{2(x-1)}$
$a = -1$, reflection in the x-axis
$b = 2$, horizontal stretch by a factor of $\frac{1}{2}$
$h = 1$, horizontal translation of 1 unit to the right
$k = 0$, no vertical translation

g) For $y = 1.5(0.75)^{\frac{x-4}{2}} - \frac{5}{2}$,
$a = 1.5$, vertical stretch by a factor of 1.5
$b = \frac{1}{2}$, horizontal stretch by a factor of 2
$h = 4$, horizontal translation of 4 units to the right

$k = -\frac{5}{2}$, vertical translation of $\frac{5}{2}$ units down

Section 7.2 Page 355 Question 4

a) Since the graph has been reflected in the *x*-axis, $a < 0$ and $0 < c < 1$. The graph has also been translated 2 units up, so $k = 2$. Choice **C**.

b) The graph has also been translated 1 unit to the right and 2 units down, so $h = 1$ and $k = -2$. Choice **A**.

c) Since the graph has been reflected in the *x*-axis, $a < 0$ and $c > 1$. The graph has also been translated 2 units up, so $k = 2$. Choice **D**.

d) The graph has also been translated 2 units to the right and 1 unit up, so $h = 2$ and $k = 1$. Choice **B**.

Section 7.2 Page 355 Question 5

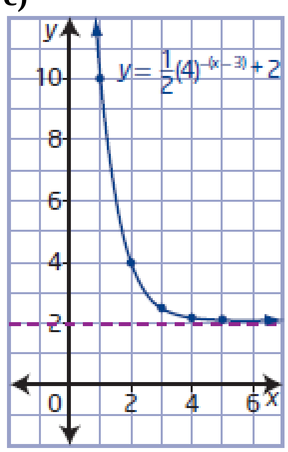
a) For $y = \frac{1}{2}(4)^{-(x-3)} + 2$,

$a = \frac{1}{2}$, vertical stretch by a factor of $\frac{1}{2}$
$b = -1$, reflection in the *y*-axis
$h = 3$, horizontal translation of 3 units to the right
$k = 2$, vertical translation of 2 units up

b)

$y = 4^x$	$y = 4^{-x}$	$y = \frac{1}{2}(4)^{-x}$	$y = \frac{1}{2}(4)^{-(x-3)} + 2$
$\left(-2, \frac{1}{16}\right)$	$\left(2, \frac{1}{16}\right)$	$\left(2, \frac{1}{32}\right)$	$\left(5, \frac{65}{32}\right)$
$\left(-1, \frac{1}{4}\right)$	$\left(1, \frac{1}{4}\right)$	$\left(1, \frac{1}{8}\right)$	$\left(4, \frac{17}{8}\right)$
(0, 1)	(0, 1)	$\left(0, \frac{1}{2}\right)$	$\left(3, \frac{5}{2}\right)$
(1, 4)	(–1, 4)	(–1, 2)	(2, 4)
(2, 16)	(–2, 16)	(–2, 8)	(1, 10)

c)

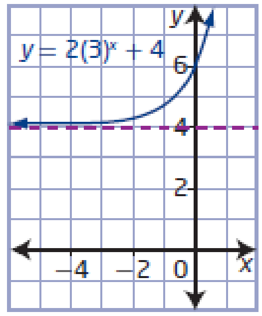


d) The domain is $\{x \mid x \in R\}$, and the range is $\{y \mid y > 2, y \in R\}$.
The equation of the horizontal asymptote is $y = 2$.
The *y*-intercept is 34.

Section 7.2 Page 355 Question 6

a) i), ii) For $y = 2(3)^x + 4$,
$a = 2$, vertical stretch by a factor of 2
$b = 1$, no horizontal stretch
$h = 0$, no horizontal translation
$k = 4$, vertical translation of 4 units up

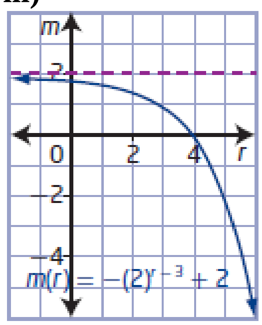
iii)



iv) The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 4, y \in \mathrm{R}\}$.
The equation of the horizontal asymptote is $y = 4$.
The y-intercept is 6.

b) i), ii) For $m(r) = -(2)^{r-3} + 2$,
$a = -1$, reflection in the x-axis
$b = 1$, no horizontal stretch
$h = 3$, horizontal translation of 3 units to the right
$k = 2$, vertical translation of 2 units up

iii)



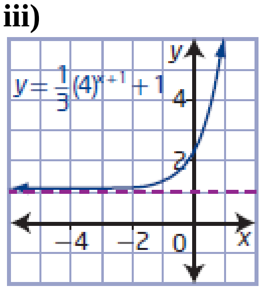
iv) The domain is $\{r \mid r \in \mathrm{R}\}$, and the range is $\{m \mid m < 2, m \in \mathrm{R}\}$.
The equation of the horizontal asymptote is $m = 2$.

The m-intercept is $\frac{15}{8}$, and the r-intercept is 4.

c) i), ii) For $y = \frac{1}{3}(4)^{x+1} + 1$,

$a = \frac{1}{3}$, vertical stretch by a factor of $\frac{1}{3}$
$b = 1$, no horizontal stretch
$h = -1$, horizontal translation of 1 unit to the left
$k = 1$, vertical translation of 1 unit up

iii)



iv) The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 1, y \in \mathrm{R}\}$.
The equation of the horizontal asymptote is $y = 1$.

The y-intercept is $\frac{7}{3}$.

d) i), ii) For $n(s) = -\frac{1}{2}\left(\frac{1}{3}\right)^{\frac{1}{4}x} - 3$,

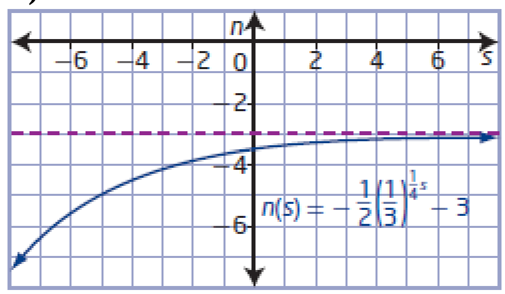
$a = -\frac{1}{2}$, vertical stretch by a factor of $\frac{1}{2}$ and a reflection in the x-axis

$b = \frac{1}{4}$, horizontal stretch by a factor of 4

$h = 0$, no horizontal translation

$k = -3$, vertical translation of 3 units down

iii)



iv) The domain is $\{s \mid s \in R\}$, and the range is $\{n \mid n < -3, n \in R\}$.
The equation of the horizontal asymptote is $n = -3$.
The n-intercept is $\frac{15}{8}$.

Section 7.2 Page 355 Question 7

a) To obtain the graph of $y = f(x - 2) + 1$, the graph of $f(x)$ must be translated 2 units to the right and 1 unit up: $y = \left(\frac{1}{2}\right)^{x-2} + 1$.

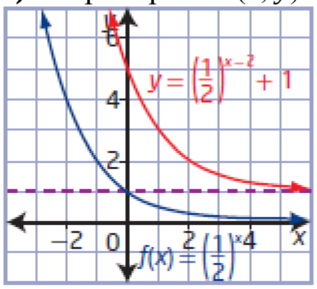
b) To obtain the graph of $y = -0.5f(x - 3)$, the graph of $f(x)$ must be vertically stretched by a factor of 0.5, reflected in the x-axis, and translated 3 units to the right: $y = -0.5(5)^{x-3}$.

c) To obtain the graph of $y = -f(3x) + 1$, the graph of $f(x)$ must be reflected in the x-axis, horizontally stretched by a factor of $\frac{1}{3}$, and translated 1 unit up: $y = \left(\frac{1}{4}\right)^{3x} + 1$.

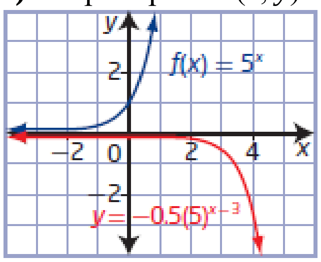
d) To obtain the graph of $y = 2f\left(-\frac{1}{3}(x-1)\right) - 5$, the graph of $f(x)$ must be vertically stretched by a factor of 2, horizontally stretched by a factor of 3, reflected in the y-axis, and translated 1 unit to the right and 5 units down: $y = 2(4)^{-\frac{1}{3}(x-1)} - 5$.

Section 7.2 Page 356 Question 8

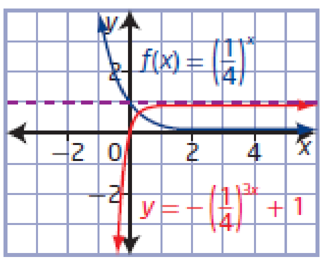
a) Map all points (x, y) on the graph of $f(x)$ to $(x + 2, y + 1)$.



