

# 5

## Relations and Functions

### BUILDING ON

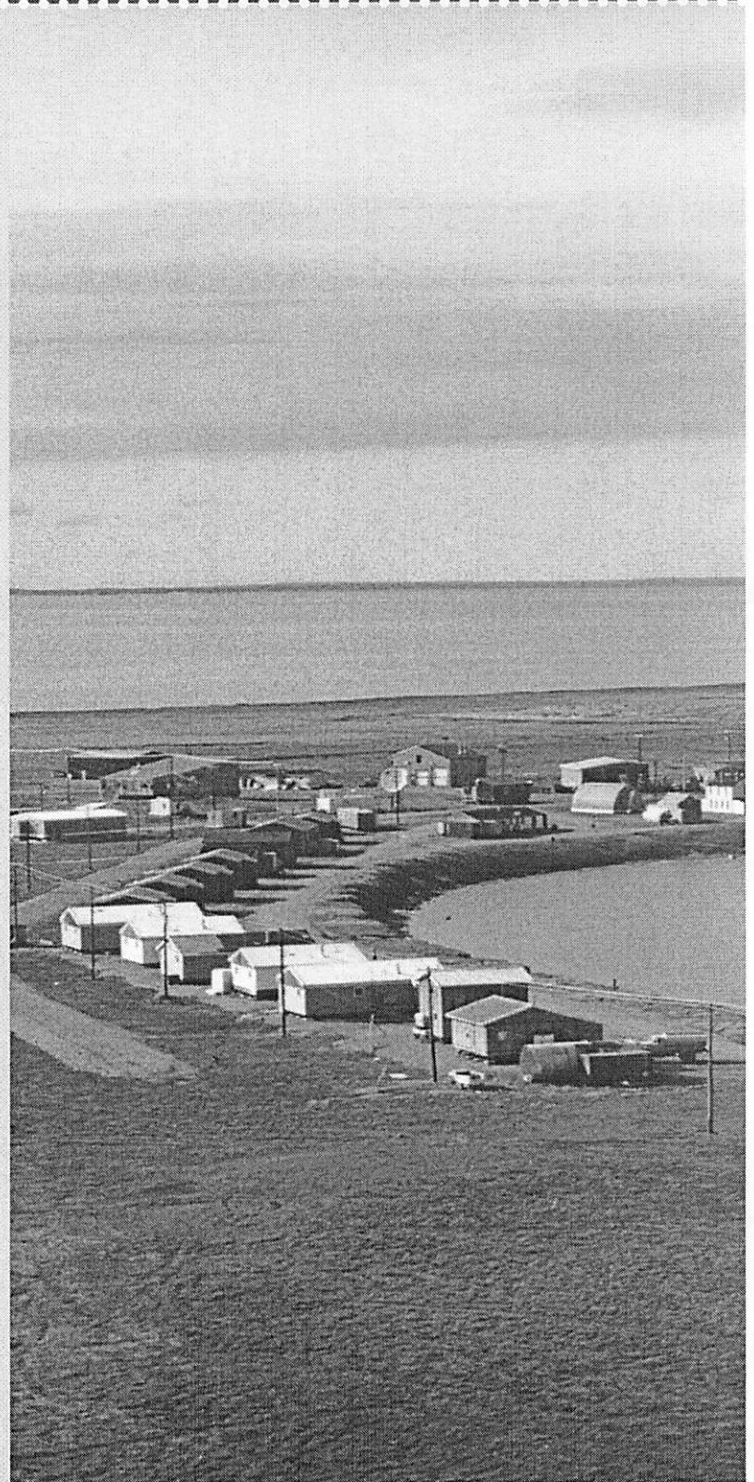
- writing equations to represent patterns in tables
- graphing and analyzing linear relations

### BIG IDEAS

- A relation associates the elements of one set with the elements of another set.
- A function is a special type of relation for which each element of the first set is associated with a unique element of the second set.
- A linear function has a constant rate of change and its graph is a non-vertical straight line.

