EXPONENTS

REMINDER: An exponent is a shortened way to represent repeated multiplication.

Examples: $3^5 = 3 \times 3 \times 3 \times 3 \times 3$
$x^2 = x \times x$
In Grade 9, you manipulated natural exponents.

EXPLORATION: While analysing repeated multiplications, we could define other exponents to keep the regurity…

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$3^5 = 3 \times 3 \times 3 \times 3 \times 3$

$\div 3$

$3^4 = 3 \times 3 \times 3 \times 3$

$3^3 = 3 \times 3 \times 3$

$3^2 = 3 \times 3$

$3^1 = 3$

$3^0 = 1$

$3^{-1} = \frac{1}{3}$

$3^{-2} = \frac{1}{3^2}$

Note : Notice that $3^0=1$, this is true for any base that is not zero…

Definition : For all whole positive numbers n, we have: $\boxed{3^{-n} = \frac{1}{3^n}}$

This rule is true for any other base (not zero).

Examples : a) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ b) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$ c) $2x^{-5} = 2 \times \frac{1}{x^5} = \frac{2}{x^5}$

This rule has many interesting consequences:

- $\left(\frac{2}{3}\right)^{-5} = \left(\frac{3}{2}\right)^{5}$

(to change the sign of an exponent, take the reciprocal of the number!)

- $\boxed{\frac{1}{x^{-5}} = \frac{x^5}{1}}$ ou $\boxed{\frac{x^{-5}}{1} = \frac{1}{x^5}}$

(if you want to switch the numerator and denominator, you need to change the sign of the exponent)

Examples: Simplify only using positive exponents and evaluate only if possible.

a) $\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^{2} = \frac{16}{9}$

b) $\frac{3}{5^{-4}} = 3 \times 5^4$

c) $\frac{2x^{-3}}{5} = \frac{2}{5x^3}$

d) $\frac{3^3x^{-3}}{2^{-3}y^2z^{-4}} = \frac{3^3 \times 2^3 z^4}{x^3 y^2} = \frac{27 \times 8 z^4}{x^3 y^2}$

$= \frac{216 z^4}{x^3 y^2}$

e) $\frac{2^5x^3}{2^8x^{-2}} = \frac{x^3x^2}{2^8\, 2^{-5}} = \frac{x^5}{2^3} = \frac{x^5}{8}$

Hwk : p 233 # 3 – 8 , 10, 16

LAW OF EXPONENTS (Reminders):

What simplifies well...

- Multiplying powers that have the same base: $3^5 \times 3^2 = 3^7$

indeed: $3^5 \times 3^2 = \underbrace{3 \times 3 \times 3 \times 3 \times 3} \times \underbrace{3 \times 3}$

$= 3^7$

- Dividing powers that have the same base: $\frac{3^5}{3^2} = 3^{5-2} = 3^3$

indeed: $\frac{3^5}{3^2} = \frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} = 3^3$

- If a power has another exponent: $(3^4)^2 = 3^{4\times 2} = 3^8$

indeed: $(3^4)^2 = 3^4 \times 3^4$
$= 3^8$

- If a product has an exponent: $(2x)^3 = 2^3x^3 = 8x^3$

indeed: $(2x)^3 = 2x \cdot 2x \cdot 2x = 2 \cdot 2 \cdot 2\, xxx$
$= 8x^3$

All of these rules continue to work the same with negative or positive exponents...

What does not simplify*... too bad...*

Adding and subtracting powers that have the same base:

$2^3 + 2^5 = 8 + 32 = 40$ $\qquad 2^3 + 2^5 \neq 2^8$

Multiplying powers that don't have the same base :

$2^3 \times 3^2 = 8 \times 9 = 72$

If a sum has an exponent:

$(x+3)^2 = x^2 + 6x + 9$ $\qquad (x+3)^2 \neq x^2 + 3^2$

Examples:

a) $3^{-5} \times 3^2 = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

b) $3^{-5} \times 9^2 = 3^{-5} \times (3^2)^2 = 3^{-5} \times 3^4 = 3^{-1} = \frac{1}{3}$

c) $((-5)^2)^{-3} = (-5)^{-6} = \frac{1}{(-5)^6} = \frac{1}{5^6}$

d) $\frac{x^{-5}}{x^{-2}} = x^{-5} \cdot x^2 = x^{-3} = \frac{1}{x^3}$

e) $(3x)^{-4} = \frac{1}{(3x)^4} = \frac{1}{3^4x^4} = \frac{1}{81x^4}$

f) $\left(2x^3y^{-2} \times \frac{1}{3}x^5y^{-3}\right)^{-1} = \left(\frac{2}{3}x^8y^{-5}\right)^{-1} = \left(\frac{2}{3}\right)^{-1} x^{-8} y^5 = \frac{3y^5}{2x^8}$

g) $\left(\frac{2x^2y^{-3}}{3x^3y}\right)^{-3} = \left(\frac{2}{3x^3x^{-2}yy^3}\right)^{-3} = \left(\frac{2}{3xy^4}\right)^{-3} = \frac{2^{-3}}{3^{-3}x^{-3}y^{-12}} = \frac{27x^3y^{12}}{8}$

h) $(2xy^{-3})^{-2}(3x^2y^{-3})^2 = 2^{-2}x^{-2}\cancel{y^6} \times 3^2x^4\cancel{y^{-6}} = \frac{9x^2}{4}$

i) $(2x-3y)^2 = 4x^2 - 12xy + 9y^2$

Hwk : p 241 # 3 – 11, 14 – 17 , 19, 21, 22

Review : worksheet + p 247 # 24, 28 – 30 , 32 + p 249 # 6, 7 + p 253 # 25, 26