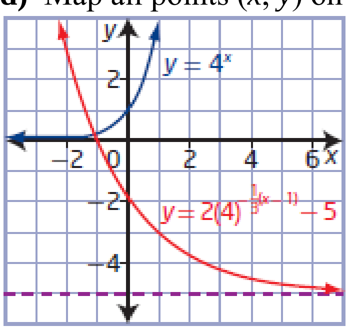
b) Map all points (x, y) on the graph of $f(x)$ to $(x + 3, –0.5y)$.



c) Map all points (x, y) on the graph of $f(x)$ to $\left(\frac{1}{3}x, -y+1\right)$.



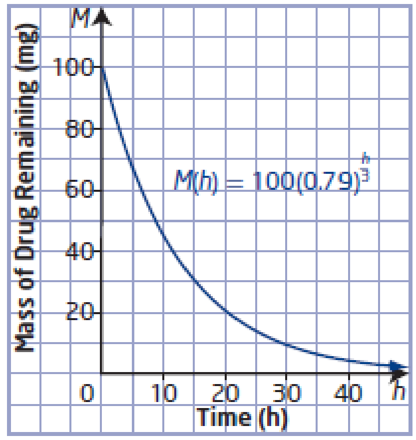
d) Map all points (x, y) on the graph of $f(x)$ to $(–3x + 1, 2y – 5)$.



Section 7.2 Page 356 Question 9

a) The number 0.79 represents the 79% of the drug remaining in the body of a dose taken. The number $\frac{1}{3}$ represents the decay rate of the dose taken. The dose decreases by 79% every $\frac{1}{3}$ h.

b)



c) The M-intercept represents the dose of 100 mg.

d) For this situation, the domain is $\{h \mid h \geq 0, h \in \mathrm{R}\}$ and the range is $\{M \mid 0 < M \leq 100, M \in \mathrm{R}\}$.

Section 7.2 Page 356 Question 10

a) Substitute $T_i = 95$ and $T_f = 20$ into

$T(t) = (T_i - T_f)(0.9)^{\frac{t}{5}} + T_f$:

$T(t) = 75(0.9)^{\frac{t}{5}} + 20$.

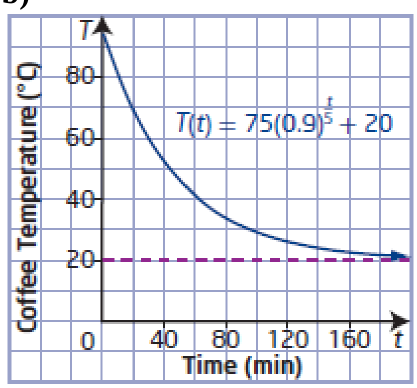
$a = 75$, vertical stretch by a factor of 75

$b = \frac{1}{5}$, horizontal stretch by a factor of 5

$h = 0$, no horizontal translation

$k = 20$, vertical translation of 20 units up

b)



c) Substitute $t = 100$.

$$T(t) = 75(0.9)^{\frac{t}{5}} + 20$$
$$T(100) = 75(0.9)^{\frac{100}{5}} + 20$$
$$T(100) = 29.1182...$$

The temperature of the coffee after 100 min is approximately 29.1 °C.

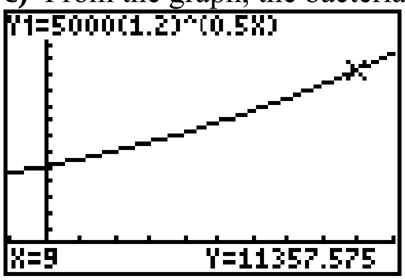
d) The equation of the horizontal asymptote is $T = 20$. This represents 20 °C, the final temperature of the coffee.

Section 7.2 Page 356 Question 11

a) For 5000 bacteria, a = 5000. For an increase of 20%, c = 1.2. For an increase that happens every 2 days, $b = \frac{1}{2}$. Then, the transformed exponential function for this situation is $P = 5000(1.2)^{\frac{x}{2}}$.

b) a = 5000, vertical stretch by a factor of 5000
$b = \frac{1}{2}$, horizontal stretch by a factor of 2
h = 0, no horizontal translation
k = 0, no vertical translation

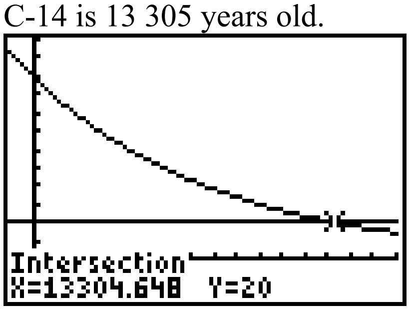
c) From the graph, the bacteria population after 9 days is approximately 11 357.



Section 7.2 Page 356 Question 12

a) The initial percent of C-14 in an organism is 100%, so a = 100. For half-life, $c = \frac{1}{2}$. Since the half-life of C-14 is about 5730 years, $b = \frac{1}{5730}$. Then, the transformed exponential function that represents the percent, P, of C-14 remaining after t years is $P = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$.

b) From the graph, the approximate age of a dead organism that has 20% of original C-14 is 13 305 years old.



Section 7.2 Page 357 Question 13

a) Let A represent the area covered by the bacteria. Let t represent time, in hours. The doubling time for the area is 10 h, so $c = 2$ and $b = \frac{1}{10}$. Since the initial area was 100 cm^2, $a = 100$. Then, $A = 100(2)^{\frac{t}{10}}$.

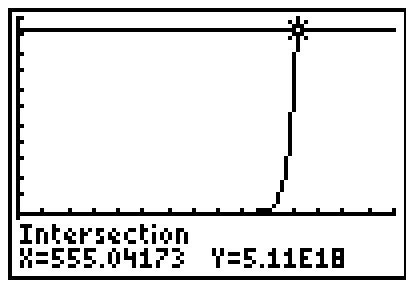
Substitute $t = 24$,

$$\begin{aligned} A &= 100(2)^{\frac{t}{10}} \\ &= 100(2)^{\frac{24}{10}} \\ &= 527.803... \end{aligned}$$

By Tuesday morning (24 h later), the bacteria covers an area of approximately 527.8 cm^2.

b) Determine the surface area of Earth: 6378 km = 637 800 000 cm

$$\begin{aligned} SA &= 4\pi r^2 \\ &= 4\pi(637\ 800\ 000)^2 \\ &\approx 5.11 \times 10^{18} \end{aligned}$$



Graph $A = 100(2)^{\frac{t}{10}}$ and $A = 5.11 \times 10^{18}$. It would take these bacteria about 555 h to cover the surface of Earth.

Section 7.2 Page 357 Question 14

a) Let P represent the fox population. Let t represent time, in years.
The initial fox population was 325 15 years ago, so $a = 325$ and $h = –15$. The population doubled in 15 years, so $b = \frac{1}{15}$ and $c = 2$. Then, $P = 325(2)^{\frac{1}{15}(t+15)}$.

Substitute $t = 20$,

$$\begin{aligned} P &= 325(2)^{\frac{1}{15}(t+15)} \\ &= 325(2)^{\frac{1}{15}(20+15)} \\ &= 1637.897... \end{aligned}$$

The fox population in 20 years will be about 1637.

b) Example: Disease or lack of food can change the rate of growth of the foxes. Exponential growth suggests that the population will grow without bound, and therefore the fox population will grow beyond the possible food sources, which is not good if not controlled.

Section 7.2 Page 357 Question C1

Example: The graph of an exponential function of the form $y = c^x$ has a horizontal asymptote at $y = 0$. Since $y \neq 0$, the graph cannot have an x-intercept.

Section 7.2 Page 357 Question C2

a) Example: For a function of the form $y = a(c)^{b(x - h)} + k$, the parameters a and k can affect the x-intercept. If $a > 0$ and $k < 0$ or $a < 0$ and $k > 0$, then the graph of the exponential function will have an x-intercept.

b) Example: For a function of the form $y = a(c)^{b(x - h)} + k$, the parameters a, h, and k can affect the y-intercept. The point $(0, y)$ on the graph of $y = c^x$ gets mapped to $(h, ay + k)$.

