2.6 Applying the Trigonometric Ratios

LESSON FOCUS

Use a primary trigonometric ratio to solve a problem modelled by a right triangle.

Make Connections

Double-decker buses with wheelchair access ramps are used in Victoria, BC. When the bus is lowered, the extended ramp allows entry to the bus at about 4 in. above the sidewalk level. The ramp is about 3 ft. 3 in. long. How could you determine the angle of inclination of the ramp?

Construct Understanding

THINK ABOUT IT

Work with a partner.

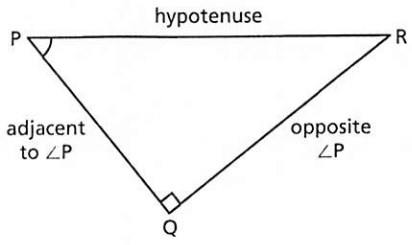
Anwar is designing a wheelchair accessibility ramp for his sister. He knows these data:

- The ramp will rise 1 ft. from the level ground to the door of the house.
- The horizontal distance from the start of the ramp at the sidewalk to the door is 20 ft.
- The building code states that the angle of inclination of the ramp must be less than 5°.

Determine whether Anwar's design will comply with the building code.

Solving a triangle means to determine the measures of all the angles and the lengths of all the sides in the triangle.

When we calculate the measures of all the angles and all the lengths in a right triangle, we **solve the triangle.** We can use any of the three primary trigonometric ratios to do this.



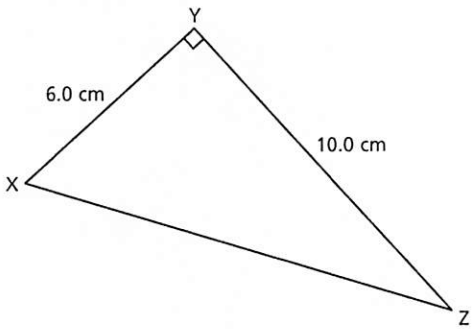
$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\sin P = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos P = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Example 1 Solving a Right Triangle Given Two Sides

Solve $\triangle XYZ$. Give the measures to the nearest tenth.



SOLUTIONS

Method 1

Determine the length of XZ first.
Use the Pythagorean Theorem in right $\triangle XYZ$.

$XZ^2 = 6.0^2 + 10.0^2$

$XZ^2 = 36.00 + 100.00$

$XZ^2 = 136.00$

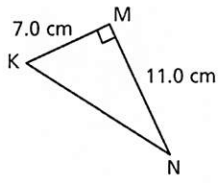
$XZ = \sqrt{136}$

$XZ = 11.6619\ldots$

XZ is approximately 11.7 cm.

CHECK YOUR UNDERSTANDING

1. Solve this triangle. Give the measures to the nearest tenth.



[Answers: KN ≐ 13.0 cm; ∠K ≐ 57.5°; ∠N ≐ 32.5°]

Determine the measure of $\angle Z$.

Since YZ is adjacent to $\angle Z$ and XZ is the hypotenuse, use the cosine ratio.

$$\cos Z = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos Z = \frac{YZ}{XZ}$$

$$\cos Z = \frac{10.0}{\sqrt{136}}$$

$$\angle Z = 30.9637...°$$

So, $\angle X = 90° - \angle Z$

The acute angles in a right triangle have a sum of 90°.

$$\angle X = 59.0362...°$$

Which other trigonometric ratio could you have used in Method 1? Why might it be better to use this ratio?

Method 2

Determine the angle measures first. Determine the measure of $\angle Z$ in right $\triangle XYZ$.

Since YZ is adjacent to $\angle Z$ and XY is opposite $\angle Z$, use the tangent ratio.

$$\tan Z = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Z = \frac{XY}{YZ}$$

$$\tan Z = \frac{6.0}{10.0}$$

$$\tan Z = 0.6$$

$$\angle Z = 30.9637...°$$

So, $\angle X = 90° - \angle Z$

The acute angles in a right triangle have a sum of 90°.

$$\angle X = 59.0362...°$$

Determine the length of XZ.

Since YZ is adjacent to $\angle Z$ and XZ is the hypotenuse, use the cosine ratio.

$$\cos Z = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos Z = \frac{YZ}{XZ}$$

$$\cos 30.9637...° = \frac{10.0}{XZ}$$

Solve the equation for XZ. Multiply both sides by XZ.

$$XZ \cos 30.9637...° = 10.0$$

Divide both sides by $\cos 30.9637...°$

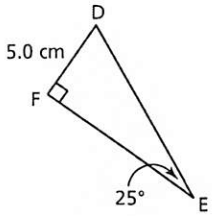
$$XZ = \frac{10.0}{\cos 30.9637...°}$$

$$XZ = 11.6614...$$

XZ is approximately 11.7 cm.
$\angle X$ is approximately 59.0° and
$\angle Z$ is approximately 31.0°.

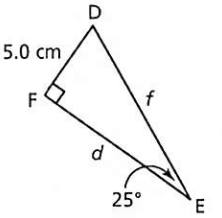
Example 2 Solving a Right Triangle Given One Side and One Acute Angle

Solve this triangle. Give the measures to the nearest tenth where necessary.



