**3.1 – Factors and Multiples of Natural Numbers**

A prime number is a number that can be divided by exactly 2 natural numbers: 1 and itself.

Examples: 2, 3, 5, 7, 11, 13, 17, 19, 29, 31….

All natural numbers can be decomposed into a product of prime numbers. We call this the decomposition into prime factors or prime factorization.

Example: $324=2×2×3×3×3×3=2^{2}×3^{4}$

 $\rightarrow $ we divide the number by 2 as many times as possible, then 3, then 5 …

Your Turn: 2646

Applications:

1) Fractions:

* Simplify fractions:

Examples:

* $\frac{324}{2646}=\frac{2^{2}×3^{4}}{2×3^{3}×7^{2}}=\frac{2×3}{7^{2}}=\frac{6}{49}$
* $\frac{1500}{625}=\frac{3×4×5^{3}}{5^{4}}=\frac{3×4}{5}=\frac{12}{5}$
* Find the smallest common denominator:

Examples:

* $\frac{1}{324}+\frac{1}{2646}=\frac{1\left(×7^{2}\right)}{2^{2}×3^{4}\left(×7^{2}\right)}+\frac{1\left(×2×3\right)}{2×3^{3}×7^{2}\left(×2×3\right)}=\frac{49}{15876}+\frac{6}{15876}=\frac{55}{15876}$

This cannot be simplified further because $\frac{55}{15876}=\frac{5×11}{2^{2}×3^{4}×7^{2}}$.

* $\frac{7}{1500}-\frac{3}{625}=\frac{7\left(×5\right)}{3×4×5^{3}\left(×5\right)}-\frac{3\left(×3×4\right)}{5^{4}\left(×3×4\right)}=\frac{35-36}{7500}=-\frac{1}{7500}$

2) GCF: the greatest common factor

$ \rightarrow $ the biggest number that divides two given numbers

 Examples:

* GCF (12, 18)$=6$
* GCF (5040; 7560) $=?$

$5040=2^{4}×3^{2}×5×7$

$7560=2^{3}×3\^3×5×7$

Therefore, GCF (5040, 7560) $=2^{3}×3^{2}×5×7=2520$

 Example:

* We have two different ribbons measuring 24cm and 42cm. We need to cut them into pieces of identical length. The length of each piece of ribbon must be a whole number and we want the pieces to be as long as possible without any waste. What length does each piece of ribbon need to be?

🡪 The length of the pieces of ribbon must be a divider for both 24 and 42.
 It also needs to be the largest number possible.
 Therefore, we are looking for the GCF (24, 42)

$\left.\begin{array}{c}24=2×2×2×3\\42=2×3×7\end{array}\right\} GCF \left(24, 42\right)=2×3=6cm$

5) LCM: the least common multiple

 Examples:

* + LCM (12, 18)$=36$
	+ LCM (540, 504) $=?$

$\left.\begin{array}{c}540=2^{2}×3^{3}×5\\504=2^{3}×3^{2}×7\end{array}\right\} LCM=2^{3}×3^{3}×5×7=7560$

* + We have rectangles that are 16cm by 40cm. Using the least rectangles as possible, we want to arrange them to form a square. What is the smallest square that we can make?



LCM (16, 40) $= ?$

$\left.\begin{array}{c}16=2^{4}\\40=2^{3}×5\end{array}\right\} LCM=2^{4}×5=80$

Hwk: p.140 #5, 6, 8, 9a, 10, 11a, 12 – 22

**3.7 – Multiplying Polynomials**

Expanding means transforming a product into a sum.

$$\left(a+b\right)\left(c+d\right)=ac+ad+bc+db$$

Each term in the first set of brackets multiplies each term in the second set.

Examples:

1. $\left(x+3\right)\left(x-5\right)=x^{2}-5x+3x-15=x^{2}-2x-15$
2. $-5x\left(x+2\right)=-5x^{2}-10x$
3. $\left(x+1\right)\left(x^{2}-3x+5\right)=x^{3}-3x^{2}+5x+x^{2}-3x+5=x^{3}-2x^{2}+2x+5$

Sometimes, there can be multiple products to develop in a single expression.

Examples:

1. $\left(3x-1\right)\left(x+2\right)-5\left(x^{2}-3\right)=3x^{2}+6x-x-2-5x^{2}+15=-2x^{2}+5x+13$
2. $(x+1)(x+2)(x+3)$

$=(x^{2}+2x+x+2)(x+3)$

$=(x^{2}+3x+2)(x+3)$

$=x^{3}+3x^{2}+3x^{2}+9x+2x+6$

$=x^{3}+6x^{2}+11x+6$

Attention: If there is a “$-$“ sign before brackets…

Example:

$\left(x+3\right)\left(2x+1\right)-(x-4)(3x+5)$

$=2x^{2}+x+6x+3-(3x^{2}+5x-12x-20)$

$=2x^{2}+7x+3-(3x^{2}-7x-20$)

$=2x^{2}+7x+3-3x^{2}+7x+20$

$=-x^{2}+14x+23$

Hwk: p.186 #4, 5, 15 – 21

Special situations:

* Perfect Squares: $\left(a+b\right)^{2}=a^{2}+2ab+b^{2}$

Ex. $\left(3x-5\right)^{2}=9x^{2}-30x+25$

* Product of Conjugates: $\left(a+b\right)\left(a-b\right)=a^{2}-b^{2}$

Ex. $\left(3x+5\right)\left(3x-5\right)=9x^{2}-25$

Hwk: p.194 #4 p.186 # 18, 6a, 7a

**3.3 – Factoring a Polynomials with a Common Factor**

We must always see if there is a greatest common factor for all the terms of the polynomial.

