

# 5.7 Interpreting Graphs of Linear Functions



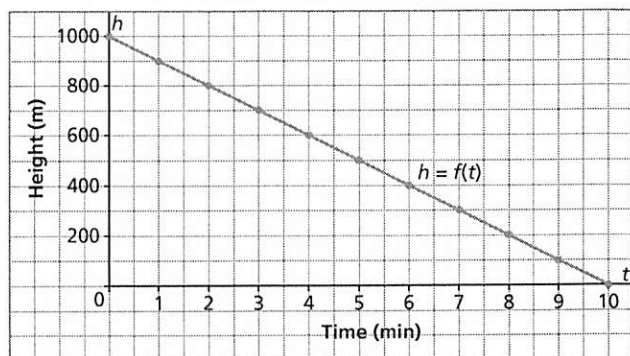
## LESSON FOCUS

Use intercepts, rate of change, domain, and range to describe the graph of a linear function.

## Make Connections

Float planes fly into remote lakes in Canada's Northern wilderness areas for ecotourism. This graph shows the height of a float plane above a lake as the plane descends to land.

Height of a Float Plane



Where does the graph intersect the vertical axis? What does this point represent?

Where does the graph intersect the horizontal axis?  
What does this point represent?

What is the rate of change for this graph? What does it represent?

# Construct Understanding

## TRY THIS

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Work in a group.

You will need grid paper.

Dogsled tours are run between Armstrong cabin and Irving cabin.  
The cabins are 100 km apart.

Dogsled team 1 travels at an average speed of 20 km/h and starts its tour at Armstrong cabin.

Dogsled team 2 travels at an average speed of 25 km/h and starts its tour at Irving cabin.

One pair of students chooses team 1 and the other pair chooses team 2.

- A. Copy and complete the table to show the distance from Irving cabin at different times on the tour.

Team 1

Time (h)	Distance from Irving Cabin (km)
0	100
1	

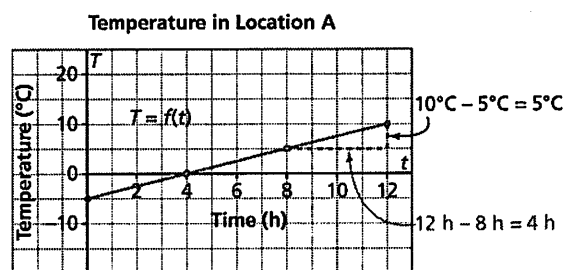
Team 2

Time (h)	Distance from Irving Cabin (km)
0	0
1	

- B. Draw a graph to show the distance from Irving cabin as a function of time.
- C. Share your results with the other pair of students.
- How are the graphs the same? How are they different?
  - Identify where each graph intersects the vertical and horizontal axes. What do these points represent?
  - Determine the rate of change for each graph. What does it represent?
  - What are the domain and range for each graph?

Any graph of a line that is not vertical represents a function.  
We call these functions **linear functions**.

Each graph below shows the temperature,  $T$  degrees Celsius, as a function of time,  $t$  hours, for two locations.



The point where the graph intersects the horizontal axis has coordinates  $(4, 0)$ . The **horizontal intercept** is 4. This point of intersection represents the time, after 4 h, when the temperature is  $0^\circ\text{C}$ .

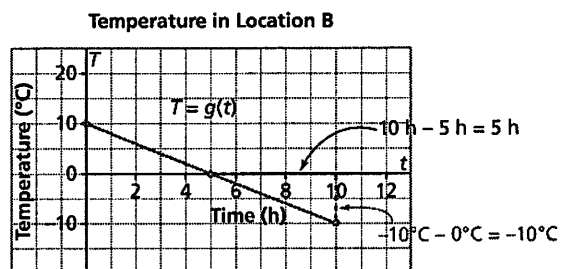
The point where the graph intersects the vertical axis has coordinates  $(0, -5)$ . The **vertical intercept** is  $-5$ . This point of intersection represents the initial temperature,  $-5^\circ\text{C}$ .

The *domain* is:  $0 \leq t \leq 12$

The *range* is:  $-5 \leq T \leq 10$

The *rate of change* is:  $\frac{\text{change in } T}{\text{change in } t} = \frac{5^\circ\text{C}}{4\text{ h}}$   
 $= 1.25^\circ\text{C/h}$

The rate of change is positive because the temperature is increasing over time.