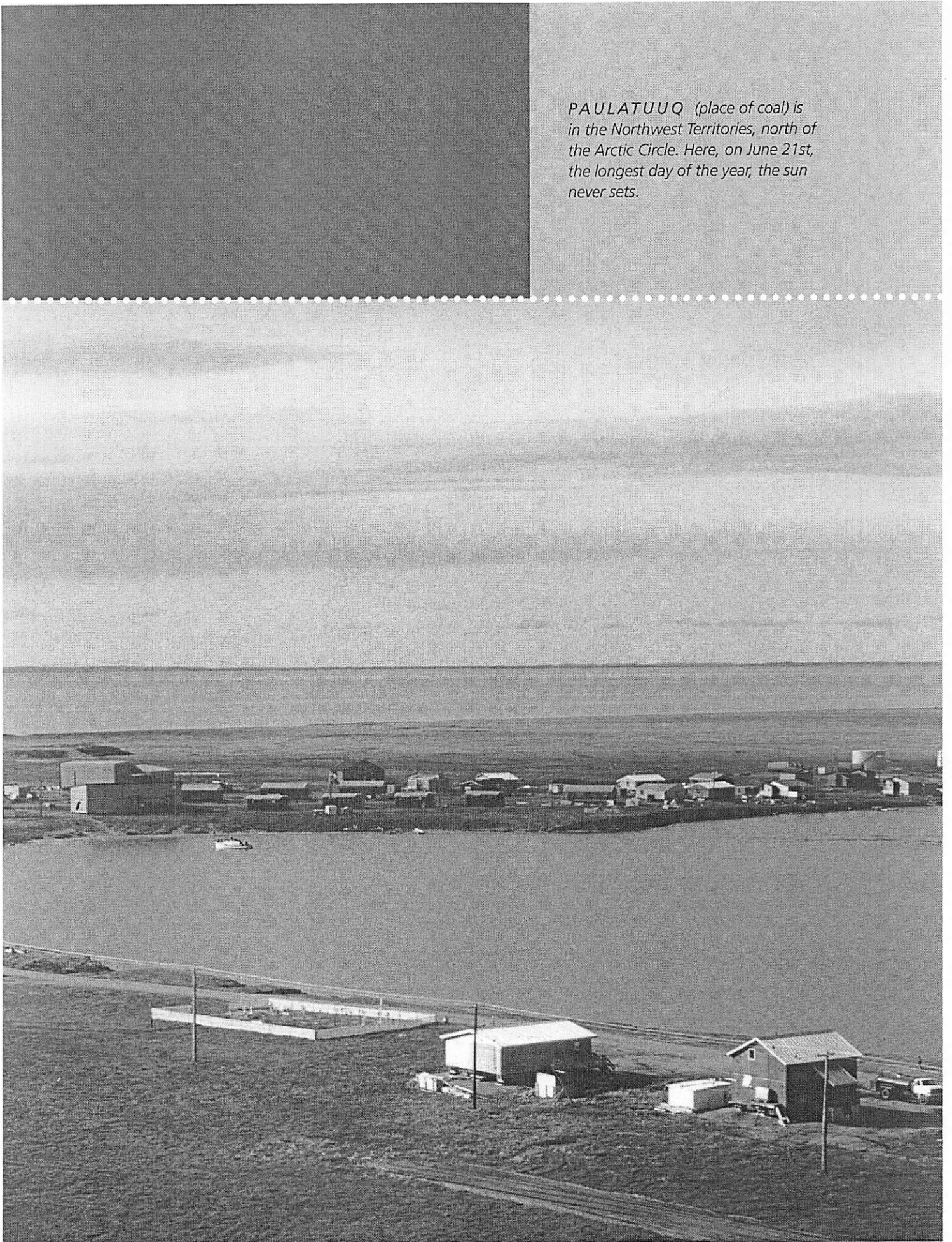
### NEW VOCABULARY

relation  
arrow diagram  
function  
domain  
range  
function notation  
rate of change  
linear function  
vertical intercept  
horizontal intercept





*PAULATUUQ (place of coal) is in the Northwest Territories, north of the Arctic Circle. Here, on June 21st, the longest day of the year, the sun never sets.*



