

5.5 Graphs of Relations and Functions



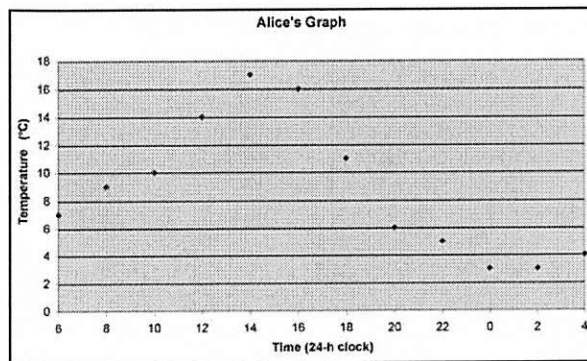
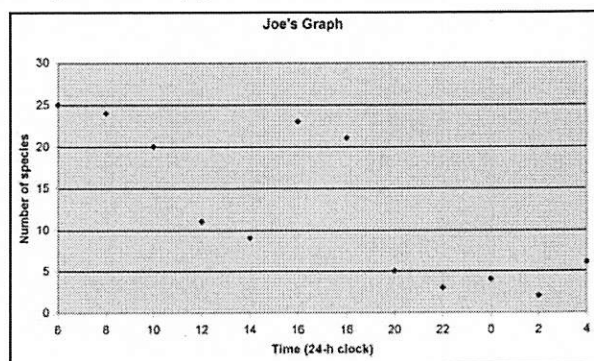
LESSON FOCUS

Determine the properties of the graphs of relations and functions.

The great horned owl is Alberta's provincial bird.

Make Connections

In an environmental study in Northern Alberta, Joe collected data on the numbers of different species of birds he heard or saw in a 1-h period every 2 h for 24 h. Alice collected data on the temperature in the area at the end of each 1-h period. They plotted their data:



Does each graph represent a relation? A function? How can you tell?

Which of these graphs should have the data points connected? Explain.

Construct Understanding

TRY THIS

Work with a partner.

You will need grid paper.

A. Each of you chooses one of these tasks:

- A sugar cube has a volume of 5 cm^3 and a mass of 4 g. Graph the mass of sugar as a function of the number of sugar cubes from 0 to 5 sugar cubes.
- Five cubic centimetres of loose sugar also has a mass of 4 g. Graph the mass of sugar as a function of the volume of sugar from 0 to 25 cm^3 of loose sugar.

B. Share your results. How are your graphs alike?
How are they different?

C. Work together:

- Identify the dependent variable and independent variable for each function. How did you decide on which axis to graph each variable?
- How did you decide whether to connect the points?
- Are there any restrictions on the domain and range? Explain.

We can represent the function that associates every whole number with its double in several ways.

Using a table of values:

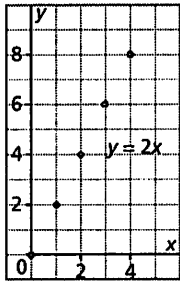
Whole Number, x	Double the Number, y
0	0
1	2
2	4
3	6
4	8

The table continues for all whole numbers.

The domain is the set of whole numbers.

The range is the set of even whole numbers.

Using a graph:



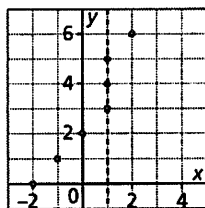
We know the relation $y = 2x$ is a function because each value of x associates with exactly one value of y , and each ordered pair has a different first element.

The *domain* of a function is the set of values of the independent variable; for the graph above, the domain is the x -values.

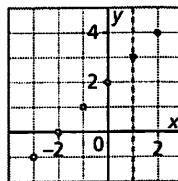
The *range* of a function is the set of values of the dependent variable; for the graph above, the range is the y -values.

When the domain is restricted to a set of discrete values, the points on the graph are not connected.

A relation that is not a function has two or more ordered pairs with the same first coordinate. So, when the ordered pairs of the relation are plotted on a grid, a vertical line can be drawn to pass through more than one point.



A function has ordered pairs with different first coordinates. So, when the ordered pairs of the function are plotted on a grid, any vertical line drawn will always pass through no more than one point.



How would the graph change if both x and y were real numbers?

How can you tell the domain and range from the graph?

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Vertical Line Test for a Function

A graph represents a function when no two points on the graph lie on the same vertical line.