SOLUTION

Label a diagram.
Determine the measure of ∠D first.
In right △DEF:



$$\angle D + \angle E = 90°$$

$$\angle D = 90° - 25°$$

$$\angle D = 65°$$

Determine the length of EF. Since EF is opposite ∠D and DF is adjacent to ∠D, use the tangent ratio.

$$\tan D = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan D = \frac{d}{e}$$

$$\tan 65° = \frac{d}{5.0}$$ Solve the equation for d. Multiply both sides by 5.0.

$$5.0 \tan 65° = d$$

$$d = 10.7225\ldots$$

EF is approximately 10.7 cm.

Use the sine ratio to calculate the length of DE.

$$\sin E = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin E = \frac{e}{f}$$

$$\sin 25° = \frac{5.0}{f}$$ Solve the equation for f. Multiply both sides by f.

$$f \sin 25° = 5.0$$ Divide both sides by sin 25°.

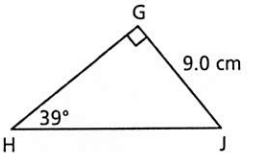
$$f = \frac{5.0}{\sin 25°}$$

$$f = 11.8310\ldots$$

DE is approximately 11.8 cm.

CHECK YOUR UNDERSTANDING

2. Solve this triangle. Give the measures to the nearest tenth where necessary.

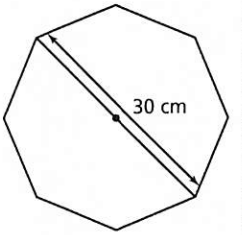


[Answers: ∠J = 51°; GH ≐ 11.1 cm; HJ ≐ 14.3 cm]

What is the advantage of determining the unknown angle before the unknown sides?

Example 3 Solving a Problem Using the Trigonometric Ratios

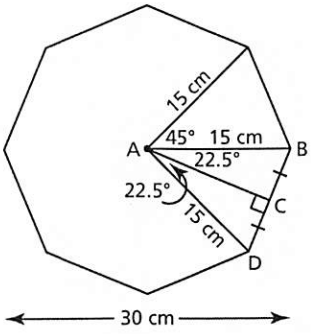
A small table has the shape of a regular octagon. The distance from one vertex to the opposite vertex, measured through the centre of the table, is approximately 30 cm. There is a strip of wood veneer around the edge of the table. What is the length of this veneer to the nearest centimetre?



SOLUTION

To determine the length of veneer, calculate the perimeter of the surface of the table.

Since the surface of the table is a regular octagon, congruent isosceles triangles are formed by drawing line segments from the centre of the surface to each vertex.



In each triangle, the angle at the centre is:

$360° \div 8 = 45°$

The line segment from the centre of the octagon to the centre of each side of the octagon bisects each central angle and is perpendicular to the side.

So, in right $\triangle ABC$,

$\angle A = 22.5°$ and $AB = 15$ cm

$$\sin A = \frac{BC}{AB}$$ Solve the equation for BC. Multiply both sides by 15.

$$\sin 22.5° = \frac{BC}{15}$$

$$15 \sin 22.5° = BC$$

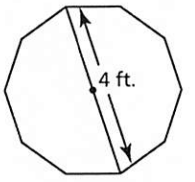
Since $BC = 15 \sin 22.5°$, then $BD = 2(15 \sin 22.5°)$, and $BD = 30 \sin 22.5°$

And, the perimeter of the octagon is:
$8(BD) = 8(30 \sin 22.5°)$
$8(BD) = 91.8440\ldots$

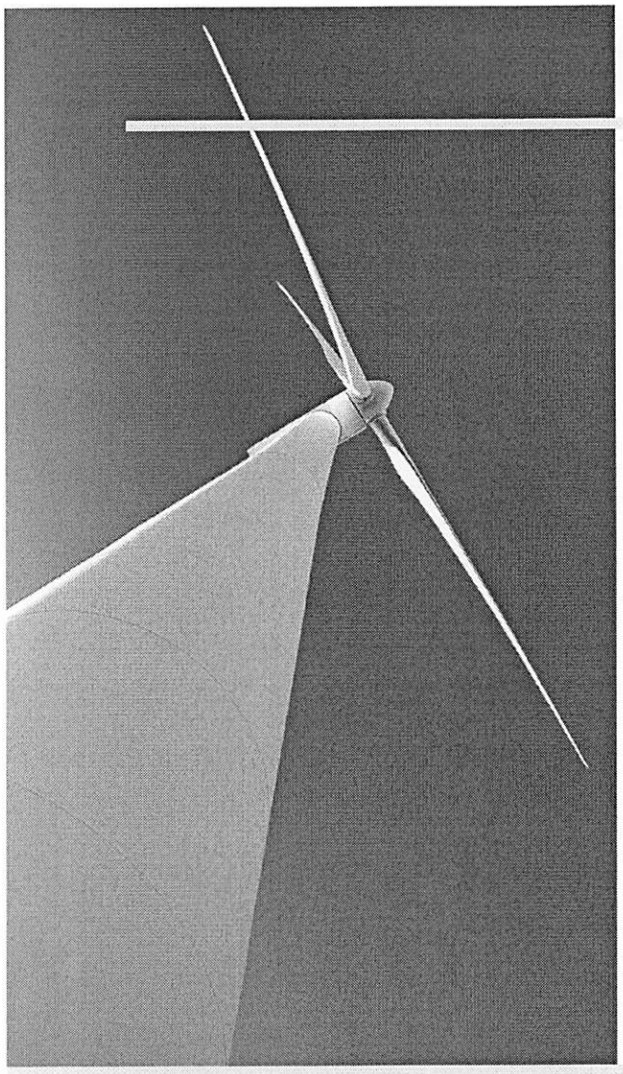
The length of veneer required is approximately 92 cm.