The point where the graph intersects the horizontal axis has coordinates  $(5, 0)$ . The **horizontal intercept** is 5. This point of intersection represents the time, after 5 h, when the temperature is  $0^\circ\text{C}$ .

The point where the graph intersects the vertical axis has coordinates  $(0, 10)$ . The **vertical intercept** is 10. This point of intersection represents the initial temperature,  $10^\circ\text{C}$ .

The *domain* is:  $0 \leq t \leq 10$

The *range* is:  $-10 \leq T \leq 10$

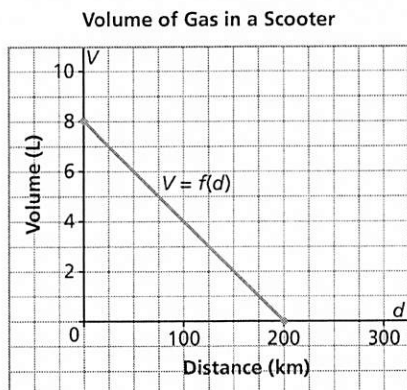
The *rate of change* is:  $\frac{\text{change in } T}{\text{change in } t} = \frac{-10^\circ\text{C}}{5\text{ h}}$   
 $= -2^\circ\text{C/h}$

The rate of change is negative because the temperature is decreasing over time.

## Example 1

### Determining Intercepts, Domain, and Range of the Graph of a Linear Function

This graph shows the fuel consumption of a scooter with a full tank of gas at the beginning of a journey.



- Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.
- What are the domain and range of this function?

### SOLUTION

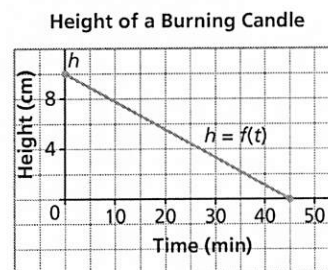
- On the vertical axis, the point of intersection has coordinates (0, 8). The vertical intercept is 8. This point of intersection represents the volume of gas in the tank when the distance travelled is 0 km; that is, the capacity of the gas tank: 8 L

On the horizontal axis, the point of intersection has coordinates (200, 0). The horizontal intercept is 200. This point of intersection is the distance travelled until the volume of gas is 0 L; that is, the distance the scooter can travel on a full tank of gas: 200 km

- The domain is the set of possible values of the distance travelled:  
 $0 \leq d \leq 200$   
The range is the set of possible values of the volume of fuel:  
 $0 \leq V \leq 8$

### CHECK YOUR UNDERSTANDING

- This graph shows how the height of a burning candle changes with time.



- Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.
- What are the domain and range of this function?

[Answers: a) (0, 10), 10; (45, 0), 45  
b) domain:  $0 \leq t \leq 45$ ; range:  
 $0 \leq h \leq 10$ ]

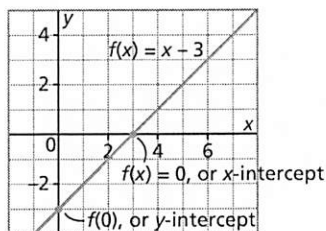
Are there any restrictions on the domain and range? Explain.

What is the fuel consumption in litres per 100 km?

We can use the intercepts to graph a linear function written in function notation.

To determine the  $y$ -intercept, evaluate  $f(x)$  when  $x = 0$ ; that is, evaluate  $f(0)$ .

To determine the  $x$ -intercept, determine the value of  $x$  when  $f(x) = 0$ .



The  $x$ -coordinate of the point where a graph intersects the  $x$ -axis is called the  **$x$ -intercept**, or the **horizontal intercept**.

The  $y$ -coordinate of the point where a graph intersects the  $y$ -axis is called the  **$y$ -intercept**, or the **vertical intercept**.

## Example 2 Sketching a Graph of a Linear Function in Function Notation

Sketch a graph of the linear function  $f(x) = -2x + 7$ .

### SOLUTION

$$f(x) = -2x + 7$$

Since the function is linear, its graph is a straight line.

Determine the  $y$ -intercept:

$$\text{When } x = 0,$$

$$f(0) = -2(0) + 7$$

$$f(0) = 7$$

Determine the  $x$ -intercept:

$$\text{When } f(x) = 0,$$

$$0 = -2x + 7$$

$$-7 = -2x + 7 - 7$$

$$-7 = -2x$$

$$x = \frac{-7}{-2}$$

$$x = \frac{7}{2}$$

Determine the coordinates of a third point on the graph.