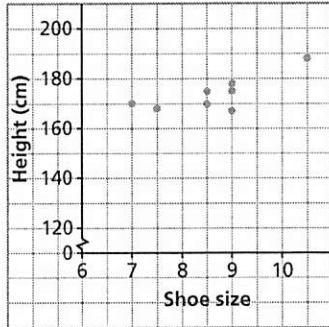
Place a ruler vertically on a graph, then slide the ruler across the graph.

If one edge of the ruler always intersects the graph at no more than one point, the graph represents a function.

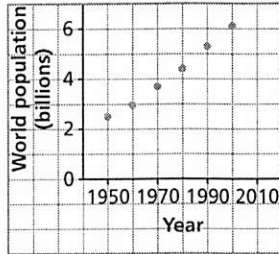
Example 1 Identifying whether a Graph Represents a Function

Which of these graphs represents a function? Justify the answer.

a) Height against Shoe Size



b) World Population

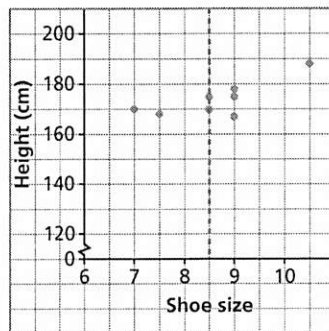


SOLUTION

Use the vertical line test for each graph.

- a) This graph does not represent a function because two points lie on the same vertical line.

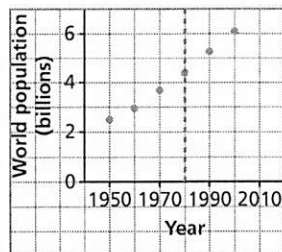
Height against Shoe Size



- b) This graph does represent a function.

Any vertical line drawn on the graph passes through 0 points or 1 point.

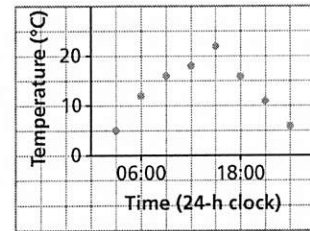
World Population



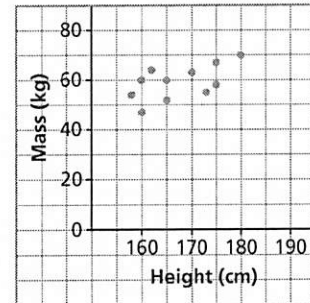
CHECK YOUR UNDERSTANDING

1. Which of these graphs represents a function? Justify your answer.

a) Outside Temperature over a 24-h Period



b) Masses of Students against Height



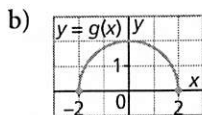
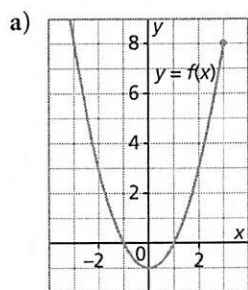
- [Answers: a) function
b) not a function]

How does the vertical line test relate to the definition of a function?

Example 2

Determining the Domain and Range of the Graph of a Function

Determine the domain and range of the graph of each function.



SOLUTION

- a) The dot at the right end of the graph indicates that the graph stops at that point.

There is no dot at the left end of the graph, so the graph continues to the left.

The domain is the set of x -values of the function.

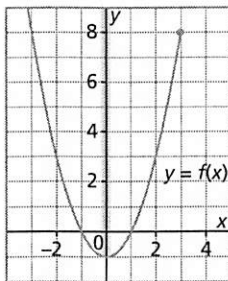
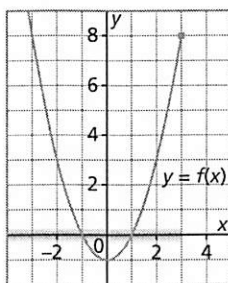
Visualize the shadow of the graph on the x -axis.

The domain is the set of all real numbers less than or equal to 3; that is, $x \leq 3$.

The range is the set of y -values of the function.

Visualize the shadow of the graph on the y -axis.

The range is the set of all real numbers greater than or equal to -1 ; that is, $y \geq -1$.



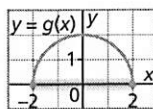
- b) The dot at each end of the graph indicates that the graph stops at that point.