CHECK YOUR UNDERSTANDING

3. A window has the shape of a regular decagon. The distance from one vertex to the opposite vertex, measured through the centre of the window, is approximately 4 ft. Determine the length of the wood moulding material that forms the frame of the window, to the nearest foot.



[Answer: approximately 12 ft.]



THE WORLD OF MATH

Profile: Renewable Energy

Aboriginal and northern communities are committed to developing sustainable energy from sources such as wind turbines, solar panels, geothermal, and hydroelectric projects.

Weather Dancer 1 is a 900 KW wind turbine situated on Piikani Nation land in southern Alberta. First commissioned in 2001, it was developed and is run as a joint venture by the Piikani Indian Utility Corporation and EPCOR, a City of Edmonton power company. Weather Dancer 1 generates 9960 MWh of carbon-dioxide free power each year. It was named in honour of Okan (Sun Dance), a traditional ceremony of the Blackfoot that renews their relationship with the life forces of nature.

How could you use trigonometry to determine the length of a blade of a wind turbine?

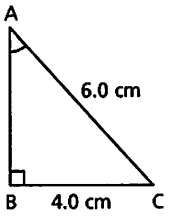
Discuss the Ideas

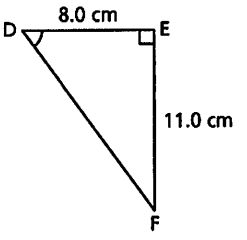
1. When we solve a right triangle, sometimes we determine the measure of an unknown angle before we determine the length of an unknown side and sometimes we reverse these calculations. How would you decide which measure to calculate first?
2. Can we solve a right triangle if we are given only the measures of the two acute angles? Explain.

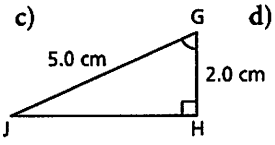
Exercises

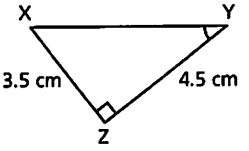
A

3. To determine the measure of each indicated angle, which trigonometric ratio would you use? Why?

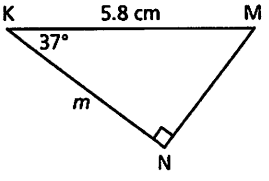
a) 

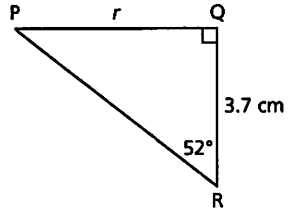
b) 

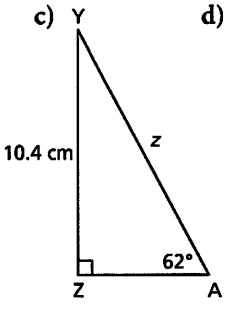
c) 

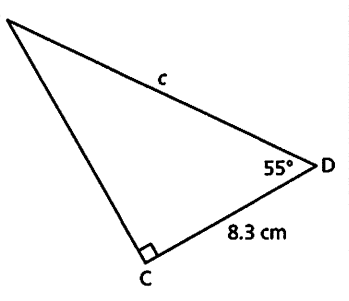
d) 

4. Determine the length of each indicated side to the nearest tenth of a centimetre. Which trigonometric ratio did you use? Why?

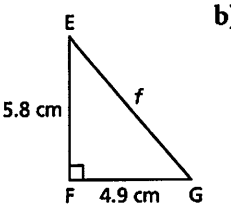
a) 

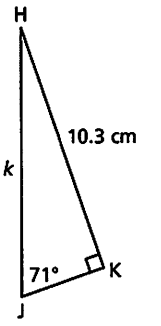
b) 

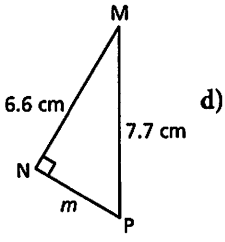
c) 

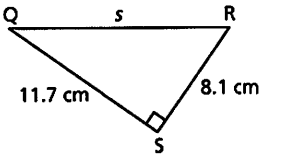
d) 

5. To determine the length of each indicated side, which strategy would you use? Why?

a) 

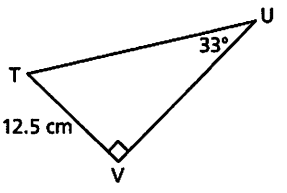
b) 

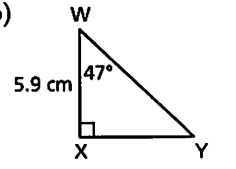
c) 

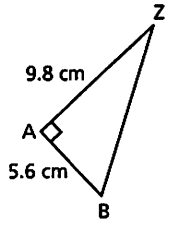
d) 

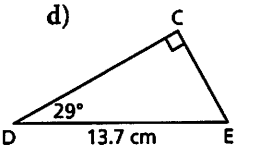
B

6. Solve each right triangle. Give the measures to the nearest tenth.

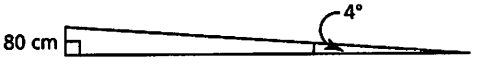
a) 

b) 

c) 

d) 

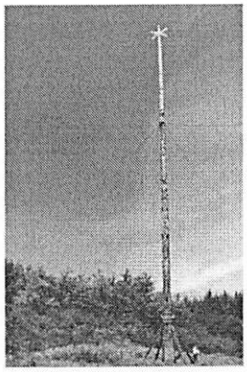
7. An architect draws this diagram of a wheelchair entrance ramp for a building.



a) Determine the length of the ramp.
b) Determine the horizontal distance the ramp will take up.