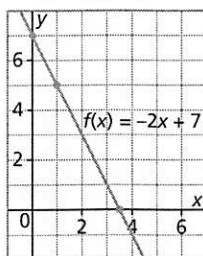
$$\text{When } x = 1,$$

$$f(1) = -2(1) + 7$$

$$f(1) = 5$$

Plot the points  $(0, 7)$ ,  $(\frac{7}{2}, 0)$ ,

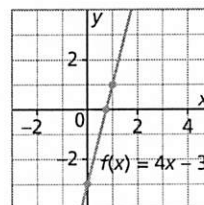
and  $(1, 5)$ , then draw a line through them.



### CHECK YOUR UNDERSTANDING

2. Sketch a graph of the linear function  $f(x) = 4x - 3$ .

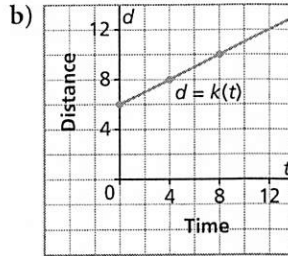
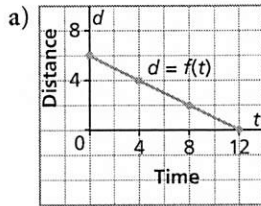
Answer:



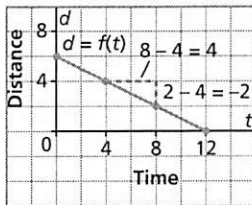
What other strategy could you use to graph the function? Which strategy would be more efficient?

**Example 3****Matching a Graph to a Given Rate of Change and Vertical Intercept**

Which graph has a rate of change of  $\frac{1}{2}$  and a vertical intercept of 6? Justify the answer.

**SOLUTION**

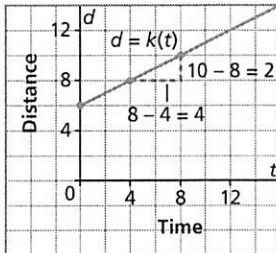
a) The graph of  $d = f(t)$  has a vertical intercept of 6.



The rate of change is:  $\frac{-2}{4} = -\frac{1}{2}$

So, it is not the correct graph.

b) The graph of  $d = k(t)$  has a vertical intercept of 6.

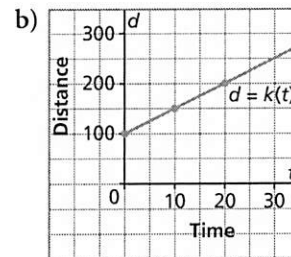
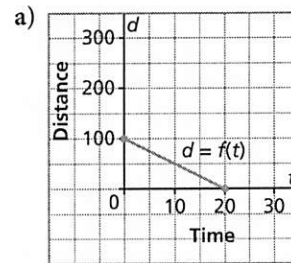


The rate of change is:  $\frac{2}{4} = \frac{1}{2}$

So, this is the correct graph.

**CHECK YOUR UNDERSTANDING**

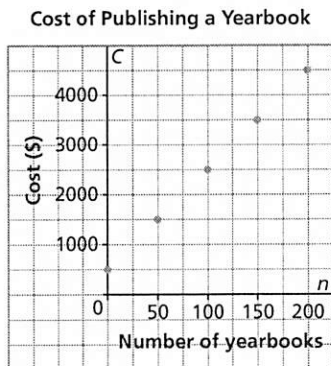
3. Which graph has a rate of change of  $-5$  and a vertical intercept of 100? Justify your answer.



[Answer: the graph in part a]

## Example 4 Solving a Problem Involving a Linear Function

This graph shows the cost of publishing a school yearbook for Collège Louis-Riel in Winnipeg.



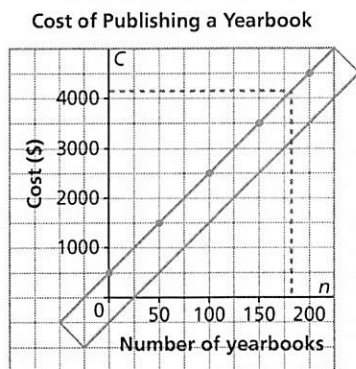
The budget for publishing costs is \$4200. What is the maximum number of books that can be printed?

