MATH LAB

2.3 Measuring an Inaccessible Height

LESSON FOCUS

Determine a height that cannot be measured directly.

Make Connections

Tree farmers use a *clinometer* to measure the angle between a horizontal line and the line of sight to the top of a tree. They measure the distance to the base of the tree. How can they then use the tangent ratio to calculate the height of the tree?

Construct Understanding

TRY THIS

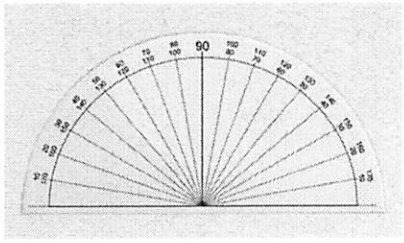
Work with a partner.

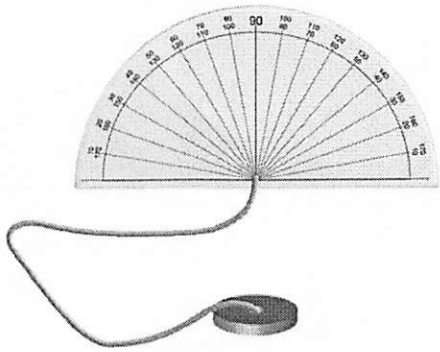
You will need:

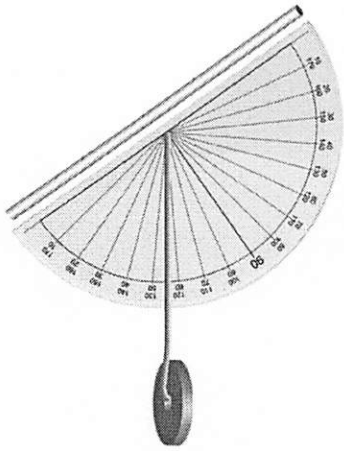
- an enlarged copy of a 180° protractor
- scissors
- a measuring tape or 2 metre sticks
- a piece of heavy cardboard big enough for you to attach the paper protractor
- a drinking straw
- glue
- adhesive tape
- a needle and thread
- a small metal washer or weight
- grid paper

A. Make a drinking straw clinometer:

- Glue or tape the paper protractor to the cardboard. Carefully cut it out.
- Use the needle to pull the thread through the cardboard at the centre of baseline of the protractor. Secure the thread to the back of the cardboard with tape. Attach the weight to the other end of the thread.
- Tape the drinking straw along the baseline of the protractor for use as a sighting tube.



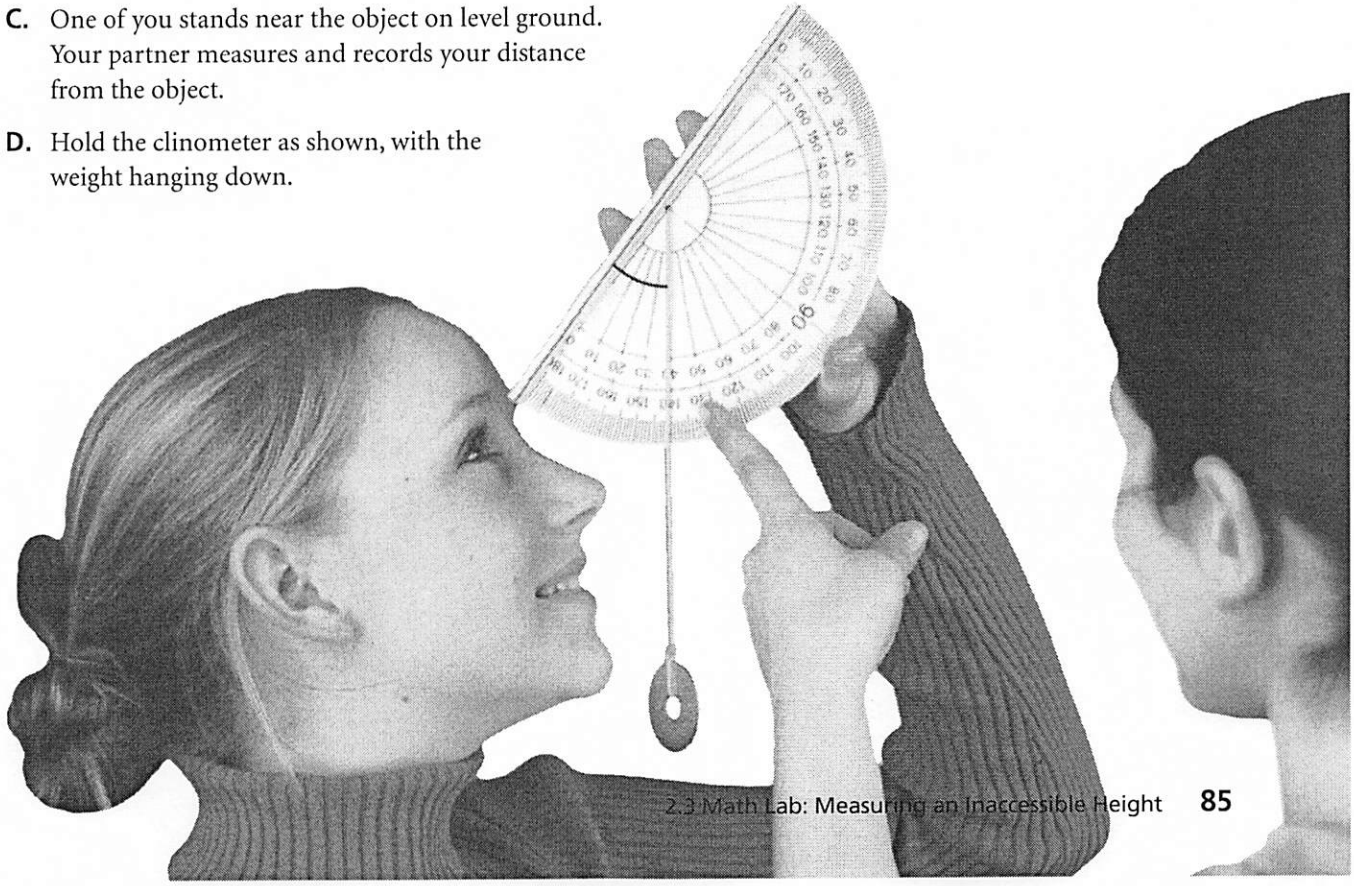




B. With your partner, choose a tall object whose height you cannot measure directly; for example, a flagpole, a totem pole, a tree, or a building.