# 5.1 Representing Relations

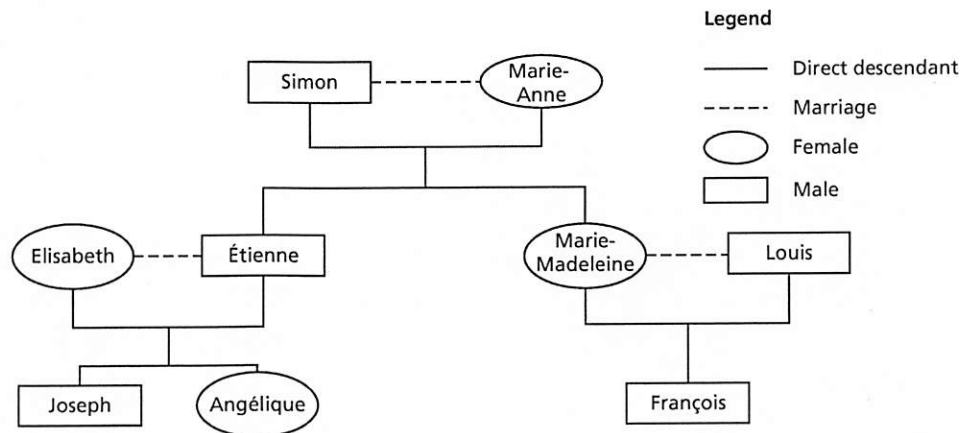
## LESSON FOCUS

Represent relations in different ways.



## Make Connections

This family tree shows relations within a family.



How is Joseph related to Simon?

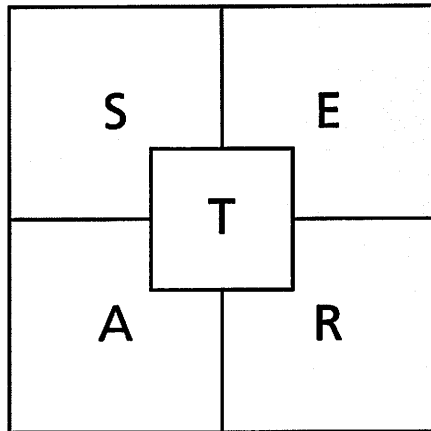
How are Angélique and François related?

How does the family tree show these relations?

# Construct Understanding

## TRY THIS

- A. Use the letters in the square below to write as many words as you can with 3 or more letters.
- Each word must contain the letter in the centre square.
  - A letter can only be used once in each word.
  - Plurals cannot be used.



- B. Use the letters in the square and the words you wrote. Pair a letter with a word to show the association “is a letter of the word.” Write as many pairs as you can.
- C. Use the same association. Suppose you reverse the order of the items in the pairs. Does the association make sense? Explain.

A *set* is a collection of distinct objects.

An *element* of a set is one object in the set.

A **relation** associates the elements of one set with the elements of another set.

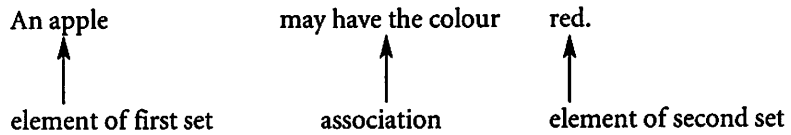
One way to write a set is to list its elements inside braces.

For example, we can write the set of natural numbers from 1 to 5 as:

{1, 2, 3, 4, 5}

The order of the elements in the set does not matter.

Consider the set of fruits and the set of colours.  
 We can associate fruits with their colours.  
 For example:



What other relations could you create using the same association between fruits and colours?

So, this set of ordered pairs is a relation:

{(apple, red), (apple, green), (blueberry, blue), (cherry, red), (huckleberry, blue)}

Here are some other ways to represent this relation:

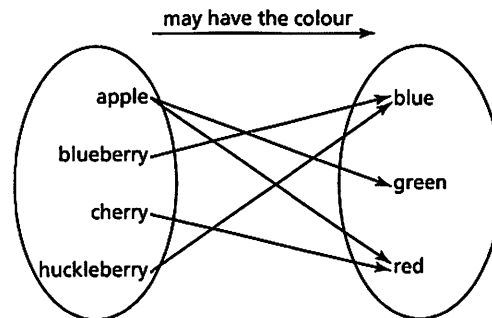
• a table

Fruit	Colour
apple	red
apple	green
blueberry	blue
cherry	red
huckleberry	blue

The heading of each column describes each set.

• an arrow diagram

The two ovals represent the sets.  
 Each arrow associates an element of the first set with an element of the second set.



The order of the words in the ordered pairs, the columns in the table, and the ovals in the arrow diagram is important. It makes sense to say, “an apple may have the colour red,” but it makes no sense to say, “red may have the colour apple.” That is, a relation has direction from one set to the other set.



## Example 1 Representing a Relation Given as a Table

Northern communities can be associated with the territories they are in. Consider the relation represented by this table.