### SOLUTIONS

#### Method 1

To estimate the number of yearbooks that can be printed for \$4200, use the graph.

From 4200 on the  $C$ -axis, draw a horizontal line to the graph, then a vertical line to the  $n$ -axis.



Use a straightedge to help.

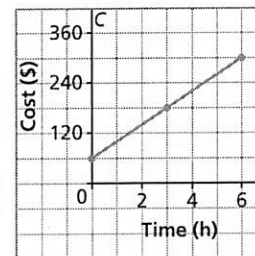
From the graph, about 180 yearbooks can be printed.

(Solution continues.)

### CHECK YOUR UNDERSTANDING

4. This graph shows the total cost for a house call by an electrician for up to 6 h work.

Cost of an Electrician's House Call



The electrician charges \$190 to complete a job. For how many hours did she work?

[Answer:  $3\frac{1}{4}$  h]

Why are the points on this graph not joined?

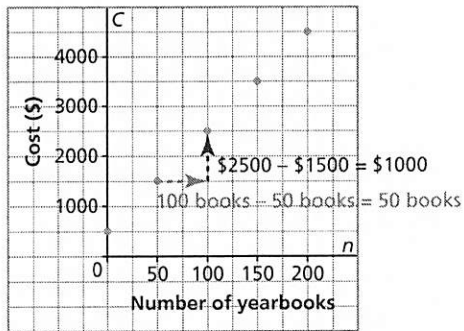
What are the domain and range of this function?

## Method 2

The set-up cost is the cost when the number of books printed is 0. This is the vertical intercept of the graph, which is 500. The set-up cost is \$500.

The increase in cost for each additional book printed is the rate of change of the function. Determine the change in each variable.

Cost of Publishing a Yearbook



The graph shows that for every 50 books printed, the cost increases by \$1000.

The rate of change is:  $\frac{\$1000}{50 \text{ books}} = \$20/\text{book}$

The increase in cost for each additional book published is \$20.

An equation that represents this situation is:  $C = 20n + 500$

To determine the maximum number of yearbooks that can be printed, use the equation:

$$\begin{aligned} C &= 20n + 500 && \text{Substitute: } C = 4200 \\ 4200 &= 20n + 500 && \text{Solve for } n. \\ 4200 - 500 &= 20n + 500 - 500 \\ 3700 &= 20n \\ \frac{3700}{20} &= \frac{20n}{20} \\ 185 &= n \end{aligned}$$

The maximum number of yearbooks that can be printed is 185.



What is an advantage of using each method?



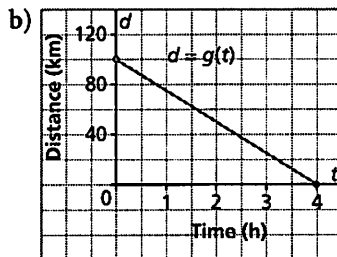
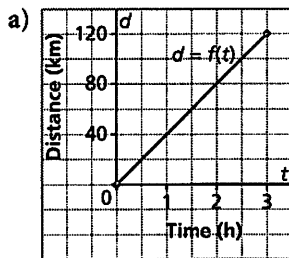
## Discuss the Ideas

1. What information do the vertical and horizontal intercepts provide about a linear function? Use an example to explain.
2. How can you tell from a graph whether a linear function has a positive or negative rate of change?
3. When a situation can be described by a linear function, why doesn't it matter which pair of points you choose to determine the rate of change?

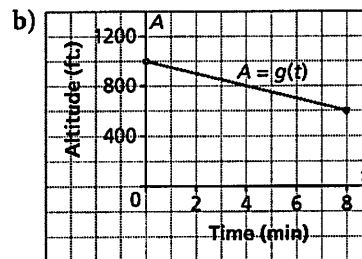
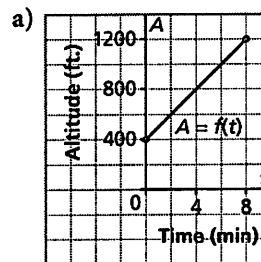
## Exercises

### A

4. Each graph below shows distance,  $d$  kilometres, as a function of time,  $t$  hours. For each graph:
- i) Determine the vertical and horizontal intercepts. Write the coordinates of the points where the graph intersects the axes.
  - ii) Determine the rate of change.
  - iii) Determine the domain and range.