C. One of you stands near the object on level ground. Your partner measures and records your distance from the object.

D. Hold the clinometer as shown, with the weight hanging down.



How does the acute angle between the thread and the straw relate to the angle of inclination of the straw?

E. Look at the top of the object through the straw. Your partner records the acute angle indicated by the thread on the protractor.

F. Your partner measures and records how far your eye is above the ground.

G. Sketch a diagram with a vertical line segment representing the object you want to measure. Label:

- your distance from the object
- the vertical distance from the ground to your eyes
- the angle of inclination of the straw

H. Change places with your partner. Repeat Steps B to G.

What other strategy could you use to determine the height of the object?

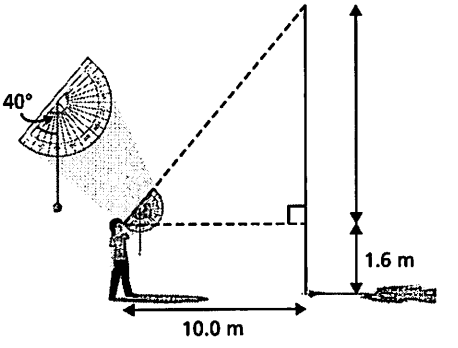
I. Use your measurements and the tangent ratio to calculate the height of the object.

J. Compare your results with those of your partner. Does the height of your eye affect the measurements? The final result? Explain.

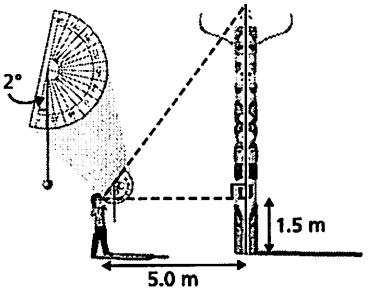
Assess Your Understanding

1. Explain how the angle shown on the protractor of your clinometer is related to the angle of inclination that the clinometer measures.

2. A tree farmer stood 10.0 m from the base of a tree. She used a clinometer to sight the top of the tree. The angle shown on the protractor scale was 40°. The tree farmer held the clinometer 1.6 m above the ground. Determine the height of the tree to the nearest tenth of a metre. The diagram is *not* drawn to scale.



3. Use the information in the diagram to calculate the height of a totem pole observed with a drinking-straw clinometer. Give the answer to the nearest metre. The diagram is *not* drawn to scale.



Keep your clinometer for use in the Review.

CHECKPOINT 1

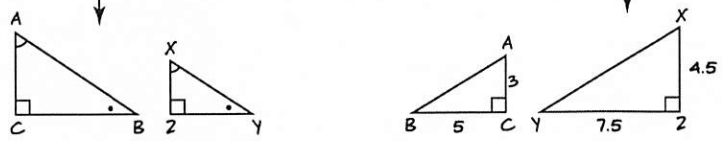
Connections

Similar Right Triangles

$\triangle ABC \sim \triangle XYZ$

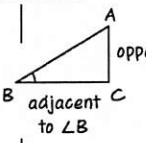
Corresponding angles are equal.

Corresponding sides are proportional.



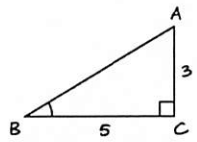
Tangent Ratio

$\tan B = \frac{\text{opposite}}{\text{adjacent}}$



Determine an angle given two legs.

Determine a side given an angle and a leg.

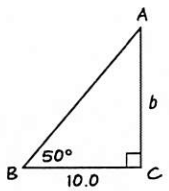


$\tan B = \frac{\text{opposite}}{\text{adjacent}}$

$\tan B = \frac{3}{5}$

$\tan B = 0.6$

$\angle B \doteq 31°$

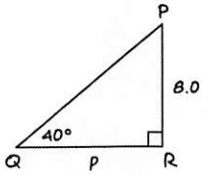


$\tan B = \frac{\text{opposite}}{\text{adjacent}}$

$\tan 50° = \frac{b}{10}$

$b = 10 \tan 50°$

$b \doteq 11.9$



$\tan Q = \frac{\text{opposite}}{\text{adjacent}}$

$\tan 40° = \frac{8}{p}$

$p = \frac{8}{\tan 40°}$

$p \doteq 9.5$

or

$\angle P = 90° - \angle Q$

$\angle P = 50°$

$\tan P = \frac{\text{opposite}}{\text{adjacent}}$

$\tan 50° = \frac{p}{8}$

$p = 8 \tan 50°$

$p \doteq 9.5$

Concept Development

In Lesson 2.1

- You applied what you know about similar right triangles to develop the concept of the **tangent ratio.**
- You used the tangent ratio **to determine an acute angle** in a right triangle when you know the lengths of the legs.

In Lesson 2.2

- You showed how to determine the **length of a leg** in a right triangle when you know the measures of an acute angle and the other leg.

In Lesson 2.3

- You applied the tangent ratio to a real-world measurement problem.