Section 7.3 Solving Exponential Equations

Section 7.3 Page 364 Question 1

a) $$\begin{aligned} 4^6 &= (2^2)^6 \\ &= 2^{12} \end{aligned}$$

b) $$\begin{aligned} 8^3 &= (2^3)^3 \\ &= 2^9 \end{aligned}$$

c) $$\begin{aligned} \left(\frac{1}{8}\right)^2 &= \left(\left(\frac{1}{2}\right)^3\right)^2 \\ &= \left(2^{-3}\right)^2 \\ &= 2^{-6} \end{aligned}$$

d) $16 = 2^4$

Section 7.3 Page 364 Question 2

a) 2^3 and $4^2 = 2^3$

b) $$\begin{aligned} 9^x &= (3^2)^x \text{ and } 27 = 3^3 \\ &= 3^{2x} \end{aligned}$$

c) $\left(\frac{1}{2}\right)^{2x}$ and $$\begin{aligned} \left(\frac{1}{4}\right)^{x-1} &= \left(\left(\frac{1}{2}\right)^2\right)^{x-1} \\ &= \left(\frac{1}{2}\right)^{2x-2} \end{aligned}$$

d) $$\begin{aligned} \left(\frac{1}{8}\right)^{x-2} &= \left(2^{-3}\right)^{x-2} \\ &= 2^{-3x+6} \end{aligned}$$ and $$\begin{aligned} 16^x &= (2^4)^x \\ &= 2^{4x} \end{aligned}$$

Section 7.3 Page 364 Question 3

a) $\left(\sqrt{16}\right)^2 = (4)^2$
$= 4^2$

b) $\sqrt[3]{16} = \sqrt[3]{4^2}$
$= 4^{\frac{2}{3}}$

c) $\sqrt{16}\left(\sqrt[3]{64}\right)^2 = 4(4)^2$
$= 4(4^2)$
$= 4^3$

d) $\left(\sqrt{2}\right)^8\left(\sqrt[4]{4}\right)^4 = 2^4(4)$
$= 4^2(4)$
$= 4^3$

Section 7.3 Page 364 Question 4

a) $2^{4x} = 4^{x+3}$
$2^{4x} = (2^2)^{x+3}$
$2^{4x} = 2^{2x+6}$
Equate the exponents.
$4x = 2x + 6$
$2x = 6$
$x = 3$

Check:

Left Side	Right Side
2^{4x}	4^{x+3}
$= 2^{4(3)}$	$= 4^{3+3}$
$= 2^{12}$	$= 4^6$
$= 4096$	$= 4096$

Left Side = Right Side
The solution is $x = 3$.

b) $25^{x-1} = 5^{3x}$
$(5^2)^{x-1} = 5^{3x}$
$5^{2x-2} = 5^{3x}$
Equate the exponents.
$2x - 2 = 3x$
$x = -2$

Check:

Left Side	Right Side
25^{x-1}	5^{3x}
$= 25^{-2-1}$	$= 5^{3(-2)}$
$= 25^{-3}$	$= 5^{-6}$
$= \dfrac{1}{15\,625}$	$= \dfrac{1}{15\,625}$

Left Side = Right Side
The solution is $x = -2$.

c) $3^{w+1} = 9^{w-1}$
$3^{w+1} = (3^2)^{w-1}$
$3^{w+1} = 3^{2w-2}$
Equate the exponents.
$w + 1 = 2w - 2$
$w = 3$

Check:

Left Side	Right Side
3^{w+1}	9^{w-1}
$= 3^{3+1}$	$= 9^{3-1}$
$= 3^4$	$= 9^2$
$= 81$	$= 81$

Left Side = Right Side
The solution is $w = 3$.

d) $36^{3m-1} = 6^{2m+5}$

$(6^2)^{3m-1} = 6^{2m+5}$

$6^{6m-2} = 6^{2m+5}$

Equate the exponents.

$6m - 2 = 2m + 5$

$4m = 7$

$m = \dfrac{7}{4}$

Check:

Left Side	Right Side
36^{3m-1}	6^{2m+5}
$= 36^{3\left(\frac{7}{4}\right)-1}$	$= 6^{2\left(\frac{7}{4}\right)+5}$
$= 36^{\frac{21}{4}-1}$	$= 6^{\frac{7}{2}+5}$
$= 36^{\frac{17}{4}}$	$= 6^{\frac{17}{2}}$
$= 4\,114\,202.164...$	$= 4\,114\,202.164...$

Left Side = Right Side

The solution is $m = \dfrac{7}{4}$.

Section 7.3 Page 364 Question 5

a) $4^{3x} = 8^{x-3}$

$(2^2)^{3x} = (2^3)^{x-3}$

$2^{6x} = 2^{3x-9}$

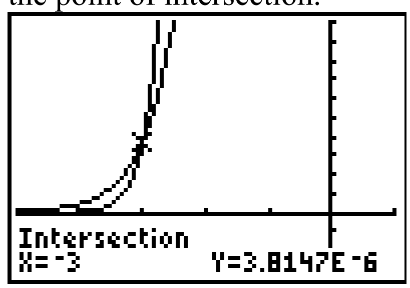
Equate the exponents.

$6x = 3x - 9$

$3x = -9$

$x = -3$

Check: Graph $y = 4^{3x}$ and $y = 8^{x-3}$ and find the point of intersection.



The solution is $x = -3$.

b) $27^x = 9^{x-2}$

$(3^3)^x = (3^2)^{x-2}$

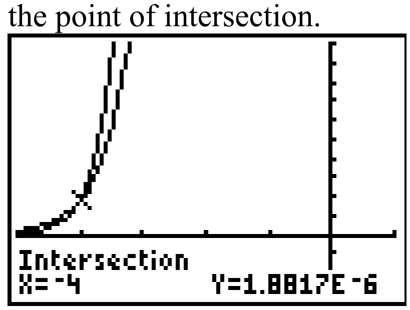
$3^{3x} = 3^{2x-4}$

Equate the exponents.

$3x = 2x - 4$

$x = -4$

Check: Graph $y = 27^x$ and $y = 9^{x-2}$ and find the point of intersection.



The solution is $x = -4$.

c) $125^{2y-1} = 25^{y+4}$

$(5^3)^{2y-1} = (5^2)^{y+4}$

$5^{6y-3} = 5^{2y+8}$

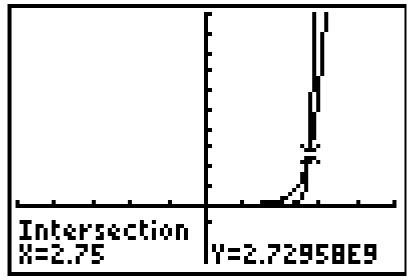
Equate the exponents.

$6y - 3 = 2y + 8$

$4y = 11$

Check: Graph $y = 125^{2x-1}$ and $y = 25^{x+4}$ and find the point of intersection.

$y = \frac{11}{4}$



The solution is $y = \frac{11}{4}$.

d) $16^{2k-3} = 32^{k+3}$

$(2^4)^{2k-3} = (2^5)^{k+3}$

$2^{8k-12} = 2^{5k+15}$

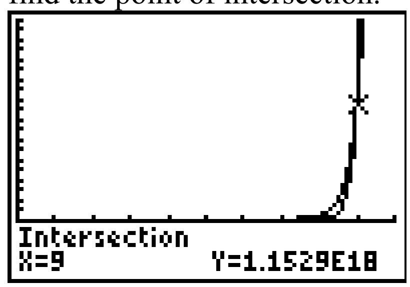
Equate the exponents.

$8k - 12 = 5k + 15$

$3k = 27$

$k = 9$

Check: Graph $y = 16^{2k-3}$ and $y = 32^{k+3}$ and find the point of intersection.



The solution is $k = 9$.

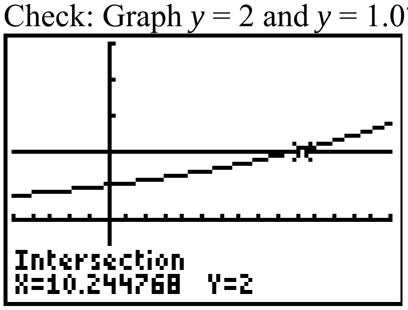
Section 7.3 Page 364 Question 6

a) Use systematic trial to solve $2 = 1.07^x$.

x	1.07^x	Mathematical Reasoning
12	2.252…	Try $x = 12$. This gives a value greater than 2. Try a lesser value.
10	1.967…	Too low. The correct value is between 10 and 12, but much closer to 10. Try 10.3.
10.3	2.007…	Close, but too high.
10.2	1.993…	This is very close and a reasonable approximation.

The solution is $x \approx 10.2$.

Check: Graph $y = 2$ and $y = 1.07^x$ and find the point of intersection.



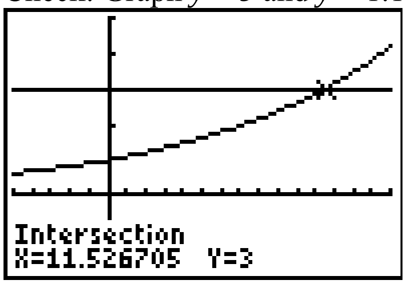
The solution is $x \approx 10.2$.

b) Use systematic trial to solve $3 = 1.1^x$.

x	1.1^x	**Mathematical Reasoning**
12	3.138…	Try $x = 12$. This gives a value greater than 3. Try a lesser value.
11	2.853…	Too low. The correct value is between 11 and 12. Try 11.6.
11.6	3.021…	Close, but too high.
11.5	2.992…	This is very close and a reasonable approximation.

The solution is $x \approx 11.5$.

Check: Graph $y = 3$ and $y = 1.1^x$ and find the point of intersection.