Expand

$$3\left(2x+5\right)=6x+15$$

Factor

Examples:

* $3x^{2}+9x-6=3\left(x^{2}+3x-2\right)$
* $15x^{2}-10x+5=5\left(3x^{2}-2x+1\right)$
* $3xy^{4}+12xy^{2}-6xy^{3}=3xy^{2}\left(y^{2}+4-2y\right)=3xy^{2}(y^{2}-2y+4)$

Notice: The expanded form is a sum of terms. The factored form is a product of factors.

Hwk:p.155 # 8, 10, 12 – 18 without worrying about the tiles

**3.5 – Factoring Polynomials in the form** $x^{2}+bx+c$

There are polynomials like $x^{2}-5x+6$ where the term “$x^{2}$” has no visible coefficient.

Method: $x^{2}-5x+6$

First, we must find two numbers with a product of 6 and a sum of -5

$\left.\begin{array}{c}×6\\+-5\end{array}\right\}-2 and 3$

Therefore: $x^{2}-5x+6=(x-2)(x+3)$

Examples: Factor $x^{2}-2x-15$

$\left.\begin{array}{c}×-15\\+-2\end{array}\right\}-5 and 3$

Therefore: $x^{2}-2x-15=(x-5)(x+3)$

Your Turn:

$x^{2}+8x+15= $

$x^{2}+x-2=$

Important:

* The first step when factoring is to look for a common factor.

Ex. $-4x^{2}-16x+128=-4\left(x^{2}+4x-32\right)=-4(x+8)(x-4)$

* Not all polynomials can be factored!

Ex. $x^{2}+x+1$

Hwk: p.166 # 9 – 17 , 19

**3.6 – Factoring Polynomials in the Form:** $ax^{2}+bx+c$

When the term “$x^{2}$” has a visible coefficient….

Method:

 $2x^{2}+11x+12$ ­­$\left.\begin{array}{c}×25\\+11\end{array}\right\}8 and 3$

We use the numbers of *a* and *c* to separate the middle term into two.

$2x^{2}+8x$ $+$ $3x+12$

We regroup the terms and look for what they have in common.

$2x\left(x+4\right)+3(x+4)$

If we did all the steps correctly, the 2 brackets will be identical.

$(2x+3)(x+4)$

Note: If you had decomposed 11*x* into $3x+8x$ instead of ­$8x+3x$, you would have gotten the same result.

Example 1: Factor $3x^{2}+8x+4$

$\left.\begin{array}{c}×12\\+8\end{array}\right\} 6 and 2$

$3x^{2}+8x+4=3x^{2}+6x+2x+4$

 $=3x\left(x+2\right)+2(x+2)$

 $=(2x+2)(x+2)$

Your turn: $2x^{2}+5x+3$

Example 2: $6x^{2}-7x+2$

$\left.\begin{array}{c}×12\\+-7\end{array}\right\}-4 and-3$

$6x^{2}-7x+2=6x^{2}-4x-3x+2 $

 $=2x\left(3x-2\right)-1(3x-2)$

 $=(2x-1)(3x-2)$

Note: You must always look for a common factor first.

Example 3: $45x^{2}+15x-30$

If we do not see the common factor, then we will have to use these very big numbers. This is much harder to factor and we will still have to factor more at the end.

$15\left(3x^{2}+x-2\right)$

$\left.\begin{array}{c}×-6\\+1\end{array}\right\} 3 et-2$

­­$=15(3x^{2}+3x-2x-2)$

$=15\left(3x\left(x+1\right)-2\left(x+1\right)\right)$

$=15(3x-2)(x+1)$

Example 4: $3x^{2}+5xy-2y^{2}$

$\left.\begin{array}{c}×-6\\+5\end{array}\right\} 6 and-1$

$3x^{2}+6xy-xy-2y^{2}$

$=3x\left(x+2y\right)-y\left(x+2y\right) $

$=\left(3x-y\right)\left(x+2y\right)$

Hwk: p.177 # 8 – 10, 12, 13, 14b, 15 – 23 (do not use the algebra tiles to answer questions)

**3.8 – Factoring Special Polynomials**

There are two types of “special” polynomials:

* Perfect Squares: factored like $\left(3x+5\right)^{2}$
* Difference of squares: in the form $a^{2}-b^{2}$

**I – Perfect Squares**

Reminder: $\left(x+3\right)^{2}=x^{2}+6x+9$

Application:

1. Factor $x^{2}+8x+16$

We see that $x^{2}$ is the square of $x$, 16 is the square of 4 and $8x$ is the double product.

Therefore: $x^{2}+8x+16=\left(x+4\right)^{2}$

We could use the method that we learned in 3.5, but this method is faster if we recognise the perfect square.

1. Factor $4x^{2}-12x+9$

$4x^{2}$ is the square of $2x$, 9 is the square of 3 and $12x$ is the double product.

Therefore: $4x^{2}-12x+9=\left(2x-3\right)^{2}$

Note: Like always, we must first look for a common factor.

Ex. $3x^{2}-30x+75=3\left(x^{2}-10x+25\right)=3\left(x-5\right)^{2}$

**II – Differences of Squares**

Reminder: $\left(x+3\right)\left(x-3\right)=x^{2}-3x+3x-9=x^{2}-9$

Application:

1. Factor: $x^{2}-16=\left(x+4\right)\left(x-4\right)$
2. Factor: $9x^{2}-1=(3x+1)(3x-1)$

Attention: A SUM of squares CANNOT be factored!

Ex. $x^{2}+4$

Note: Do not forget to look for a common factor.

Example: $3x^{2}-12=3\left(x^{2}-4\right)=3(x+2)(x-2)$

Hwk: p.194 # 4 - 21