

1.6

Solving Investment Portfolio Problems

YOU WILL NEED

- spreadsheet software
- financial application on a graphing calculator

EXPLORE...

- Describe three different investments that would result in \$30 000 in 10 years.

portfolio

One or more investments held by an individual investor or by a financial organization.

GOAL

Analyze, compare, and design investment portfolios that meet specific financial goals.

INVESTIGATE the Math

André is 17 years old and in Grade 12. He plans to start a 4-year history degree next year, focusing on the Métis of Western Canada. His goal is to be a curator at Le Musée de Saint-Boniface in Winnipeg. He has the following investments in his post-secondary education savings **portfolio** :

- Starting when he was born, his parents deposited \$100 at the end of each month into a savings account, earning an average annual interest rate of 3%, compounded monthly.
- On his eighth birthday, his grandparents bought him a 10-year \$5000 GIC that earned 4%, compounded annually.

André plans to redeem both investments now and combine them into one investment account that earns 4.2%, compounded quarterly, for one year until he starts school.

He expects to withdraw money from his investment account each year to cover major expenses, such as tuition, rent, and books. He also plans to work part-time while at university and during the summers.



Le Musée de Saint-Boniface is housed in Winnipeg's oldest building. The museum's collection includes items of cultural significance to both Métis and Francophone communities of Manitoba.

? Will André have enough money to pay for 4 years of university?

- A. How much will André's parents' investment be worth when he redeems it?
- B. How much will his grandparents' investment be worth when he redeems it?
- C. How much will André's new investment account be worth when he starts school?

- D. Suppose that André withdraws a lump sum from his investment account at the beginning of first year to cover his major first-year expenses. His summer job and part-time work during the year will pay for the rest of his first-year expenses. When he starts second year, his investment account has a value of \$27 446.17. How much was his withdrawal at the beginning of first year?
- E. Suppose that André withdraws the same amount for his second-year expenses. How much will he have in the account when he starts third year?
- F. Suppose that André withdraws the same amount for his third-year expenses. Will he have enough left to withdraw the same amount again for his fourth-year expenses?
- G. Suppose that André decides not to work part-time during fourth year because he wants to focus on his studies. What interest rate, compounded quarterly, would his account need to earn during his third year so that he can withdraw \$11 500 for his fourth year?

Reflecting

- H. When solving this problem, you determined
- the future value of a single investment,
 - the future value of an investment involving regular payments,
 - the present value of an investment, and
 - the interest rate required for a single investment to grow to a certain future value.
- Identify where you determined each value.

APPLY the Math

EXAMPLE 1

Determining the future value and doubling time of an investment portfolio

Phyllis started to build an investment portfolio for her retirement.

- She purchased a \$500 Canada Savings Bond (CSB) at the end of each year for 10 years. The first five CSBs earned a fixed rate of 4.2%, compounded annually. The next five CSBs earned a fixed rate of 4.6% compounded annually.
 - Three years ago, she also purchased a \$4000 GIC that earned 6%, compounded monthly.
- a) What was the value of Phyllis's portfolio 10 years after she started to invest?
- b) Phyllis found a savings account that earned 4.9%, compounded semi-annually. She redeemed her portfolio and invested all the money in the savings account. About how long will it take her to double her money?



Sal's Solution

a)

	A	B	C	D	E
1	Year	P (\$)	i	n	A (\$)
2	1	500	0.042	9	724.0682
3	2	500	0.042	8	694.8831
4	3	500	0.042	7	666.8744
5	4	500	0.042	6	639.9946
6	5	500	0.042	5	614.1983
7	6	500	0.046	4	598.5449
8	7	500	0.046	3	572.2227
9	8	500	0.046	2	547.058
10	9	500	0.046	1	523
11	10	500	0.046	0	500
12				Sum:	6080.85

The CSB part of the portfolio has a value of \$6080.85.

\$4000 at 6% compounded monthly for 3 years:

$$A = P(1 + i)^n$$

$$A = 4000 \left(1 + \frac{0.06}{12} \right)^{3(12)}$$

$$A = 4786.722\dots$$

The GIC part of the portfolio has a value of \$4786.72.

$$\text{Portfolio value} = 6080.85 + 4786.72$$

$$\text{Portfolio value} = 10\,867.57$$

Phyllis's portfolio is worth \$10 867.57.

b) Use the Rule of 72 to estimate the doubling time.

$$\frac{72}{4.9} = 14.694\dots$$

The portfolio will take about 14.5 years to double.