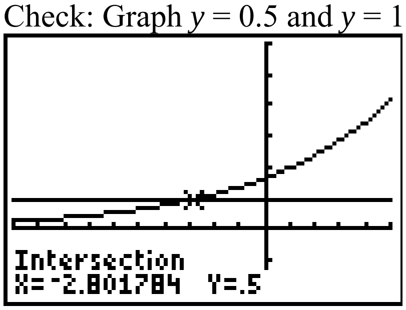
The solution is $x \approx 11.5$.

c) Use systematic trial to solve $0.5 = 1.2^{x-1}$.

x	1.2^{x-1}	**Mathematical Reasoning**
–4	0.4018…	Try $x = –4$. This gives a value lesser than 0.5. Try a greater value.
–2	0.5787…	Too high. The correct value is between –4 and –2. Try –3.
–3	0.4822…	Too low.
–2.9	0.4911…	Close, but too low.
–2.8	0.5001…	This is very close and a reasonable approximation.

The solution is $x \approx –2.8$.

Check: Graph $y = 0.5$ and $y = 1.2^{x-1}$ and find the point of intersection.



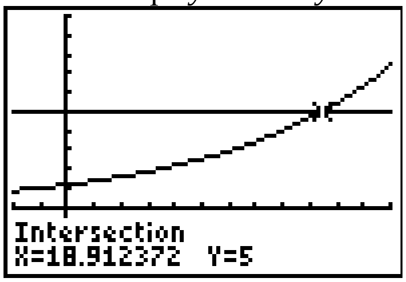
The solution is $x \approx –2.8$.

d) Use systematic trial to solve $5 = 1.08^{x+2}$.

x	1.08^{x+2}	**Mathematical Reasoning**
20	5.436…	Try $x = 20$. This gives a value greater than 5. Try a lesser value.
18	4.660…	Too low. The correct value is between 18 and 20. Try 19.
19	5.033…	Close, but too high.
18.9	4.995…	This is very close and a reasonable approximation.

The solution is $x \approx 18.9$.

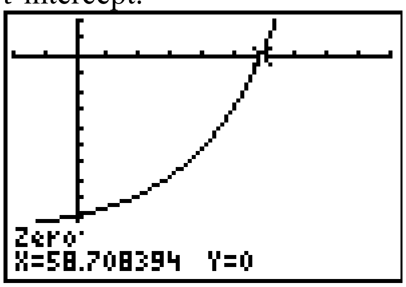
Check: Graph $y = 5$ and $y = 1.08^{x+2}$ and find the point of intersection.



The solution is $x \approx 18.9$.

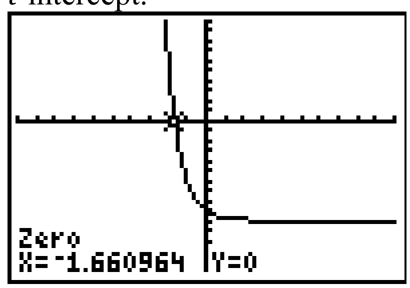
Section 7.3 Page 364 Question 7

a) Graph $y = 10(1.04)^t - 100$ and find the t-intercept.



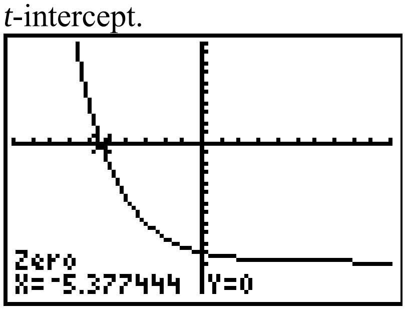
The solution is $t \approx 58.71$.

b) Graph $y = \left(\frac{1}{2}\right)^{2t} - 10$ and find the t-intercept.



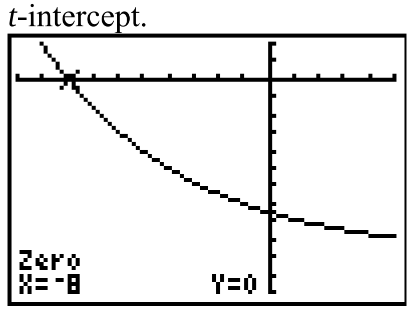
The solution is $t \approx -1.66$.

c) Graph $y = \left(\frac{1}{4}\right)^{\frac{t}{3}} - 12$ and find the t-intercept.



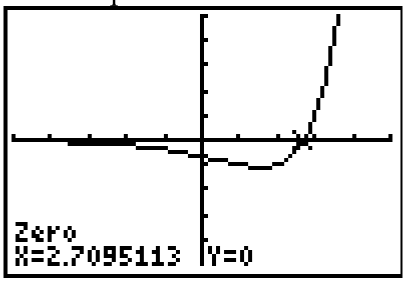
The solution is $t \approx -5.38$.

d) Graph $y = 25\left(\frac{1}{2}\right)^{\frac{t}{4}} - 100$ and find the t-intercept.



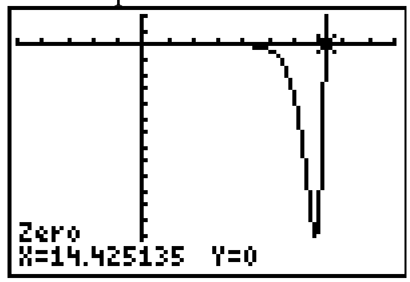
The solution is $t = -8$.

e) Graph $y = 3^{t-1} - 2^{t}$ and find the t-intercept.



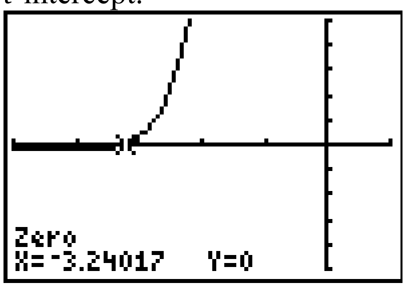
The solution is $t \approx 2.71$.

f) Graph $y = 5^{t-2} - 4^{t}$ and find the t-intercept.



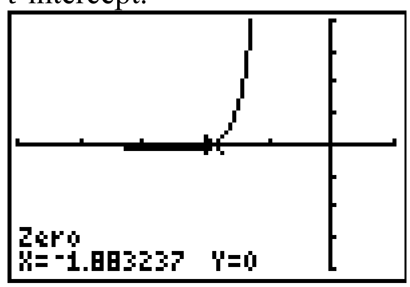
The solution is $t \approx 14.43$.

g) Graph $y = 8^{t+1} - 3^{t-1}$ and find the t-intercept.



The solution is $t \approx -3.24$.

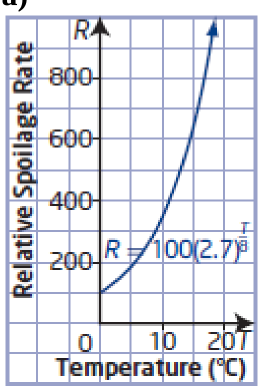
h) Graph $y = 7^{2t+1} - 4^{t-2}$ and find the t-intercept.



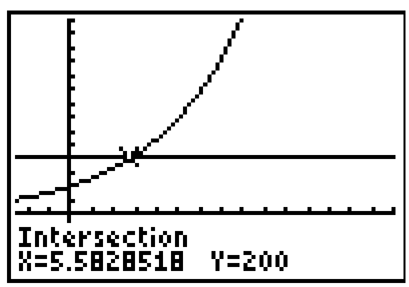
The solution is $t \approx -1.88$.

Section 7.3 Page 364 Question 8

a)



b) Graph $R = 100(2.7)^{\frac{T}{8}}$ and $R = 200$ and find the point of intersection.



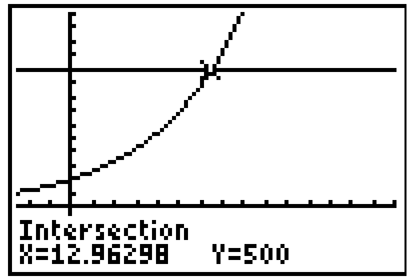
The temperature at which the relative spoilage rate doubles to 200 is approximately 5.6 °C.

c) Substitute $T = 15$.

$$\begin{aligned} R &= 100(2.7)^{\frac{T}{8}} \\ &= 100(2.7)^{\frac{15}{8}} \\ &= 643.883... \end{aligned}$$

The relative spoilage rate at 15 °C is approximately 643.

d) Graph $R = 100(2.7)^{\frac{T}{8}}$ and $R = 500$ and find the point of intersection.



The maximum storage temperature is approximately 13.0 °C.