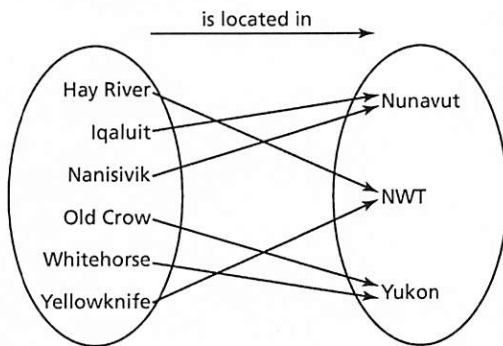
Community	Territory
Hay River	NWT
Iqaluit	Nunavut
Nanisivik	Nunavut
Old Crow	Yukon
Whitehorse	Yukon
Yellowknife	NWT

- a) Describe this relation in words.  
 b) Represent this relation:  
 i) as a set of ordered pairs  
 ii) as an arrow diagram

### SOLUTION

- a) The relation shows the association “is located in” from a set of northern communities to a set of territories. For example, Hay River is located in the NWT.
- b) i) The communities are the first elements in the ordered pairs. The territories are the second elements in the ordered pairs. The ordered pairs are: {(Hay River, NWT), (Iqaluit, Nunavut), (Nanisivik, Nunavut), (Old Crow, Yukon), (Whitehorse, Yukon), (Yellowknife, NWT)}
- ii) The communities are written in the first set of the arrow diagram.

The territories are written in the second set; each territory is written only once.



### CHECK YOUR UNDERSTANDING

1. Animals can be associated with the classes they are in.

Animal	Class
ant	Insecta
eagle	Aves
snake	Reptilia
turtle	Reptilia
whale	Mammalia

- a) Describe this relation in words.  
 b) Represent this relation:  
 i) as a set of ordered pairs  
 ii) as an arrow diagram

[Answers: a) The relation shows the association “belongs to the class” between a set of animals and a set of classes. b) i) {(ant, Insecta), (eagle, Aves), (snake, Reptilia), (turtle, Reptilia), (whale, Mammalia)}]

Why is the direction of the arrows in the arrow diagram important?

How are the representations the same?

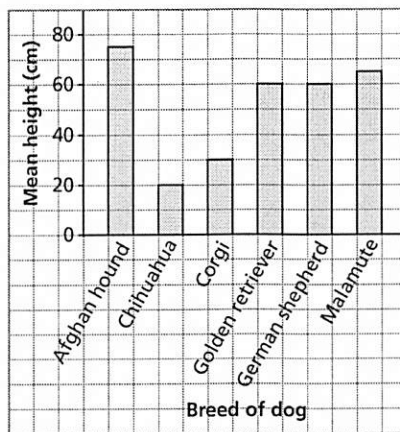
When the elements of one or both sets in a relation are numbers, the relation can be represented as a bar graph.

## Example 2 Representing a Relation Given as a Bar Graph

Different breeds of dogs can be associated with their mean heights. Consider the relation represented by this graph. Represent the relation:

- as a table
- as an arrow diagram

Mean Heights of Different Breeds of Dogs



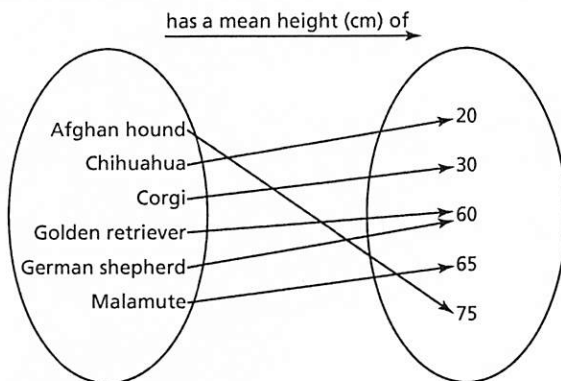
### SOLUTION

- The association is: “has a mean height of”

In the table, write the breeds of dogs in the first column and the mean heights in centimetres in the second column.

Breed of Dog	Mean Height (cm)
Afghan hound	75
Chihuahua	20
Corgi	30
Golden retriever	60
German shepherd	60
Malamute	65

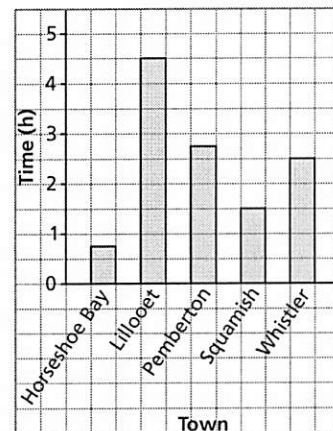
- In the arrow diagram, write the breeds of dogs in the first set and the mean heights in centimetres in the second set.