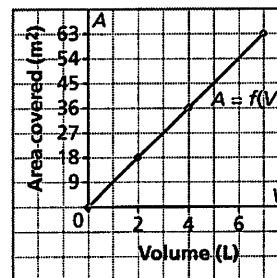


5. Each graph shows the altitude,  $A$  feet, of a small plane as a function of time,  $t$  minutes. For each graph:
- i) Determine the vertical intercept. Write the coordinates of the point where the graph intersects the axis.
  - ii) Determine the rate of change.
  - iii) Determine the domain and range.



### B

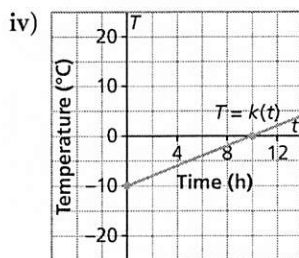
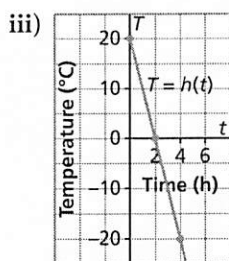
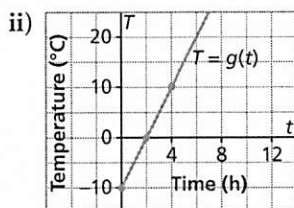
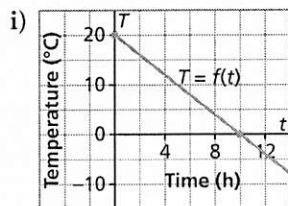
6. Sketch a graph of each linear function.
- a)  $f(x) = 4x + 3$
  - b)  $g(x) = -3x + 5$
  - c)  $h(x) = 9x - 2$
  - d)  $k(x) = -5x - 2$
7. This graph shows the area,  $A$  square metres, that paint covers as a function of its volume,  $V$  litres.



- a) What is the rate of change? What does it represent?
- b) What area is covered by 6 L of paint?
- c) What volume of paint would cover 45 m<sup>2</sup>?

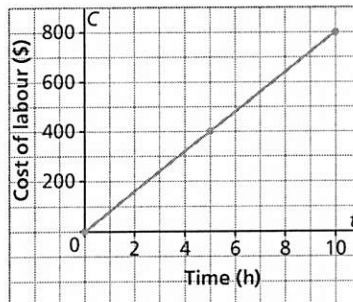
8. The graphs below show the temperature,  $T$  degrees Celsius, as a function of time,  $t$  hours, at different locations.

- Which graph has a rate of change of  $5^\circ\text{C}/\text{h}$  and a vertical intercept of  $-10^\circ\text{C}$ ?
- Which graph has a rate of change of  $-10^\circ\text{C}/\text{h}$  and a vertical intercept of  $20^\circ\text{C}$ ?



9. St. Adolphe, Manitoba, is located in the flood plain of the Red River. To help prevent flooding, backhoes were used to build dikes around houses and farms in the town. This graph shows the labour costs for running a backhoe.

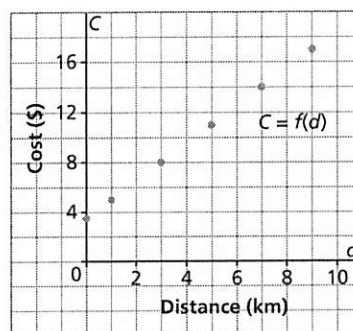
Cost of Running a Backhoe



- Determine the vertical and horizontal intercepts. Write the coordinates of the point where the graph intersects the axes. Describe what the point represents.
- Determine the rate of change. What does it represent?
- Write the domain and range.
- What is the cost to run the backhoe for 7 h?
- For how many hours is the backhoe run when the cost is  $\$360$ ?



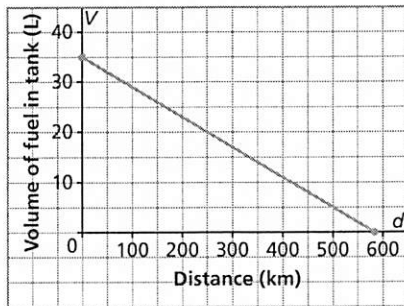
10. This graph shows the cost for a cab at Eagle Taxi Cabs. The cost,  $C$  dollars, is a function of the distance travelled,  $d$  kilometres.