Section 7.3 Page 364 Question 9

Let N represent the number of bacteria. Let t represent time, in hours.
The initial bacteria count is 2000, so $a = 2000$. The bacteria double every 0.75 h, so $b = \frac{4}{3}$ and $c = 2$. Then, $N = 2000(2)^{\frac{4}{3}t}$.
Substitute $N = 32\,000$,

$$N = 2000(2)^{\frac{4}{3}t}$$
$$32\,000 = 2000(2)^{\frac{4}{3}t}$$
$$16 = (2)^{\frac{4}{3}t}$$
$$2^4 = (2)^{\frac{4}{3}t}$$

Equate the exponents.

$$\frac{4}{3}t = 4$$
$$t = 3$$

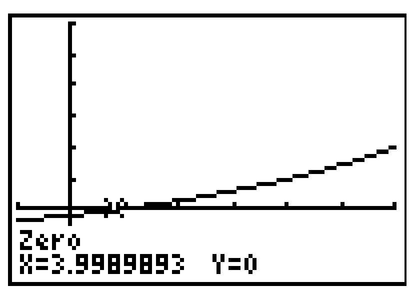
After 3 h the bacteria will be 32 000.

Section 7.3 Page 364 Question 10

Use the formula $A = P(1 + i)^n$, where $A = 7000$, $P = 6000$, and $i = 0.0393$.

$$A = P(1 + i)^n$$
$$7000 = 6000(1 + 0.0393)^n$$
$$\frac{7}{6} = 1.0393^n$$

Graph $y = 1.0393^x - \frac{7}{6}$ and find the x-intercept.



Simionie would have to invest his money in a GIC for 4 years.

Section 7.3 Page 365 Question 11

a) Use the formula $A = P(1 + i)^n$, where $P = 1000$ and $i = 0.02$: $A = 1000(1.02)^n$.

b) Substitute $n = 16$, the number of compounding periods in 4 years.

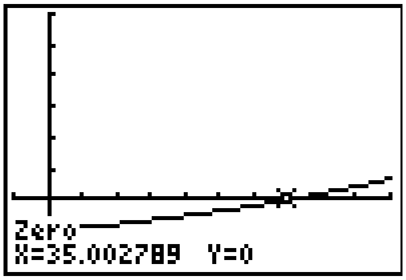
$$
\begin{aligned}
A &= 1000(1.02)^n \\
&= 1000(1.02)^{16} \\
&= 1372.785\ldots
\end{aligned}
$$

The value of the investment after 4 years is \$1372.79.

c) Substitute $A = 2000$.

$$
\begin{aligned}
A &= 1000(1.02)^n \\
2000 &= 1000(1.02)^n \\
2 &= 1.02^n
\end{aligned}
$$

Graph $y = 1.02^x - 2$ and find the x-intercept.



From the graph, it appears that it will take 35 compounding periods, or 8.75 years, for the investment to double in value. However, substituting $n = 35$ into the original function $A = 1000(1.02)^n$ results in a value of \$1999.89. So, it will take 36 compounding periods, or 9 years, for the investment to actually double in value.

Section 7.3 Page 365 Question 12

a) Let the initial sample of Co-60 be 1, so $a = 1$. For half-life, $c = \frac{1}{2}$. Since the half-life of Co-60 is about 5.3 years, $b = \frac{1}{5.3}$. Then, the exponential function that represents this situation is $m = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$, where m is the amount of Co-60 remaining after t years.

b) Substitute $t = 26.5$.

$$
\begin{aligned}
m &= \left(\frac{1}{2}\right)^{\frac{t}{5.3}} \\
&= \left(\frac{1}{2}\right)^{\frac{26.5}{5.3}} \\
&= \left(\frac{1}{2}\right)^{5} \\
&= \frac{1}{32}
\end{aligned}
$$

The fraction of a sample of Co-60 that will remain after 26.5 years is $\frac{1}{32}$.

c) Substitute $m = \frac{1}{512}$.

$$m = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$$

$$\frac{1}{512} = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$$

$$\left(\frac{1}{2}\right)^{9} = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$$

Equate the exponents.

$$\frac{t}{5.3} = 9$$
$$t = 47.7$$

It will take a sample of Co-60 47.7 years to decay to $\frac{1}{512}$ of its original mass.

Section 7.3 Page 365 Question 13

a) Use the formula $A = P(1 + i)^n$, where $P = 500$ and $i = 0.033$: $A = 500(1.033)^n$.

b) Substitute $n = 10$, the number of compounding periods in 5 years.

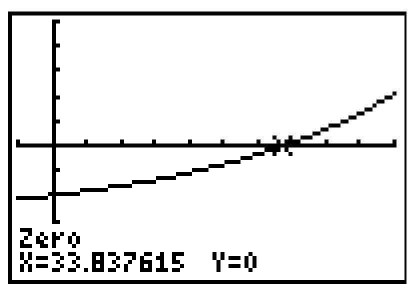
$$\begin{aligned} A &= 500(1.033)^n \\ &= 500(1.033)^{10} \\ &= 691.788\ldots \end{aligned}$$

The value of the investment after 5 years is \$691.79.

c) Substitute $A = 1500$.

$$\begin{aligned} A &= 500(1.033)^n \\ 1500 &= 500(1.033)^n \\ 3 &= 1.033^n \end{aligned}$$

Graph $y = 1.033^x - 3$ and find the x-intercept.



It will take 34 compounding periods, or 17 years, for the investment to triple in value.

Section 7.3 Page 365 Question 14

Use the formula $A = P(1 + i)^n$, where $A = 20\ 000$, $i = 0.035$, and $n = 36$.

$$A = P(1 + i)^n$$
$$20\ 000 = P(1 + 0.035)^{36}$$
$$P = \frac{20\ 000}{1.035^{36}}$$
$$P = 5796.654\ldots$$

Glenn and Arlene will need to invest \$5796.65 today.

Section 7.3 Page 365 Question 15

a) i) $2^{3x} > 4^{x+1}$

$2^{3x} > (2^2)^{x+1}$

$2^{3x} > 2^{2x+2}$

Equate the exponents.

$3x > 2x + 2$

$x > 2$

ii) $81^x < 27^{2x+1}$

$(3^4)^x < (3^3)^{2x+1}$

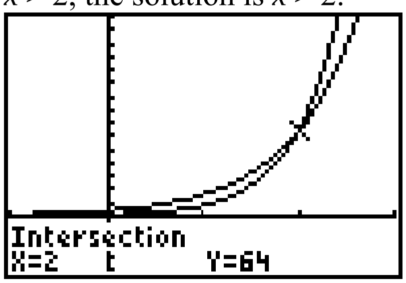
$3^{4x} < 3^{6x+3}$

Equate the exponents.

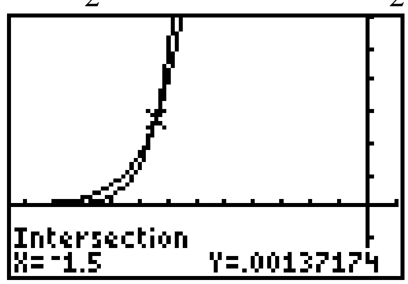
$4x < 6x + 3$

$x > -\frac{3}{2}$

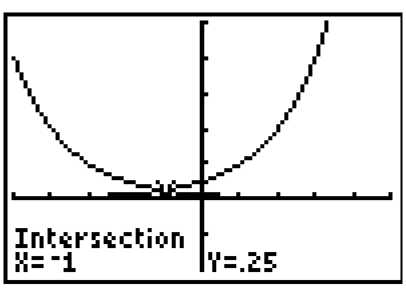
b) i) Since the graph of $y = 2^{3x}$ is greater than (above) the graph of $y = 4^{x+1}$ when $x > 2$, the solution is $x > 2$.



ii) Since the graph of $y = 81^x$ is less than (below) the graph of $y = 27^{2x+1}$ when $x > -\frac{3}{2}$, the solution is $x > -\frac{3}{2}$.



c) Example: Solve the inequality $\left(\frac{1}{2}\right)^{x+3} > 2^{x-1}$.



Since the graph of $y = \left(\frac{1}{2}\right)^{x+3}$ is greater than (above) the graph of $y = 2^{x-1}$ when $x < -1$, the solution is $x < -1$.

Section 7.3 Page 365 Question 16

$4^{2x} + 2(4^x) – 3 = 0$
$(4^x)^2 + 2(4^x) – 3 = 0$
$(4^x + 3)(4^x – 1) = 0$
$4^x + 3 = 0$ or $4^x – 1 = 0$
$4^x = –3$ $\quad$ $4^x = 1$
$x = 0$

Since the value 4^x is always greater than zero, there is no real value of x for which $4^x = –3$. The real solution is $x = 0$.

Section 7.3 Page 365 Question 17

$$\begin{aligned} 4^x – 4^{x-1} &= 24 \\ 4^x – 4^x(4)^{-1} &= 24 \\ 4^x(1 – 4^{-1}) &= 24 \\ 4^x(0.75) &= 24 \\ 4^x &= 32 \\ 2^{2x} &= 2^5 \end{aligned}$$

Equate the exponents.

$2x = 5$

$x = \frac{5}{2}$

Substitute $x = \frac{5}{2}$ into $(2^x)^x$.

$$\begin{aligned} (2^x)^x &= \left(2^{\frac{5}{2}}\right)^{\frac{5}{2}} \\ &= 2^{\frac{25}{4}} \\ &= 76.109... \end{aligned}$$

The value of $(2^x)^x$ is approximately 76.1.