### CHECK YOUR UNDERSTANDING

- Different towns in British Columbia can be associated with the average time, in hours, that it takes to drive to Vancouver. Consider the relation represented by this graph.

Average Travel Time to Vancouver



Represent the relation:

- as a table
- as an arrow diagram

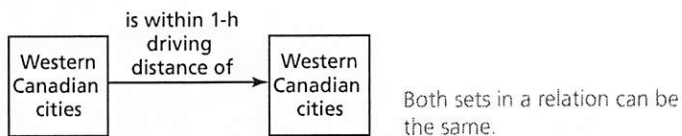
Answer: a)

Town	Average Time (h)
Horseshoe Bay	0.75
Lillooet	4.5
Pemberton	2.75
Squamish	1.5
Whistler	2.5

Sometimes a relation contains so many ordered pairs that it is impossible to list all of them or to represent them in a table.

### Example 3 Identifying a Relation from a Diagram

In this diagram:



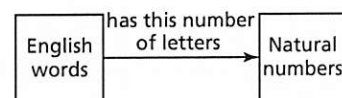
- Describe the relation in words.
- List 2 ordered pairs that belong to the relation.

#### SOLUTION

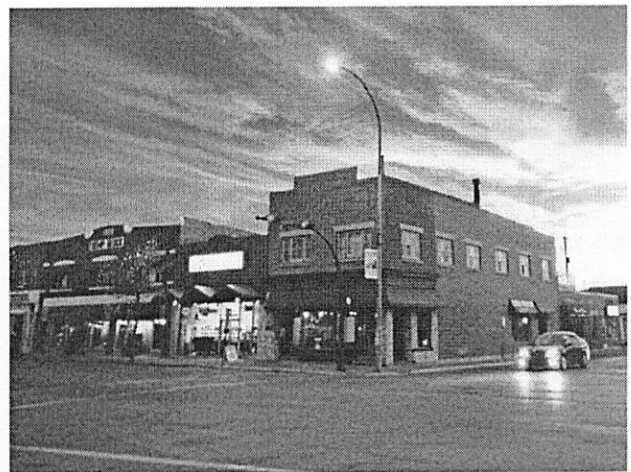
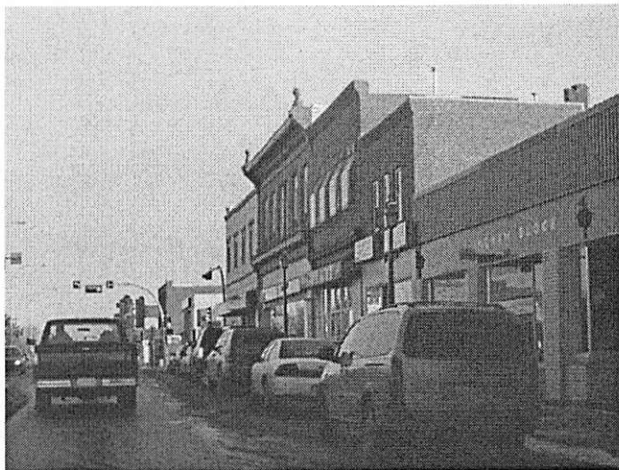
- The relation shows the association “is within 1-h driving distance” from a set of cities in Western Canada to a set of cities in Western Canada.
- Two ordered pairs that belong to the relation are:  
(Innisfail AB, Olds AB), (Kelowna B.C., Vernon B.C.)

#### CHECK YOUR UNDERSTANDING

- In the diagram below:
  - Describe the relation in words.
  - List 2 ordered pairs that belong to the relation.



[Sample Answers: a) The relation shows the association “has this number of letters” from a set of English words to a set of natural numbers. b) (mathematics, 11) and (language, 8)]



### Discuss the Ideas

- What are the advantages and disadvantages of the different ways you can represent a relation?
- Why is the order of the elements in an ordered pair important? Give an example.