Give the measures to the nearest centimetre.

8. The world's tallest totem pole is in Alert Bay, B.C., home of the Nimpkish First Nation. Twenty feet from the base of the totem pole, the angle of elevation of the top of the pole is 83.4°. How tall is the totem pole to the nearest foot?



9. A helicopter leaves its base, and flies 35 km due west to pick up a sick person. It then flies 58 km due north to a hospital.

a) When the helicopter is at the hospital, how far is it from its base to the nearest kilometre?

b) When the helicopter is at the hospital, what is the measure of the angle between the path it took due north and the path it will take to return directly to its base? Write the angle to the nearest degree.

10. A road rises 1 m for every 15 m measured along the road.

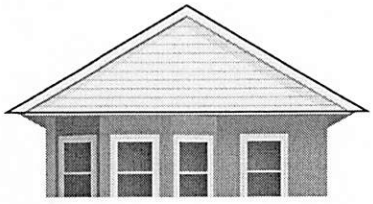
a) What is the angle of inclination of the road to the nearest degree?

b) How far does a car travel horizontally when it travels 15 m along the road? Give the answer to the nearest tenth of a metre.

11. A roof has the shape of an isosceles triangle with equal sides 7 m long and base 12 m long.

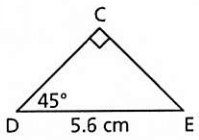
a) What is the measure of the angle of inclination of the roof to the nearest degree?

b) What is the measure of the angle at the peak of the roof to the nearest degree?



12. Determine the perimeter and area of each shape. Give the measures to the nearest tenth.

a)



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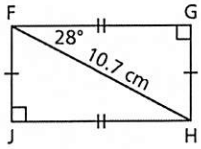
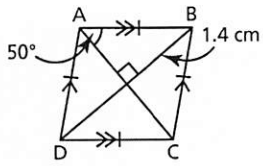


13. Determine the perimeter of this rhombus to the nearest tenth of a centimetre.



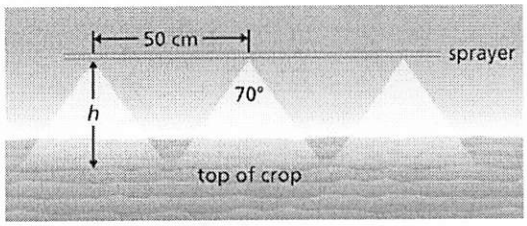
14. A candle has the shape of a right prism whose bases are regular polygons with 12 sides. On each base, the distance from one vertex to the opposite vertex, measured through the centre of the base, is approximately 2 in. The candle is 5 in. high.

a) What is the area of the base, to the nearest square inch?

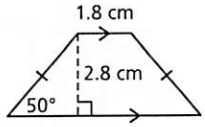
b) What is the volume of wax in the candle, to the nearest cubic inch?

C

15. To irrigate crops, a farmer uses a boom sprayer pulled by a tractor. The nozzles are 50 cm apart and spray at an angle of 70°. To the nearest centimetre, how high should the sprayer be placed above the crops to ensure that all the crops are watered?



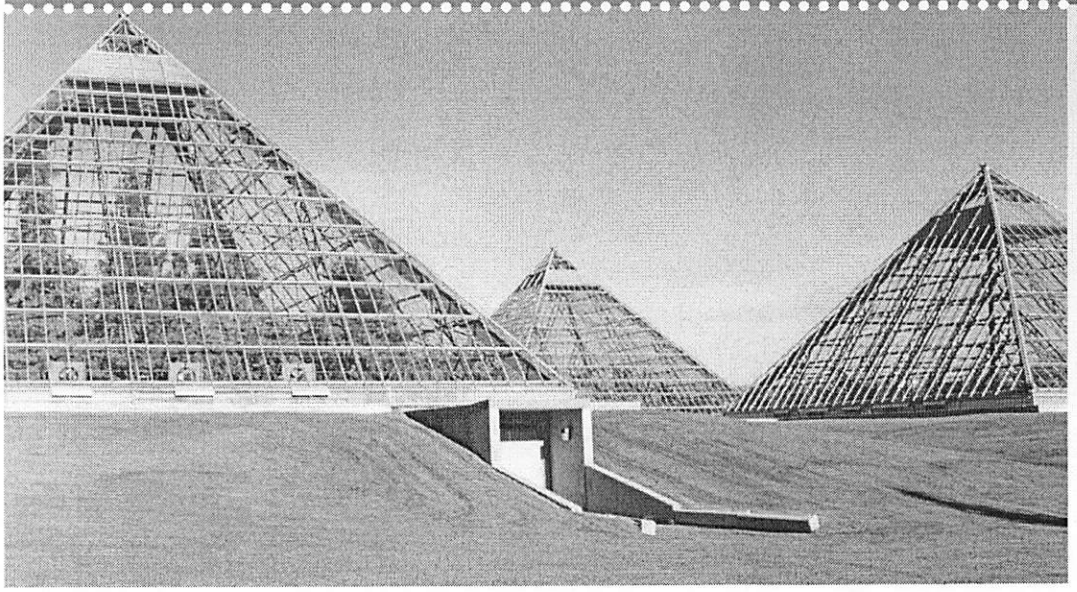
16. Determine the perimeter and area of this isosceles trapezoid. Give the measures to the nearest tenth.