The domain is the set of x -values of the function.

Visualize the shadow of the graph on the x -axis.

The domain is the set of real numbers between -2 and 2 , including these numbers; that is, $-2 \leq x \leq 2$.

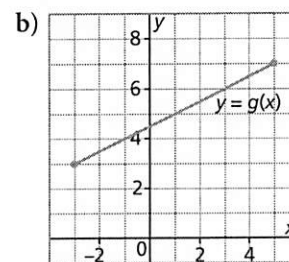
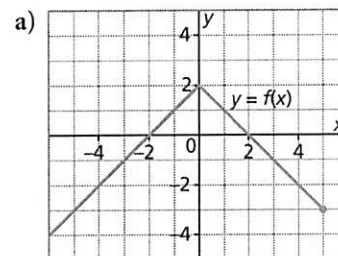
We say: “ x is greater than or equal to -2 and less than or equal to 2 .”



(Solution continues.)

CHECK YOUR UNDERSTANDING

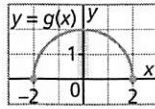
2. Determine the domain and range of the graph of each function.



[Answers: a) $x \leq 5$; $y \leq 2$
b) $-3 \leq x \leq 5$; $3 \leq y \leq 7$]

When data are not discrete, we use inequality symbols to indicate the domain and range.

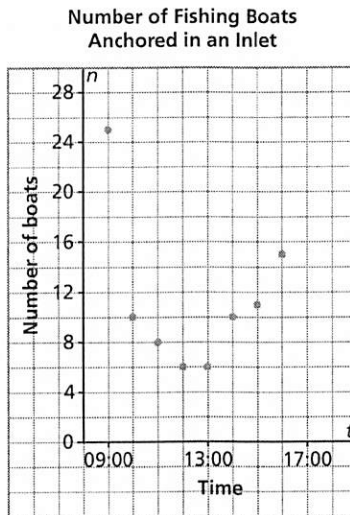
The range is the set of y -values of the function. Visualize the shadow of the graph on the y -axis. The range is the set of real numbers between 0 and 2, including these numbers; that is, $0 \leq y \leq 2$.



Example 3 Determining the Domain and Range of the Graph of a Situation

This graph shows the number of fishing boats, n , anchored in an inlet in the Queen Charlotte Islands as a function of time, t .

- Identify the dependent variable and the independent variable. Justify the choices.
- Why are the points on the graph not connected? Explain.
- Determine the domain and range of the graph.



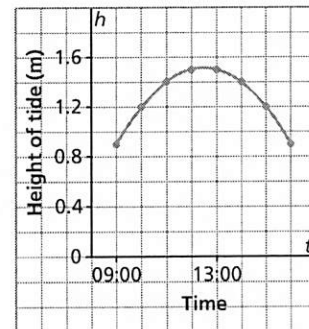
SOLUTION

- The number of fishing boats is a function of time. Since the number of boats, n , depends on the time of day, the dependent variable is n and the independent variable is t .
- The points on the graph are not connected because the number of boats is restricted to a whole number. This means that most values between the points are not valid; for example, between 10:00 and 11:00, the number of boats decreases from 10 to 8. We may plot a point at $n = 9$, if we know a corresponding time, but no other point is valid between 10 and 8 because we cannot have a fractional number of boats.
- The domain is the set of times; that is, $\{09:00, 10:00, 11:00, 12:00, 13:00, 14:00, 15:00, 16:00\}$
The range is the set of the numbers of boats; that is, $\{6, 8, 10, 11, 15, 25\}$

CHECK YOUR UNDERSTANDING

- This graph shows the approximate height of the tide, h metres, as a function of time, t , at Port Clements, Haida Gwaii on June 17, 2009.

Height of Tide at Port Clements, June 17, 2009

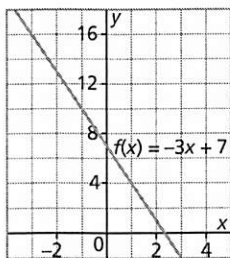


- Identify the dependent variable and the independent variable. Justify your choices.
- Why are the points on the graph connected? Explain.
- Determine the domain and range of the graph.

[Answers: a) h, t
c) $09:00 \leq t \leq 16:00$;
 $0.9 \leq h \leq 1.5$]

Example 4**Determining Domain Values and Range Values from the Graph of a Function**

Here is a graph of the function $f(x) = -3x + 7$.