Section 7.3 Page 365 Question 18

Use the formula $PMT = PV\left[\frac{i}{1-(1+i)^{-n}}\right]$ with $PMT = 831.90$, $PV = 150\ 000$, and $i = 0.0025$.

$$PMT = PV\left[\frac{i}{1-(1+i)^{-n}}\right]$$

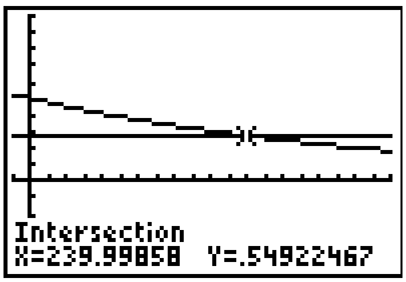
$$831.90 = 150\ 000\left[\frac{0.0025}{1-(1+0.0025)^{-n}}\right]$$

$$0.005\ 546 = \left[\frac{0.0025}{1-(1.0025)^{-n}}\right]$$

$$1-(1.0025)^{-n} = \frac{0.0025}{0.005\ 546}$$

$$(1.0025)^{-n} = 1-\frac{0.0025}{0.005\ 546}$$

Graph $y = 1.0025^{-n}$ and $y = 1-\frac{0.0025}{0.005\ 546}$ and find the point of intersection.



It will take Tyseer 240 payment periods, or 20 years, to pay off the mortgage.

Section 7.3 Page 365 Question C1

a) You can express 16^2 with a base of 4 by writing 16 as 4^2 and simplifying.

$$\begin{aligned}16^2 &= (4^2)^2\\ &= 4^4\end{aligned}$$

b) Example: You can express 16^2 with a base of 2 by writing 16 as 2^4 and simplifying.

$$\begin{aligned}16^2 &= (2^4)^2\\ &= 2^8\end{aligned}$$

You can express 16^2 with a base of $\frac{1}{2}$ by writing 16 as $\left(\frac{1}{2}\right)^{-4}$ and simplifying.

$$\begin{aligned}16^2 &= \left(\left(\frac{1}{2}\right)^{-4}\right)^2\\ &= \left(\frac{1}{2}\right)^{-8}\end{aligned}$$

Section 7.3 Page 365 Question C2

a) $16^{2x} = 8^{x-3}$	**b)** Given equation.
$(2^4)^{2x} = (2^3)^{x-3}$	Express 16 and 8 as powers of 2.
$2^{8x} = 2^{3x-9}$	Apply the power of a power law.
$8x = 3x - 9$	Equate the exponents.
$5x = -9$	Isolate the x-term.
$x = -\frac{9}{5}$	Solve of x.

Chapter 7 Review

Chapter 7 Review Page 366 Question 1

a) For the population of a country, in millions, that grows at a rate of 1.5% per year, the graph would show the function $y = 1.015x$: graph **B**.

b) The graph of $y = 10^x$ contains the point (1, 10): graph **D**.

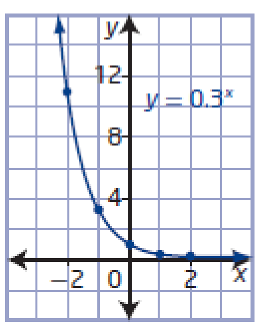
c) For Tungsten-187, a radioactive isotope that has a half-life of 1 day, the graph would show the function $y = \left(\frac{1}{2}\right)^x$: graph **A**.

d) The graph of $y = 0.2^x$ contains the point (–1, 5): graph **C**.

Chapter 7 Review Page 366 Question 2

a)

x	y
–2	$11.\overline{1}$
–1	$3.\overline{3}$
0	1
1	0.3
2	0.09



b) The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y > 0, y \in \mathrm{R}\}$.
There is no x-intercept. The y-intercept is 1.
The function is decreasing for all values of x.
The equation of the horizontal asymptote is $y = 0$.

Chapter 7 Review Page 366 Question 3

There is a pattern in the ordered pairs.

x	***y***
0	1
–1	4
–2	16

As the value of x increases by 1 unit, the value of y decreases by a factor of $\frac{1}{4}$.

Therefore, for this function, $c = \frac{1}{4}$.

Use the point (–1, 4) to check the function $y = \left(\frac{1}{4}\right)^x$:

Left Side	Right Side
y	$\left(\frac{1}{4}\right)^x$
$= 4$	$= \left(\frac{1}{4}\right)^{-1}$
	$= 4$

The function equation for the graph is $y = \left(\frac{1}{4}\right)^x$.

Chapter 7 Review Page 366 Question 4

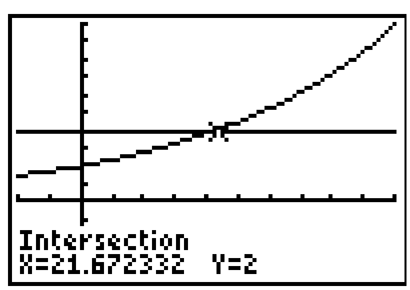
a) Since the interest rate is 3.25% per year, each year the investment grows by a factor of 103.25%, or 1.0325, as a decimal.

b) Substitute $t = 10$.

$v = 1.0325^t$
$\quad = 1.0325^{10}$
$\quad = 1.376\ldots$

The value of $1 if it is invested for 10 years will be $1.38.

c) Graph $v = 1.0325^t$ and $v = 2$ and find the point of intersection.



It will take approximately 21.7 years for the value of the dollar invested to reach $2.

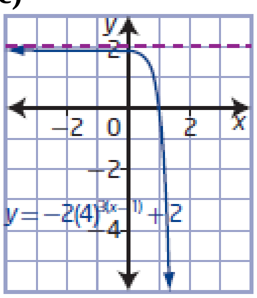
Chapter 7 Review Page 366 Question 5

a) For $y = -2(4)^{3(x-1)} + 2$,
$a = -2$, vertical stretch by a factor of 2 and reflected in the x-axis
$b = 3$, horizontal stretch by a factor of $\frac{1}{3}$
$h = 1$, horizontal translation of 1 unit to the right
$k = 2$, vertical translation of 2 units up

b)

Transformation	Parameter Value	Function Equation
horizontal stretch	$b = 3$	$y = 4^{3x}$
vertical stretch	$a = -2$	$y = -2(4)^x$
translation left/right	$h = 1$	$y = (4)^{x-1}$
translation up/down	$k = 2$	$y = 4^x + 2$

c)



d) The domain is $\{x \mid x \in \mathrm{R}\}$, and the range is $\{y \mid y < 2, y \in \mathrm{R}\}$.
The equation of the horizontal asymptote is $y = 2$.
The x-intercept is 1. The y-intercept is $\frac{63}{32}$.

Chapter 7 Review Page 367 Question 6

a) Look for a pattern in the points.

$y = 3^x$	Transformed Function
$\left(-1, \frac{1}{3}\right)$	$\left(2, \frac{1}{3}\right)$
(0, 1)	(3, 1)
(1, 3)	(4, 3)

The transformation can be described by the mapping $(x, y) \rightarrow (x + 3, y)$.
This represents a horizontal translation of 3 units to the right.

b) Look for a pattern in the points.

$y = 3^x$	**Transformed Function**
$\left(-1, \frac{1}{3}\right)$	$\left(-1, \frac{11}{3}\right)$
(0, 1)	(0, –3)
(1, 3)	(1, –1)

The transformation can be described by the mapping $(x, y) \rightarrow (x, y - 4)$.
This represents a vertical translation of 4 units down.

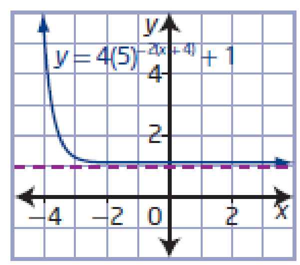
c) Look for a pattern in the points.

$y = 3^x$	**Transformed Function**
$\left(-1, \frac{1}{3}\right)$	$\left(-2, \frac{5}{3}\right)$
(0, 1)	(–1, 1)
(1, 3)	(0, –1)

The transformation can be described by the mapping $(x, y) \rightarrow (x - 1, -y + 2)$.
This represents a reflection in the x-axis and a translation of 1 unit to the right and 2 units up.

Chapter 7 Review Page 367 Question 7

a) For $f(x) = 5^x$,
• stretched vertically by a factor of 4, $a = 4$
• stretched horizontally by a factor of $\frac{1}{2}$ and reflected in the y-axis, $b = -2$
• translated 1 unit up and 4 units to the left, $k = 1$ and $h = -4$
The equation of the transformed function is $y = 4(5)^{-2(x+4)} + 1$.