Reflect

How does the information you are given about a right triangle determine the steps you take to solve the triangle? Include examples with your explanation.

2.7 Solving Problems Involving More than One Right Triangle



LESSON FOCUS

Use trigonometry to solve problems modelled by more than one right triangle.

Make Connections

The Muttart Conservatory in Edmonton has four climate-controlled square pyramids, each representing a different climatic zone. Each of the tropical and temperate pyramids is 24 m high and the side length of its base is 26 m. How do you think the architects determined the angles at which to cut the glass pieces for each face at the apex of the pyramid?

Construct Understanding

THINK ABOUT IT

Work with a partner.

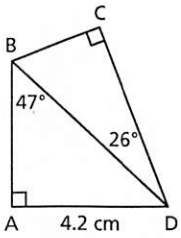
Sketch a square pyramid.
Label its height and base with the measurements above.
Draw right triangles on your sketch that would help you determine the angle between the edges of the pyramid at its apex.

How could you use trigonometry to help you determine this angle?

We can use trigonometry to solve problems that can be modelled using right triangles. When more than one right triangle is involved, we have to decide which triangle to start with.

Example 1 Calculating a Side Length Using More than One Triangle

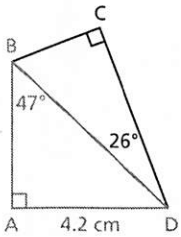
Calculate the length of CD to the nearest tenth of a centimetre.



SOLUTION

The length of CD cannot be determined in one step because we know only the measure of one angle in △BCD. So, use △ABD to calculate the length of BD.

In right △ABD, AD is opposite ∠ABD and BD is the hypotenuse.



Use the sine ratio in △ABD.

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin B = \frac{AD}{BD}$$

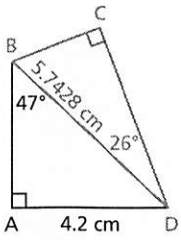
$$\sin 47° = \frac{4.2}{BD}$$ Solve for BD. Multiply both sides by BD.

$$BD \sin 47° = 4.2$$ Divide both sides by sin 47°.

$$BD = \frac{4.2}{\sin 47°}$$

$$BD = 5.7427\ldots$$

In right △BCD, CD is adjacent to ∠BDC and BD is the hypotenuse. Use the cosine ratio in △BCD.



$$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos D = \frac{CD}{BD}$$

$$\cos 26° = \frac{CD}{5.7427\ldots}$$ Solve for CD. Multiply both sides by 5.7427...

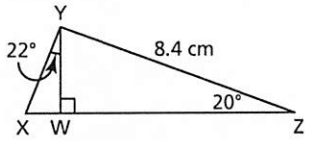
$$(5.7427\ldots)\cos 26° = CD$$

$$CD = 5.1615\ldots$$

CD is approximately 5.2 cm.

CHECK YOUR UNDERSTANDING

1. Calculate the length of XY to the nearest tenth of a centimetre.

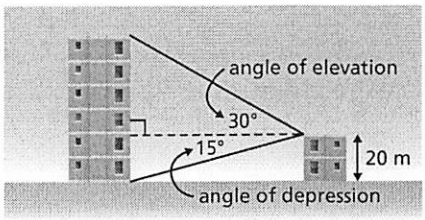


[Answer: XY ≐ 3.1 cm]

Explain how you could calculate all the unknown sides and angles of quadrilateral ABCD.

Example 2 Solving a Problem with Triangles in the Same Plane

From the top of a 20-m high building, a surveyor measured the angle of elevation of the top of another building and the **angle of depression** of the base of that building. The surveyor sketched this plan of her measurements. Determine the height of the taller building to the nearest tenth of a metre.



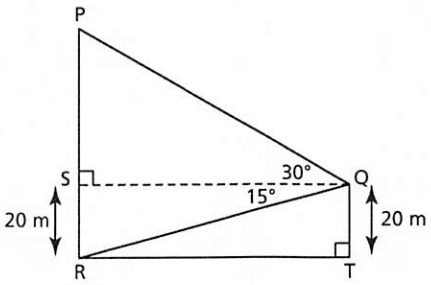
SOLUTION

Draw and label a diagram. The height of the building is represented by PR.

We cannot calculate PR in one step because it is not a side of a right triangle.

$PR = PS + SR$

In $\triangle PQS$, we know only the measure of $\angle PQS$. So, use right $\triangle QRS$ to calculate the length of SQ. QSRT is a rectangle, so $SR = QT = 20$ m.



Use the tangent ratio in $\triangle QRS$.

$$\tan Q = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Q = \frac{SR}{QS}$$

$$\tan 15° = \frac{20}{QS}$$

Solve the equation for QS. Multiply both sides by QS.

$$QS \tan 15° = 20$$

Divide both sides by tan 15°.

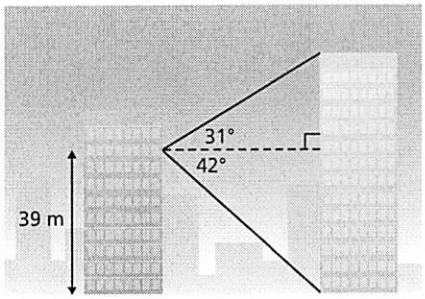
$$QS = \frac{20}{\tan 15°}$$

$$QS = 74.6410\ldots$$

(Solution continues.)