- Determine the range value when the domain value is -2 .
- Determine the domain value when the range value is 4 .

SOLUTION

The domain value is a value of x . The range value is a value of $f(x)$.

- To determine the value of $f(x)$ when $x = -2$:

Begin at $x = -2$ on the x -axis.

Draw a vertical line to the graph, then a horizontal line to the y -axis.

The line appears to intersect the y -axis at 13 .

$$\text{So, } f(-2) = 13$$

When the domain value is -2 , the range value is 13 .

- To determine the value of x when $f(x) = 4$:

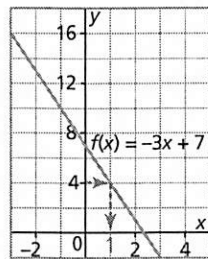
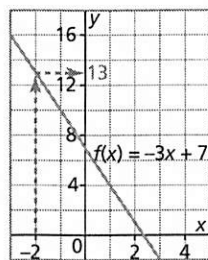
Since $y = f(x)$, begin at $y = 4$ on the y -axis.

Draw a horizontal line to the graph, then a vertical line to the x -axis.

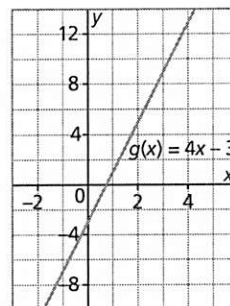
The line intersects the x -axis at 1 .

$$\text{So, when } f(x) = 4, x = 1$$

When the range value is 4 , the domain value is 1 .

**CHECK YOUR UNDERSTANDING**

- Here is a graph of the function $g(x) = 4x - 3$.



- Determine the range value when the domain value is 3 .
- Determine the domain value when the range value is -7 .

[Answers: a) 9 b) -1]

Discuss the Ideas

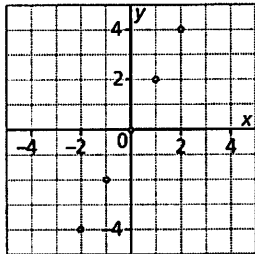
- How do you decide whether to connect the points you plot for a graph?
- What can you tell about the domain and range of a function from its graph?
- How can you identify whether a graph represents a function?

Exercises

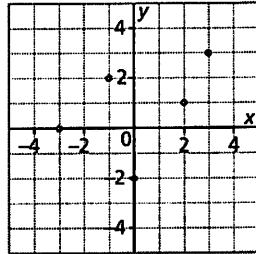
A

4. List the domain and the range of the graph of each function.

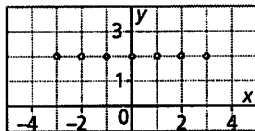
a)



b)



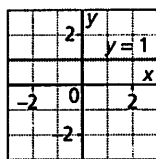
c)



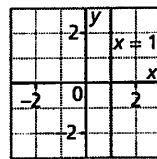
5. How can you tell that each graph in question 4 represents a function?

6. Which of these graphs represents a function? Justify your answer.

a)

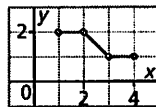


b)

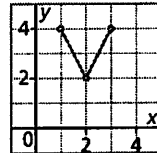


7. Match the graph of each function to its domain and range listed below.

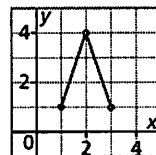
a)



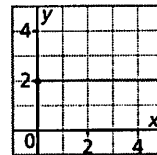
b)



c)



d)



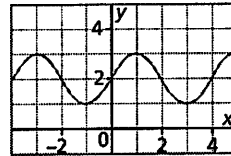
- i) domain: $1 \leq x \leq 3$; range: $2 \leq y \leq 4$
 ii) domain: $1 \leq x \leq 3$; range: $1 \leq y \leq 4$
 iii) domain: $x \geq 0$; range: $y = 2$
 iv) domain: $1 \leq x \leq 4$; range: $1 \leq y \leq 2$

B

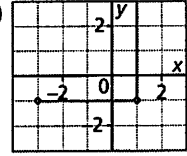
8. Which of these graphs represents a function? Justify your answer.

Write the domain and range for each graph.