# Exercises

## A

3. For each table below:
- Describe the relation in words.
  - Represent the relation:
    - as a set of ordered pairs
    - as an arrow diagram

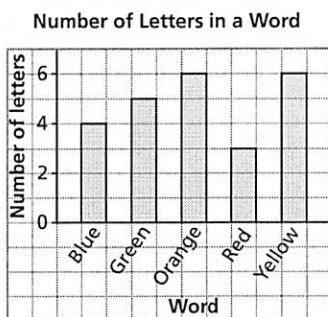
a)

Coin	Value (\$)
penny	0.01
nickel	0.05
dime	0.10
quarter	0.25
loonie	1.00
toonie	2.00

b)

Sport	Equipment
badminton	shuttlecock
badminton	racquet
hockey	puck
hockey	stick
tennis	ball
tennis	racquet
soccer	ball

4. Consider the relation represented by this graph.



Represent the relation:

- as a table
- as an arrow diagram

## B

5. This table shows some of Manitoba's francophone artists and the medium they use.

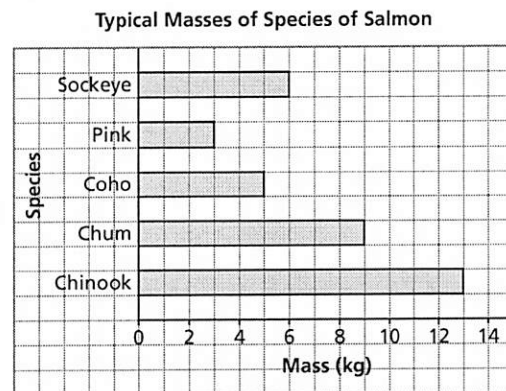
Artist	Medium
Gaëtanne Sylvester	sculpture
Hubert Thérout	painting
Huguette Gauthier	stained glass
James Culleton	painting
Nathalie Dupont	photography
Simone Hébert Allard	photography

- Describe the relation in words.
- Represent this relation:
  - as a set of ordered pairs
  - as an arrow diagram



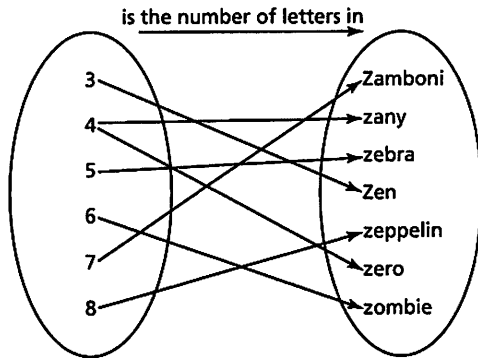
*Burning Sunset Detail* by James Culleton

6. a) Describe the relation represented by this bar graph.

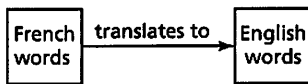


- Represent the relation as a set of ordered pairs.
- Represent the relation in a different way.

7. For a word game, words that begin with the letter Z can be difficult to find.
- a) What does this arrow diagram represent?



- b) Represent this relation in two different ways.
- c) Create an arrow diagram for words beginning with the letter X, then represent the relation in two different ways.
8. In the diagram below:
- a) Describe the relation in words.
- b) List two ordered pairs that belong to the relation.

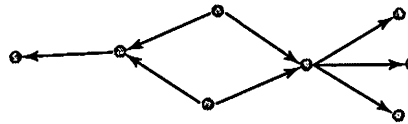


9. A digital clock displays digits from 0 to 9 by lighting up different segments in two squares. For example, the digit 2 needs 5 segments to light up, as shown.
- 
- a) List the set of ordered pairs of the form: (digit, number of segments lit up)
- b) Represent this relation in two different ways.
10. Here are some Canadian hockey players and the year they were born.
- Jennifer Botterill (1979); Jonathan Cheechoo (1980); Roberto Luongo (1979); Jordin Tootoo (1983); Hayley Wickenheiser (1978)
- For each association below, use these data to represent a relation in different ways.
- a) was born in
- b) is the birth year of

11. Choose five people in your class.
- a) Use the association “is older than” to write a relation. Represent the relation using a set of ordered pairs.
- b) Create your own association for these five people, then describe the relation in words. Represent this relation in different ways.



12. Two dice are rolled and the numbers that show are recorded.
- a) Use each association below to create a relation as a set of ordered pairs.
- i) The sum of the numbers is even.
- ii) The difference between the numbers is a prime number.
- b) In part a, does the order of the numbers in each ordered pair matter? Explain.
13. The association “is the parent of” is shown in the diagram. Each dot represents a person and each arrow maps a parent to her or his child.



In this relation:

- a) How many children are shown?
- b) How many parents are shown?
- c) How many grandparents are shown?
- Justify your answers.
14. The association “is the sister of” is shown in the diagram. Each dot represents a person and each arrow maps a sister to a sibling.
- 
- In this relation:
- a) How many females are shown?
- b) How many males are shown?
- Justify your answers.

## Reflect

Create a relation that you can describe in words. Show two different ways to represent your relation.