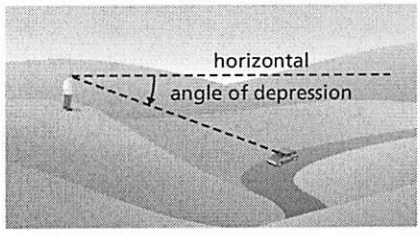
CHECK YOUR UNDERSTANDING

2. A surveyor stands at a window on the 9th floor of an office tower. He uses a clinometer to measure the angles of elevation and depression of the top and the base of a taller building. The surveyor sketches this plan of his measurements. Determine the height of the taller building to the nearest tenth of a metre.



[Answer: Approximately 65.0 m]

The **angle of depression** of an object below the horizontal is the angle between the horizontal and the line of sight from an observer.



Use the tangent ratio in $\triangle PQS$.

$$\tan Q = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Q = \frac{PS}{QS}$$

$$\tan 30° = \frac{PS}{74.6410...}$$ Solve the equation for PS. Multiply both sides by 74.6410...

$$(74.6410...) \tan 30° = PS$$

$$PS = 43.0940...$$

So, $PR = PS + SR$

$$PR = 43.0940... + 20$$

$$= 63.0940...$$

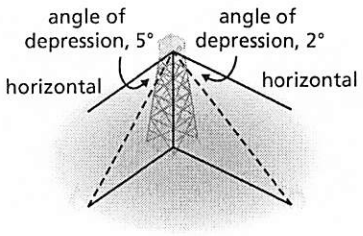
The taller building is approximately 63.1 m high.

Suppose you did not evaluate a decimal equivalent for QS. What expression would you need to use to determine the length of PS?

Sometimes the right triangles we solve are not in the same plane.

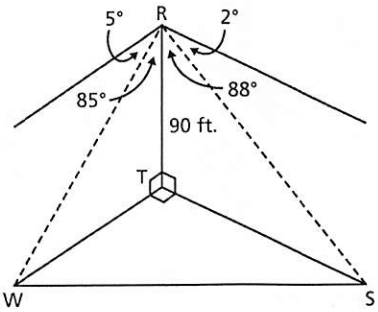
Example 3 Solving a Problem with Triangles in Different Planes

From the top of a 90-ft. observation tower, a fire ranger observes one fire due west of the tower at an angle of depression of 5°, and another fire due south of the tower at an angle of depression of 2°. How far apart are the fires to the nearest foot? The diagram is *not* drawn to scale.



SOLUTION

Label a diagram.



The fires are due south and due west of the tower, so the angle between the lines of sight, TW and TS, to the fires from the base of the tower is 90°.

Since the angles of depression are 5° and 2° respectively, the angles between the tower, RT, and the lines of sight are 85° and 88° respectively.

To calculate the distance WS between the fires, first calculate the distances, TW and TS, of the fires from the base of the tower.

Use the tangent ratio in right △RTW.

$$\tan R = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan R = \frac{WT}{RT}$$

$$\tan 85° = \frac{WT}{90}$$ Solve for WT. Multiply both sides by 90.

$$90 \tan 85° = WT$$

$$WT = 1028.7047\ldots$$

Use the tangent ratio in right △RTS.

$$\tan R = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan R = \frac{TS}{RT}$$

$$\tan 88° = \frac{TS}{90}$$ Solve for TS. Multiply both sides by 90.

$$90 \tan 88° = TS$$

$$TS = 2577.2627\ldots$$

In right △STW, use the Pythagorean Theorem.

$$SW^2 = WT^2 + TS^2$$

$$SW^2 = 1028.7047\ldots^2 + 2577.2627\ldots^2$$

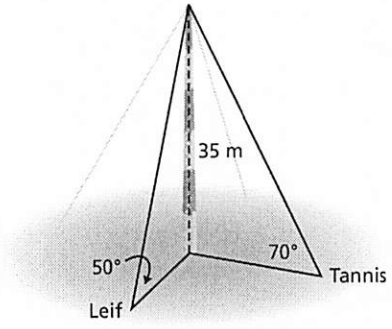
$$SW = \sqrt{1028.7047\ldots^2 + 2577.2627\ldots^2}$$

$$= 2774.9805\ldots$$

The distance between the fires is approximately 2775 ft.

CHECK YOUR UNDERSTANDING

3. A communications tower is 35 m tall. From a point due north of the tower, Tannis measures the angle of elevation of the top of the tower as 70°. Her brother Leif, who is due east of the tower, measures the angle of elevation of the top of the tower as 50°. How far apart are the students to the nearest metre? The diagram is *not* drawn to scale.



[Answer: About 32 m]

Solve Example 3 using your calculator only once. Explain why this might be more efficient and accurate than calculating intermediate lengths.

Discuss the Ideas

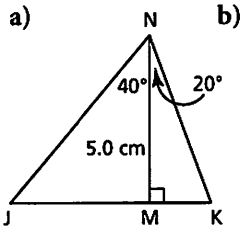
1. What do you have to think about when you draw a diagram with triangles in three dimensions?
2. When you have to solve a problem that involves two right triangles, how do you decide where to begin?