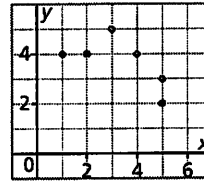
a)



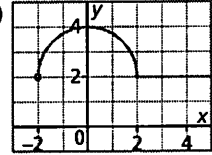
b)



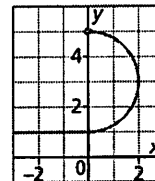
c)



d)

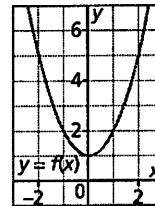


e)

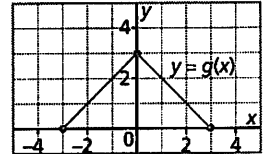


9. Determine the domain and range of the graph of each function.

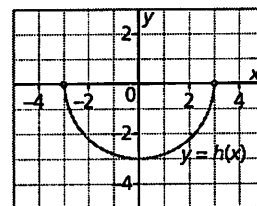
a)



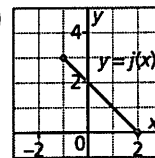
b)



c)



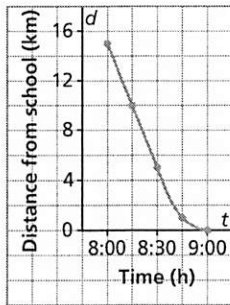
d)



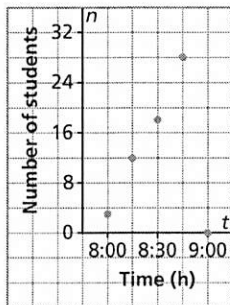
10. Suppose a student drew a graph of each function described below. For which graphs should the student connect the points? Justify your answers.
- The cost of a custom-made T-shirt is a function of the number of letters on the T-shirt.
 - The altitude of a plane is a function of the time it is in the air.
 - The mass of a baby is a function of her age.
 - The cube root of a real number is a function of the number.

11. a) What do the data in each graph represent?

i) **Graph A**
Distance of School Bus from School



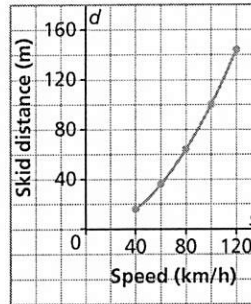
ii) **Graph B**
Number of Students on a School Bus



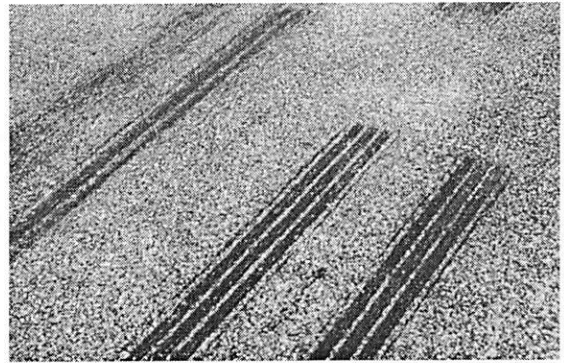
- Identify the independent and dependent variables.
- Why are the points connected on one graph but not on the other?

12. When police officers investigate a car crash, they can estimate the speed the car was travelling by measuring the skid distance.

Skid Distance of a Car

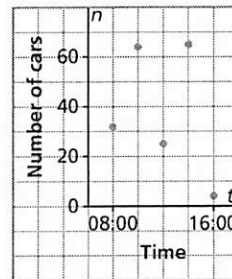


- Why are the points on the graph connected?
- Estimate the domain and range of the graph. Are there any restrictions on the domain and range? Explain.



13. This graph shows the number of cars, n , in the school parking lot as a function of time, t .

Number of Cars in the School Parking Lot



- Identify the independent and dependent variables. Justify your choices.
- Why are the points on the graph not connected?
- Estimate the domain and range of the graph. Are there any restrictions on the domain and range? Explain.

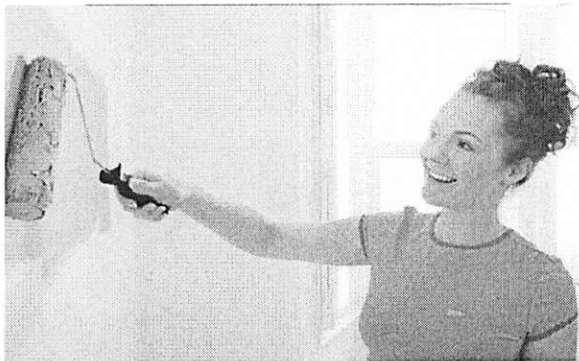
14. Paulatuuq is north of the Arctic Circle. The table shows the number of hours, h , the sun is above the horizon every 60 days from January 1st, which is day 0.