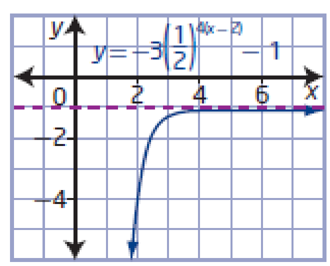


b) For $g(x) = \left(\frac{1}{2}\right)^x$,

• stretched horizontally by a factor of $\frac{1}{4}$, $b = 4$
• stretched vertically by a factor of 3 and reflected in the x-axis, $a = -3$

• translated 2 units to the right and 1 unit down, $h = 2$ and $k = -1$
The equation of the transformed function is

$y = -3\left(\frac{1}{2}\right)^{4(x-2)} - 1$.



Chapter 7 Review Page 367 Question 8

a) For $T = 190\left(\frac{1}{2}\right)^{\frac{1}{10}t}$,

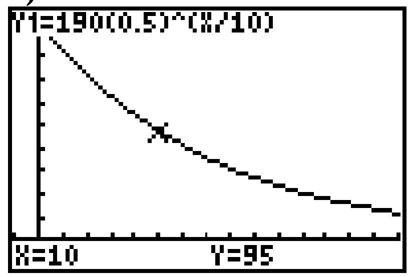
$a = 190$, vertical stretch by a factor of 190

$b = \frac{1}{10}$, horizontal stretch by a factor of 10

$h = 0$, no horizontal translation

$k = 0$, no vertical translation

b)



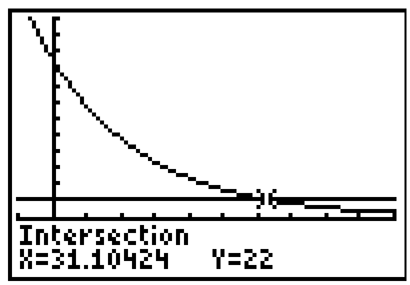
c) For this situation, the domain is $\{t \mid t \geq 0, t \in \mathrm{R}\}$ and the range is $\{T \mid 0 < T \leq 190, T \in \mathrm{R}\}$.

d) Substitute $T = 22$.

$$T = 190\left(\frac{1}{2}\right)^{\frac{1}{10}t}$$

$$\color{red}{22} = 190\left(\frac{1}{2}\right)^{\frac{1}{10}t}$$

Graph $T = 190\left(\frac{1}{2}\right)^{\frac{1}{10}t}$ and $T = 22$ and find the point of intersection.



The milk will keep fresh at 22 °C for approximately 31.1 h.

Chapter 7 Review Page 367 Question 9

a) $36 = 6^2$

b) $\frac{1}{36} = 6^{-2}$

c) $\left(\sqrt[3]{216}\right)^5 = 6^5$

Chapter 7 Review Page 367 Question 10

a)
$$\begin{aligned} 3^{5x} &= 27^{x-1} \\ 3^{5x} &= (3^3)^{x-1} \\ 3^{5x} &= 3^{3x-3} \end{aligned}$$

Equate the exponents.

$5x = 3x - 3$

$2x = -3$

$x = -\frac{3}{2}$

b)
$$\begin{aligned} \left(\frac{1}{8}\right)^{2x+1} &= 32^{x-3} \\ (2^{-3})^{2x+1} &= (2^5)^{x-3} \\ 2^{-6x-3} &= 2^{5x-15} \end{aligned}$$

Equate the exponents.

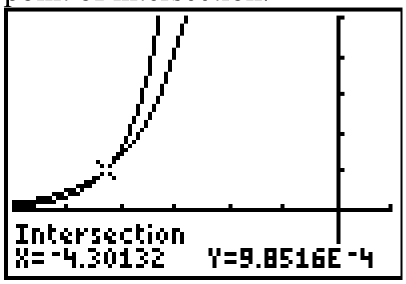
$-6x - 3 = 5x - 15$

$-11x = -12$

$x = \frac{12}{11}$

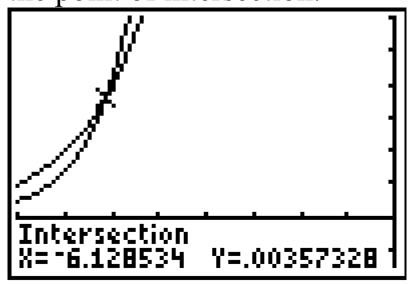
Chapter 7 Review Page 367 Question 11

a) Graph $y = 3^{x-2}$ and $y = 5^x$ and find the point of intersection.



The solution is $x \approx -4.30$.

b) Graph $y = 2^{x-2}$ and $y = 3^{x+1}$ and find the point of intersection.



The solution is $x \approx -6.13$.

Chapter 7 Review Page 367 Question 12

a) Let the initial sample of Ni-65 be 1, so $a = 1$. For half-life, $c = \frac{1}{2}$. Since the half-life of Ni-65 is 2.5 h, $b = \frac{1}{2.5}$. Then, the exponential function that represents this situation is $m = \left(\frac{1}{2}\right)^{\frac{t}{2.5}}$, where m is the amount of Ni-65 remaining after t hours.

b) Substitute $t = 10$.

$$\begin{aligned} m &= \left(\frac{1}{2}\right)^{\frac{t}{2.5}} \\ &= \left(\frac{1}{2}\right)^{\frac{10}{2.5}} \\ &= \left(\frac{1}{2}\right)^{4} \\ &= \frac{1}{16} \end{aligned}$$

The fraction of a sample of Ni-65 that will remain after 10 h is $\frac{1}{16}$.

c) Substitute $m = \frac{1}{1024}$.

$$\begin{aligned} m &= \left(\frac{1}{2}\right)^{\frac{t}{2.5}} \\ \frac{1}{1024} &= \left(\frac{1}{2}\right)^{\frac{t}{2.5}} \\ \left(\frac{1}{2}\right)^{10} &= \left(\frac{1}{2}\right)^{\frac{t}{2.5}} \end{aligned}$$

Equate the exponents.

$$\begin{aligned} \frac{t}{2.5} &= 10 \\ t &= 25 \end{aligned}$$

It will take a sample of Ni-65 25 h to decay to $\frac{1}{1024}$ of its original mass.

Chapter 7 Practice Test

Chapter 7 Practice Test Page 368 Question 1

The functions $y = 2^x$, $y = \left(\frac{2}{3}\right)^x$, and $y = 7^x$ will all have the same y-value of 1 when $x = 0$: Choice **B**.

Chapter 7 Practice Test Page 368 Question 2

To obtain the graph of $y = 3^{\frac{1}{4}(x-5)} - 2$, transform the graph of $y = 3^x$ by a horizontal stretch by a factor of 4 and a translation of 5 units to the right and 2 units down: Choice **C**.

Chapter 7 Practice Test Page 368 Question 3

Let V represent the value of the car. Let t represent time, in years.
The current value is 100 000, so a = 100 000. The value doubles every 10 years, so $b = \frac{1}{10}$ and c = 2. Then, $V = 100\,000(2)^{\frac{t}{10}}$.
Substitute t = –20,

$$\begin{aligned} V &= 100\,000(2)^{\frac{t}{10}} \\ &= 100\,000(2)^{\frac{-20}{10}} \\ &= 25\,000 \end{aligned}$$

The value of the car 20 years ago was \$25 000: Choice **B**.

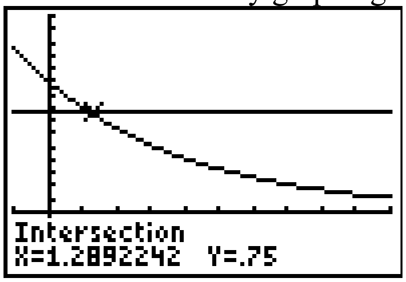
Chapter 7 Practice Test Page 368 Question 4

$$\begin{aligned} \frac{2^9}{(4^3)^2} &= \frac{2^9}{((2^2)^3)^2} \\ &= \frac{2^9}{2^{12}} \\ &= 2^{-3} \end{aligned}$$

Choice **A**.

Chapter 7 Practice Test Page 368 Question 5

Solve $0.75 = 0.8^x$ by graphing.



The glass should be approximately 1.3 mm thick: Choice **D**.

Chapter 7 Practice Test Page 368 Question 6

a) Look for a pattern in the points.

$y = 5^x$	**Transformed Function**
(0, 1)	(–3, 3)
(1, 5)	(–2, 7)
(2, 25)	(–1, 27)

The transformation can be described by the mapping $(x, y) \rightarrow (x - 3, y + 2)$.
This represents a horizontal translation of 3 units to the left and 2 units up: $h = -3$ and $k = 2$. So, the equation of the transformed function is $y = 5^{x+3} + 2$.

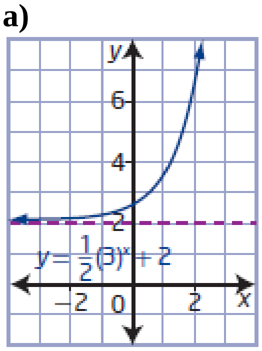
b) Look for a pattern in the points.