Exercises

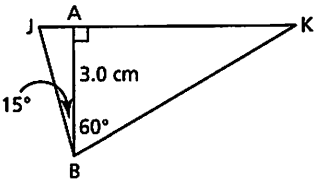
A

3. In each triangle, determine the length of JK to the nearest tenth of a centimetre.

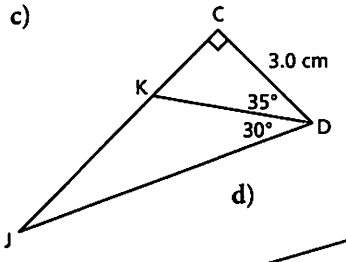
a)



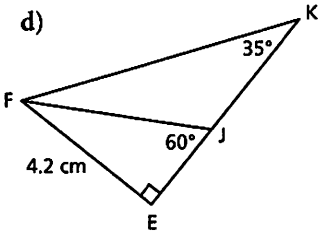
b)



c)

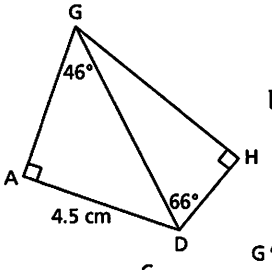


d)

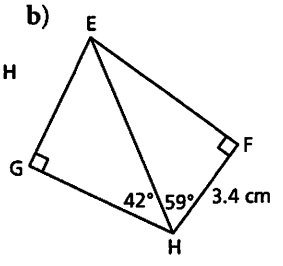


4. In each quadrilateral, calculate the length of GH to the nearest tenth of a centimetre.

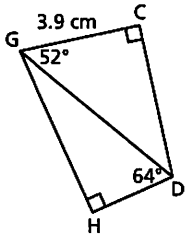
a)



b)



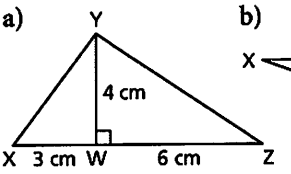
c)



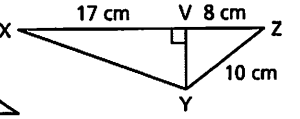
B

5. In each diagram, calculate the measure of each ∠XYZ to the nearest tenth of a degree.

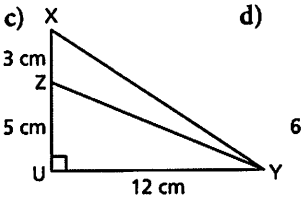
a)



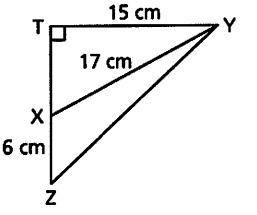
b)



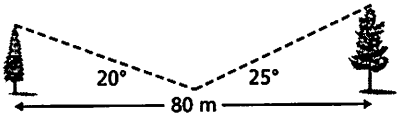
c)



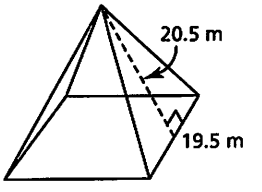
d)



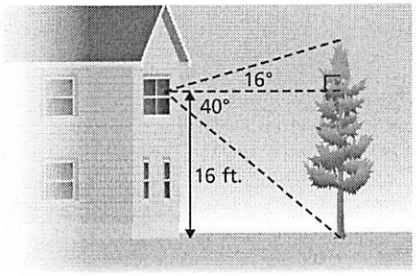
6. Two trees are 80 m apart. From a point halfway between the trees, the angles of elevation of the tops of the trees are measured. What is the height of each tree to the nearest metre?



7. At the Muttart Conservatory, the arid pyramid has 4 congruent triangular faces. The base of each face has length 19.5 m and the slant height of the pyramid is 20.5 m. What is the measure of each of the three angles in the face? Give the measures to the nearest degree.

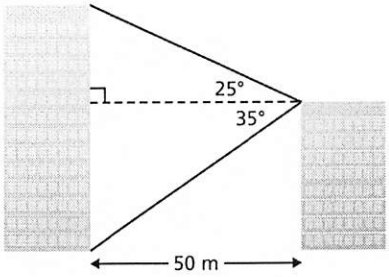


8. From a window on the second floor of her house, a student measured the angles of elevation and depression of the top and base of a nearby tree. The student knows that she made the measurements 16 ft. above the ground.



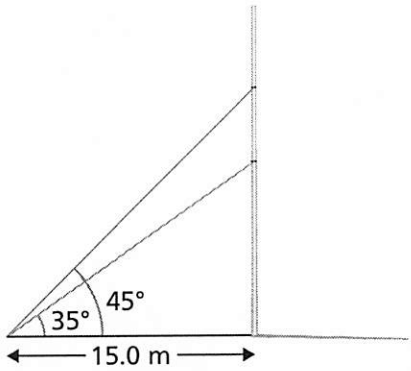
a) What is the horizontal distance between the student and the tree?
b) How tall is the tree?
Give the measures to the nearest foot.

9. Two office towers are 50 m apart. From the top of the shorter tower, the angle of depression of the base of the taller tower is 35°. The angle of elevation of the top of this tower is 25°. Determine the height of each tower to the nearest metre.



10. A rectangle has dimensions 5.5 cm by 2.8 cm. Determine the measures of the angles at the point where the diagonals intersect. What strategy did you use? Could you have determined the angle measures a different way? Explain.

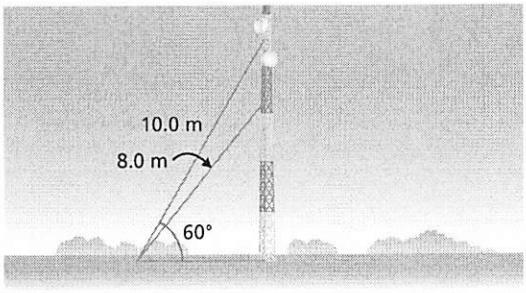
11. A student wanted to know the distance between two particular carvings on a spirit pole. She measured the angle of elevation of each carving 15.0 m from the base of the pole. The student drew the sketch below. What is the distance between the carvings to the nearest tenth of a metre?