Day	h
0	0
60	9.7
120	18.5
180	24.0
240	15.9
300	7.4
360	0

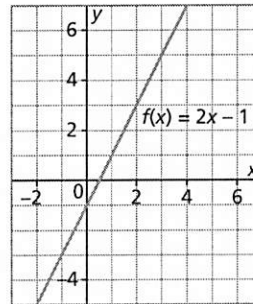
- Identify the independent variable and the dependent variable. Justify your choices.
 - Graph the data in the table. Did you connect the points? Why or why not?
 - Use the table of values and the graph to explain why this relation is a function.
15. One litre of latex paint covers approximately 8.5 m^2 and costs \$12.
- Copy and complete this table.

Volume of Paint, p (L)	0	2	4	6	8
Cost, c (\$)	0	24			
Area Covered, A (m^2)	0	17			

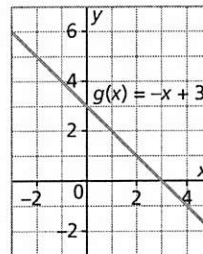
- Graph the area covered as a function of the volume of paint.
- Graph the area covered as a function of the cost.
- Write the domain and range of the functions in parts b and c.



16. This is a graph of the function $f(x) = 2x - 1$.



- Determine the range value when the domain value is 0.
 - Determine the domain value when the range value is 5.
17. This is a graph of the function $g(x) = -x + 3$.



- Determine the range value when the domain value is -2 .
 - Determine the domain value when the range value is 0.
18. Draw a graph of a function on a grid. Write the domain and range of the function. Exchange graphs with a classmate, and check that the domain and range of your classmate's graph are correct. If they are not, correct them, then explain your corrections to your classmate.
19. Sketch a graph of a function that has each domain and range.
- domain: $-2 \leq x \leq 3$; range: $1 \leq y \leq 5$
 - domain: $x \geq 1$; range: $-1 \leq y \leq 1$

20. One planetary year is the time it takes for a planet to travel once around the sun. Since the planets take different times to travel around the sun, one year on each planet is different. The distance from Earth to the sun is 1 astronomical unit. Other distances in the solar system are compared to the distance from Earth to the sun.

	Earth	Jupiter	Saturn	Uranus
Distance from Sun (astronomical units)	1	5	10	19
Planetary Year (Earth years)	1	12	29	84

- Graph planetary year as a function of distance from the sun. Did you connect the points? Explain.
- Write the domain and range of this function.

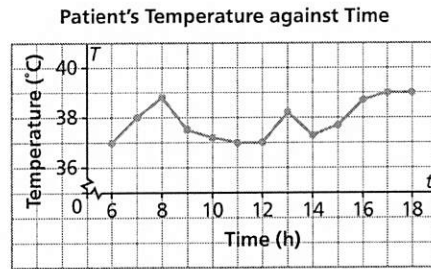
C

21. This table shows the costs to send letters within Canada in 2009.

Mass of Letter	Cost (\$)
Up to 30 g	0.54
Greater than 30 g and up to 50 g	0.98
Greater than 50 g and up to 100 g	1.18
Greater than 100 g and up to 200 g	1.96
Greater than 200 g and up to 500 g	2.75

- Graph the cost of sending a letter as a function of its mass. Did you connect the points? Explain.
- Write the domain and range of this function.

22. A hospital patient has his temperature taken every hour.



- Should the points have been connected? Give reasons for your answer.