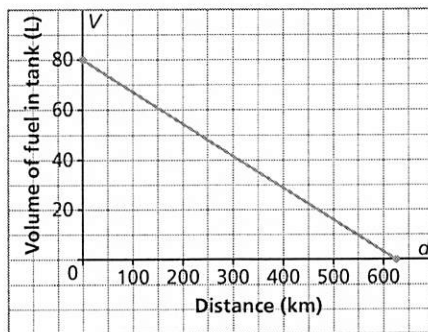
- Determine the rate of change. What does it represent?
- What is the cost when the distance is 7 km?
- What is the distance when the cost is  $\$9.50$ ?

11. A Smart car and an SUV have full fuel tanks, and both cars are driven on city roads until their tanks are nearly empty. The graphs show the fuel consumption for each vehicle.

Fuel Consumption of a Smart Car



Fuel Consumption of an SUV

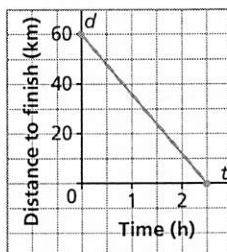


Use the graphs to explain why the Smart car is more economical to drive than the SUV.



12. This graph shows the distance to the finish line,  $d$  kilometres, as a function of time,  $t$  hours, for one dogsled in a race near Churchill, Manitoba.

Dogsled Race

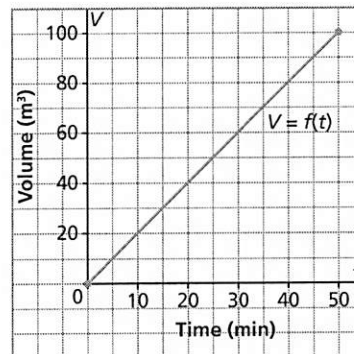


- a) What was the length of time it took the dogsled to finish the race?  
 b) What was the average speed of the dogsled?  
 c) How long was the race in kilometres?  
 d) What time did it take for the dogsled to complete  $\frac{2}{3}$  of the race?

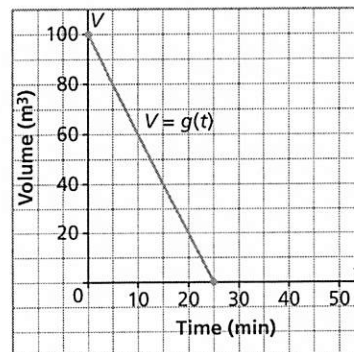


13. The capacity of each of 2 fuel storage tanks is  $100 \text{ m}^3$ . Graph A represents the volume of fuel in one tank as a function of time as the tank is filled. Graph B represents the volume of fuel in another tank as a function of time as the tank is emptied.

Graph A

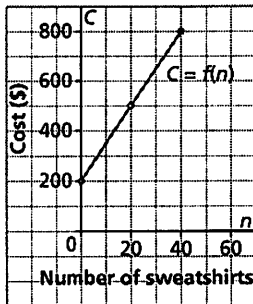


Graph B

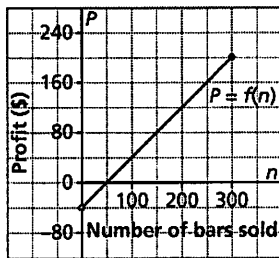


- a) Does it take longer to fill the empty tank or empty the full tank? How do you know?  
 b) In the time it takes for one tank to be half empty, about how much fuel would be in a tank that was being filled from empty?

14. Ballenas School places an order for school sweatshirts with its logo of a killer whale on the back. This graph shows the cost of the sweatshirts,  $C$  dollars, as a function of the number ordered,  $n$ .

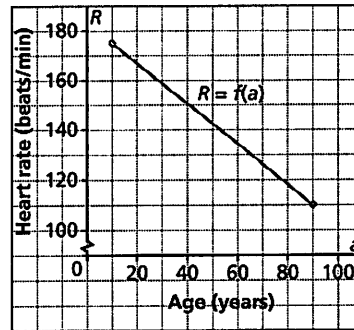


- a) The number of sweatshirts cannot be a fraction or decimal. Why do you think the points on the graph are joined?  
 b) i) About how many sweatshirts can be bought for \$700?  
 ii) Suppose one more sweatshirt was ordered. What would be the increase in cost?
15. Sketch a graph of each linear function for positive values of the independent variable.  
 a)  $f(x) = 5 - 2.5x$       b)  $g(t) = 85t$   
 c)  $h(n) = 750 + 55n$     d)  $V(d) = 55 - 0.08d$
16. Northlands School Outdoor Club had a fundraiser to help purchase snowshoes. The club had 300 power bars to sell. This graph shows the profit made from selling power bars.