12. The Legislative Building in Wascana Park, Regina, has a domed tower at its centre.

Janelle stood due south of the tower, 40 m from a point directly beneath the dome, and measured the angle of elevation of the top of the dome as 53°. Troy stood due east of the tower and measured the angle of elevation of the top of the dome as 61°.
a) How high is the top of the dome?
b) How far is Troy from a point directly beneath the dome?
c) How far apart are Janelle and Troy?
Give the measures to the nearest metre.

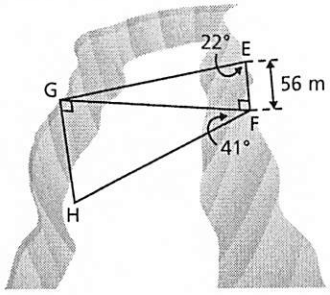
13. A communications tower has many guy wires supporting it. Two of these guy wires are 10.0 m and 8.0 m long. They are attached at the same point on the ground. The longer wire has an angle of inclination of 60°.



a) How far from the base of the tower are the wires attached on the ground?
b) What is the angle of inclination of the shorter guy wire?
c) How far apart are the points where the guy wires are attached to the tower?

Give the measures to the nearest tenth.

14. A surveyor drew the sketch below to show the measurements he took to determine the width and depth of a gorge.

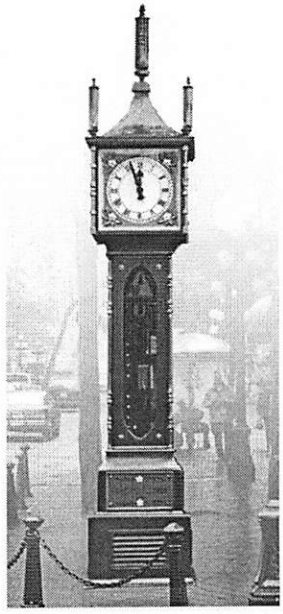


a) Determine the width, GF, of the gorge.
b) Determine the depth, GH, of the gorge.

Give the measures to the nearest metre.

15. A surveyor wants to determine the height of a cliff on the other side of the river from where she is standing. The surveyor cannot cross the river. She has a clinometer and a measuring tape. Describe how she can calculate the height of the cliff.

16. The Gastown Steam clock in Vancouver has been chiming since 1977. From a point on the ground, Connor measured the angle of elevation of the top of the clock tower as 59.5°. Monique was 3.5 m from Connor. The line joining them formed a right angle with the line joining Connor and the base of the tower. The angle between Monique's lines of sight to Connor and to the base of the tower was 40.6°.

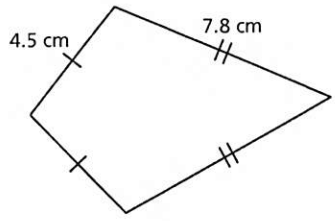


a) Sketch a diagram.
b) Determine the height of the tower to the nearest tenth of a metre.

C

17. In the kite below, the shorter diagonal is 6.8 cm long.

a) Determine the measures of the four angles of the kite to the nearest tenth of a degree.
b) Determine the length of the longer diagonal to the nearest millimetre.



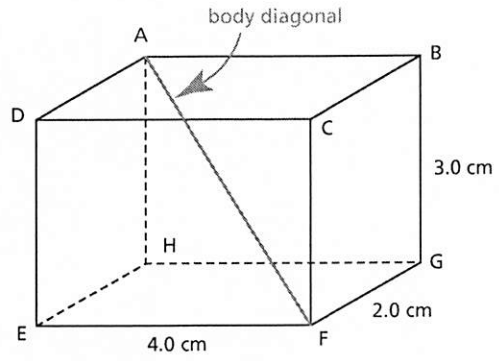
18. At the Muttart Conservatory, the tropical pyramid has 4 congruent triangular faces. The base of each face has length 25.7 m and the slant height of the pyramid is 27.2 m.

a) Sketch and label the pyramid.
b) What is the height of the pyramid to the nearest tenth of a metre?

19. a) What is the length of the body diagonal in this rectangular prism?

b) What is the measure of $\angle AFH$, the angle between the body diagonal and a diagonal of the base of the prism?

Give the measures to the nearest tenth.



20. A communications tower is supported by guy wires. One guy wire is anchored at a point that is 8.9 m from the base of the tower and has an angle of inclination of 36°. From this point, the angle of elevation of the top of the tower is 59°. How far from the top of the tower is the guy wire attached to the tower?

21. A geodesic dome is constructed by bolting together many pentagonal pyramids. Each triangular face of a pyramid is formed with two struts, each 54 in. long, and one strut that is 60 in. long. Determine the height of one of these pyramids.

Reflect Summarize the different steps used to solve right triangle problems.

THE WORLD OF MATH

Historical Moment: Claudius Ptolemy

Claudius Ptolemy, who died in 161 CE, was a mathematician, an astronomer, and a geographer. He lived in Alexandria in Roman Egypt, and wrote in Greek. As part of his interest in astronomy, he extended the tables of trigonometric ratios started by Hipparchus of Bythnia (190 – 120 BCE) and studied triangles. Ptolemy's first major work, the *Almagest,* is the only ancient comprehensive material on astronomy that survives today. His other major works were *Geographia, Harmonics,* and *Optics.*