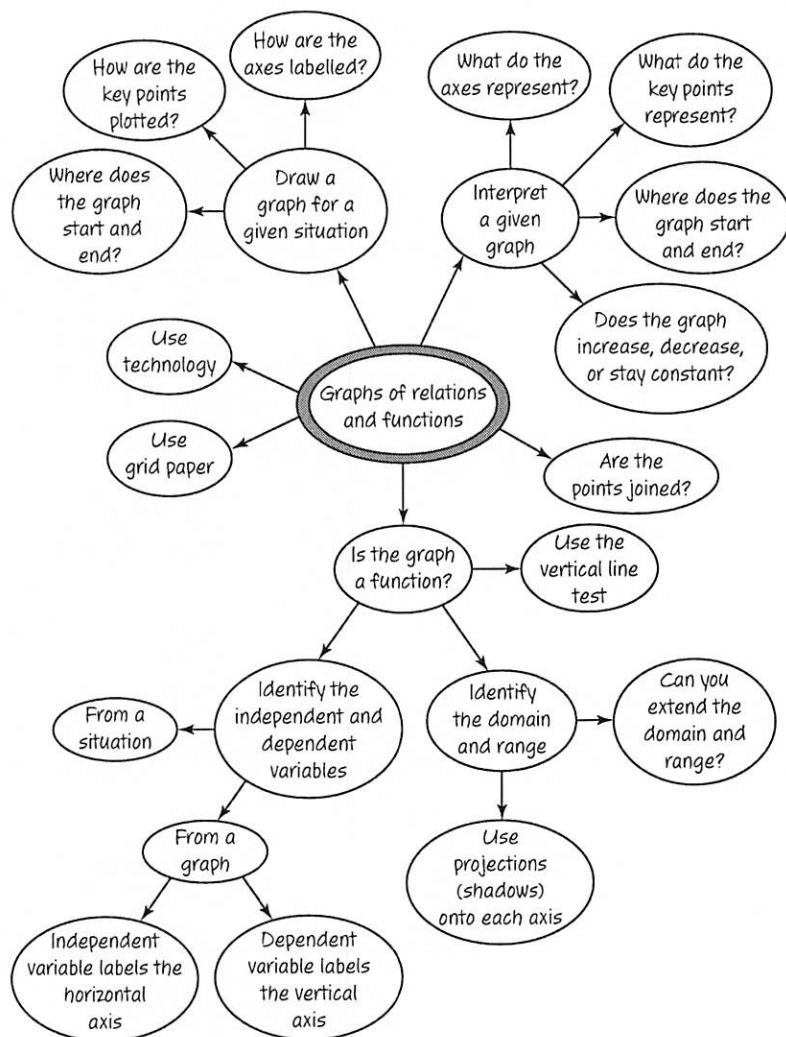
- Is this statement true? A measure of time can be any real number, so any graph with time as its independent variable should have its points connected. Explain your answer with examples.
- Payment scheme 1: A person receives 1¢ on day 1, then each day the payment is doubled. Payment Scheme 2: A person receives \$10 each day. For both payments, the total money received is a function of the number of days.
 - Make a table of values for each payment scheme.
 - Graph the data.
 - Which payment scheme would you choose if you were receiving the money for 30 days? Explain.

Reflect

Generalize and explain rules for determining whether a graph represents a function. How do you determine the domain and range of a function from its graph? Include examples in your explanation.

CHECKPOINT 2

Connections



Concept Development

In Lesson 5.3

- You applied what you know about functions to interpret graphs that represent different situations.
- You applied what you know about functions to sketch graphs that represent different situations.

In Lesson 5.4

- You generated data for a relation, then graphed and analyzed the data.

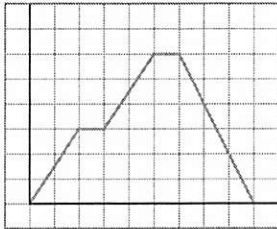
In Lesson 5.5

- You used the vertical line test on graphs to identify functions.
- You identified the independent and dependent variables of a function.
- You graphed tables of values for functions and identified their domains and ranges.
- You connected points on a graph if all real-number values of the variables were permitted.

Assess Your Understanding

5.3

1. Copy the graph below. Choose labels for each axis, then describe a situation the graph could represent. Justify your description.



5.4

2. a) Use technology or grid paper to graph these data for people up to the age of 18.
 b) Should you join the points?
 Explain your reasoning.
 c) What are the domain and range of these data?
 d) Suppose data for more people, up to the age of 18, with different masses were graphed. Would there be any restrictions on the domain and range? If your answer is yes, state the restrictions. If your answer is no, explain why no restrictions exist.

Age (years)	Mass (kg)
14	45
14	50
15	56
15	64
17	65
18	90

5.5

3. Which graphs represent functions? Justify your answer. Write the domain and range of each graph.