$y = 2^x$	**Transformed Function**
(0, 1)	$\left(1, -\frac{9}{2}\right)$
(1, 2)	(2, –5)
(2, 4)	(3, –6)

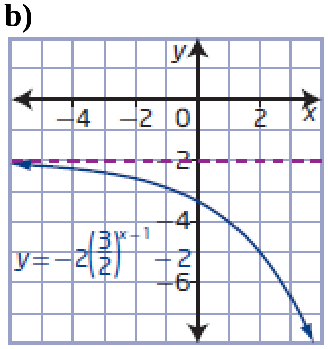
The transformation can be described by the mapping $(x, y) \rightarrow (x + 1, -0.5y - 4)$.
This represents a vertical stretch by a factor of 0.5, a reflection in the x-axis, and a translation of 1 unit to the left and 4 units down: $a = -0.5$, $h = 1$, and $k = -4$.
So, the equation of the transformed function is $y = -0.5(2)^{x-1} - 4$.

Chapter 7 Practice Test Page 369 Question 7

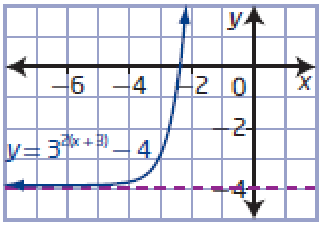
a)



b)



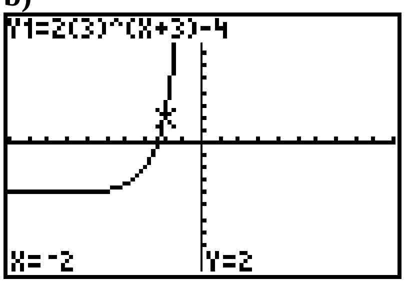
c)



Chapter 7 Practice Test Page 369 Question 8

a) The base function for $g(x) = 2(3)^{x+3} - 4$ is $f(x) = 3^x$.
For $g(x) = 2(3)^{x+3} - 4$,
$a = 2$, vertical stretch by a factor of 2
$b = 1$, no horizontal stretch
$h = -3$, horizontal translation of 3 units to the left
$k = -4$, vertical translation of 4 units down

b)



c) The domain is $\{x \mid x \in \mathrm{R}\}$, the range is $\{y \mid y > -4, y \in \mathrm{R}\}$, and the equation of the horizontal asymptote is $y = -4$.

Chapter 7 Practice Test Page 369 Question 9

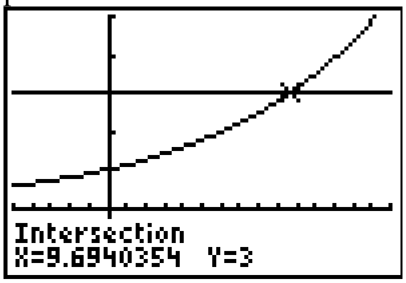
a)
$$3^{2x} = 9^{\frac{1}{2}(x-4)}$$
$$3^{2x} = (3^2)^{\frac{1}{2}(x-4)}$$
$$3^{2x} = 3^{x-4}$$
Equate the exponents.
$2x = x - 4$
$x = -4$

b)
$$27^{x-4} = 9^{x+3}$$
$$(3^3)^{x-4} = (3^2)^{x+3}$$
$$3^{3x-12} = 3^{2x+6}$$
Equate the exponents.
$3x - 12 = 2x + 6$
$x = 18$

c)
$$1024^{2x-1} = 16^{x+4}$$
$$(2^{10})^{2x-1} = (2^4)^{x+4}$$
$$2^{20x-10} = 2^{4x+16}$$
Equate the exponents.
$20x - 10 = 4x + 16$
$16x = 26$
$$x = \frac{13}{8}$$

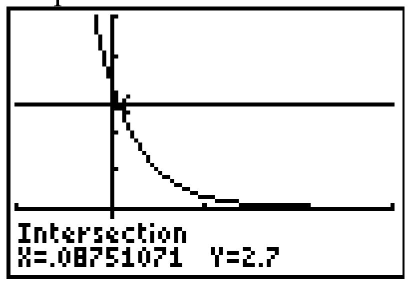
Chapter 7 Practice Test Page 369 Question 10

a) Graph $y = 3$ and $y = 1.12^x$ and find the point of intersection.



The solution is $x \approx 9.7$.

b) Graph $y = 2.7$ and $y = 0.3^{2x - 1}$ and find the point of intersection.



The solution is $x \approx 0.1$.

Chapter 7 Practice Test Page 369 Question 11

a) If the growth rate remains constant at 2.77%, then the population would have been multiplied by a factor of 1.0277 after 1 year.

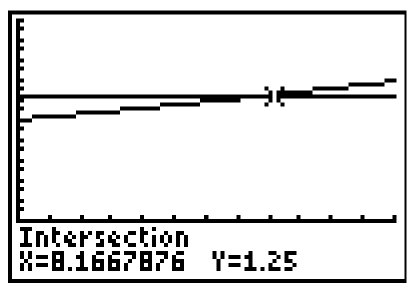
b) Let the initial percent of the population for Saskatoon in 2010 be 100, so $a = 100$. For the growth rate from part a), $c = 1.0277$. Then, the exponential function that represents this situation is $P = 100(1.0277)^t$, where P is percent of the population t years after 2010.

c) For this situation, the domain is $\{t \mid t \geq 0, t \in \mathrm{R}\}$ and the range is $\{P \mid P \geq 100, P \in \mathrm{R}\}$.

d) For Saskatoon's population to grow by 25%, substitute $P = 125$.

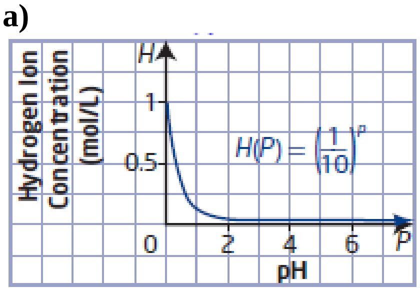
$P = 100(1.0277)^t$
$125 = 100(1.0277)^t$
$1.25 = 1.0277^t$

Graph $y = 1.25$ and $y = 1.0277^x$ and find the point of intersection.
It will take approximately 8.2 years for Saskatoon's population to grow by 25%.



Chapter 7 Practice Test Page 369 Question 12

a)



b) Substitute $P = 7$.

$$H(P) = \left(\frac{1}{10}\right)^P$$

$$H(7) = \left(\frac{1}{10}\right)^7$$

$$H(7) = 1 \times 10^{-7}$$

The hydrogen ion concentration for a pH of 7.0 is 1.0×10^{-7} [H+].

c) Determine the hydrogen ion concentration for a pH of 7.6. Substitute $P = 7.6$.

$$H(P) = \left(\frac{1}{10}\right)^P$$

$$H(7.6) = \left(\frac{1}{10}\right)^{7.6}$$

$$H(7.6) \approx 2.5 \times 10^{-8}$$

The equivalent range of hydrogen ion concentration for a pH between 7.0 and 7.6 is from 1.0×10^{-7} [H+] to 2.5×10^{-8} [H+].

Chapter 7 Practice Test Page 369 Question 13

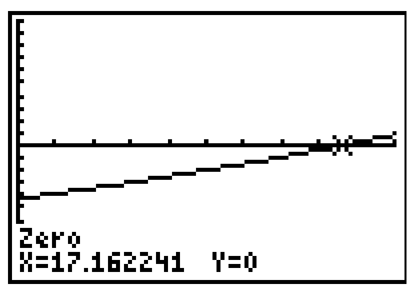
Use the formula $A = P(1 + i)^n$, where $A = 5000$, $P = 3500$ and $i = 0.021$.
Solve $5000 = 3500(1.021)^n$.

$$5000 = 3500(1.021)^n$$

$$\frac{5000}{3500} = 1.021^n$$

$$\frac{10}{7} = 1.021^n$$

Graph $y = 1.021^x - \frac{10}{7}$ and find the x-intercept.



It will take approximately 18 compounding periods, or 4.5 years, before Lucas has enough money to take the vacation he wants.

Chapter 7 Practice Test   Page 369   Question 14

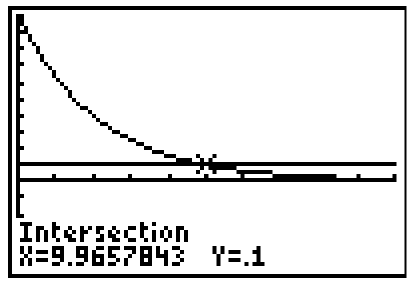
Determine when a computer purchased for \$3000 is worth 10% of its values, or \$300. Substitute $V = 300$.

$$V(t) = 3000\left(\frac{1}{2}\right)^{\frac{t}{3}}$$

$$300 = 3000\left(\frac{1}{2}\right)^{\frac{t}{3}}$$

$$0.1 = \left(\frac{1}{2}\right)^{\frac{t}{3}}$$

Graph $y = 0.1$ and $y = \left(\frac{1}{2}\right)^{\frac{x}{3}}$ and find the point of intersection.



It will take about 9.97 years, for the computer to be worth 10% of its purchase price.