- a) What is the profit on each bar sold? How do you know?  
 b) Determine the intercepts. What does each represent?  
 c) Describe the domain and range for the function. Why would you not want to list all the values in the range?

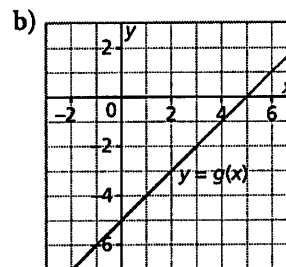
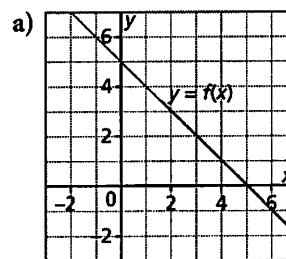
17. This graph shows the recommended maximum heart rate of a person,  $R$  beats per minute, as a function of her or his age,  $a$  years, for a stress test.



- a) Why are there no intercepts on this graph?  
 b) What is the rate of change? What does it represent?  
 c) At what age is the recommended maximum heart rate 120 beats/min?  
 d) What is the approximate recommended maximum heart rate for a person aged 70?

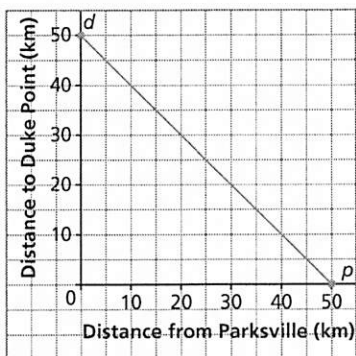


18. Two graphs that relate two real numbers  $x$  and  $y$  in different ways are shown below. For each graph:
- i) State the  $x$ - and  $y$ -intercepts.  
 ii) Use the intercepts to describe how  $x$  and  $y$  are related.



19. a) Sketch a graph of the linear function  $d = f(t)$  that satisfies these conditions:  
 $f(1.5) = 127.5$  and  $f(3.5) = 297.5$   
 b) Determine  $f(5)$ .  
 c) Determine  $t$  when  $f(t) = 212.5$ .  
 d) Suggest a context for this linear function.
20. The distance between Parksville and the Duke Point Ferry Terminal on Vancouver Island is 50 km. A person drives from Parksville to the ferry terminal.

Distance from Parksville and to Duke Point



- a) What do the intercepts represent? Why are they equal?  
 b) What is the rate of change? Why does it not have units? What does it indicate?  
 c) How would interchanging the dependent and independent variables change the graph?  
 d) Suppose the distance between two towns A and B is  $k$  kilometres. Describe the graph of the function, “Distance to A as a function of distance from B”. State the intercepts, domain and range, and the rate of change.



## Reflect

Explain why knowing the intercepts and the rate of change of the graph of a linear function may be helpful when you solve problems. Include examples in your explanation.



## THE WORLD OF MATH

### Historical Moment: Theano

Theano was one of the first known woman mathematicians. Her husband was Pythagoras, perhaps the most famous mathematician of all time. Theano lived in the 6th century B.C.E. in what is now southern Italy. She wrote many articles on mathematics, as well as on physics, medicine, astronomy, and child psychology. Her most famous work was on the development of the golden ratio and the golden rectangle.



# STUDY GUIDE

## CONCEPT SUMMARY

### Big Ideas

- A relation associates the elements of one set with the elements of another set.

- A function is a special type of relation for which each element of the first set is associated with a unique element of the second set.

- A linear function has a constant rate of change and its graph is a non-vertical straight line.

### Applying the Big Ideas

This means that:

- A relation may be represented as: a rule, a table, a set of ordered pairs, an arrow diagram, and a graph. The set of first elements is the domain and the set of related second elements is the range.
- For a function, each element of the domain is associated with exactly one element of the range.
- For a linear function, a constant change in the independent variable results in a constant change in the dependent variable, and any vertical line drawn through the graph intersects the graph at no more than one point.

### Reflect on the Chapter

- What is a relation? What is a function? Create a graphic organizer to show their common characteristics, and those that are unique.
- How can the properties of linear functions be used to solve real-world problems? Include examples with your explanation.