To figure out the value of the CSB part of the portfolio, I knew that I had to determine the future value of 10 bonds with two different interest rates. Then I had to determine the sum.

I decided that a spreadsheet would make the calculations easier.

To create the spreadsheet formula for the future values in column E, I used the following formula:

$$A = P(1 + i)^n$$

Since the bonds were purchased at the end of each year, the last bond would not earn any interest.

\$6080.85 seemed reasonable because ten \$500 CSBs would be \$5000, not counting interest.

Next, I determined the value of the GIC she purchased 3 years ago using the compound interest formula.

I added the values of the CSBs and GIC to determine the total value of the portfolio.

I rounded the estimate down because I knew that 14.694... was a bit high, due to semi-annual compounding.

Your Turn

Suppose that \$21 735.14 was the future value of a single investment of \$5000, made 31 years ago. What interest rate, compounded semi-annually, would the investment have had to earn?

EXAMPLE 2**Designing and adjusting an investment portfolio to meet a financial goal**

John is an avid sailor and dreams about competing in the Olympics. He wants to buy his own Laser sailboat in 6 years, but, in the meantime, he sails on a friend's boat. The cost of a new Laser is about \$9660, including taxes. John won \$2500 in his most recent race and can save \$50 a month from his part-time job.



- What recommendations for a portfolio of two different investments would you make, based on available investments and interest rates? Explain.
- Will the portfolio support the cost of the new Laser in 6 years?
- If not, suggest a solution for John.

Lasers are small boats with only one sail. Of the classes of boats that compete at the Olympics, Laser is the least expensive.

Kyla's Solution

- After researching available investments and interest rates, I recommend that John
 - uses the \$2500 cash to buy a 6-year GIC that earns 5.1%, compounded semi-annually, and
 - deposits his monthly savings of \$50 in a high-interest savings account that earns 4.3%, compounded monthly.
- \$2500 GIC that earns 5.1%, compounded semi-annually, for 6 years:

The present value is \$2500.

The annual interest rate is 5.1%.

The compounding frequency is 2 times per year.

The number of years is 6.

The future value is unknown.

Future value = 3381.956...

Deposits of \$50 at the end of each month, earning 4.3%, compounded monthly, for 6 years:

The regular payment amount is \$50.

The payment frequency is 12 times per year.

The number of payments is 6(12) or 72.

The payments are made at the end of each month.

The annual interest rate is 4.3%.

The compounding frequency is 12 times per year.

The future value is unknown.

Future value = 4098.726...

I wanted the cash to start earning interest right away at a good interest rate for 6 years. John also needs an account with a high interest rate, where he can deposit his monthly savings.

I used the financial application on my calculator and entered these values to determine the future value of the GIC.

I entered these values into the financial application on my calculator to determine the future value of the savings account.



$$\text{Total value} = 3381.956... + 4098.726...$$

$$\text{Total value} = 7480.682...$$

John will have \$7480.68 in 6 years, which is not enough.

c) $9660 - 7480.68 = 2179.32$

He needs \$2179.32 more.

$$\frac{2179.32}{72} = 30.268...$$

John needs to save about \$30 more each month.

$$4098.73 + 2179.32 = 6278.05$$

He needs to have a future value of \$6278.05 in his savings account.

A regular payment amount of \$76.60, or \$26.60 more each month, will give him enough money in 6 years to buy the Laser.

I divided the additional amount he needs by the number of months in 6 years to estimate how much more he will need to save each month.

If he saves \$30 more each month, he will have almost enough money, without counting interest. So, \$30 per month will be enough.

I used the financial application on my calculator and tried different regular payment amounts until I got a future value of \$6278.05.

I tried \$75, \$76, \$77, \$76.50, and finally \$76.60.

Your Turn

- What would John's monthly payments have to be if he wanted to buy the Laser in 5 years?
- Suppose that the price of the Laser increases by 15% over the 6 years. How much would his monthly deposits have to be if he still wanted to buy the Laser in 6 years?

EXAMPLE 3

Comparing the rates of return of two investment portfolios

Jason and Malique are each hoping to buy a house in 10 years. They want their money to grow so they can make a substantial down payment.

Jason's portfolio:

- A 10-year \$2000 GIC that earns 4.2%, compounded semi-annually
- A savings account that earns 1.8%, compounded weekly, where he saves \$55 every week
- A 5-year \$4000 bond that earns 3.9%, compounded quarterly, which he will reinvest in another bond at an interest rate of 4.1%

Malique's portfolio:

- A tax-free savings account (TFSA) that earns 2.2%, compounded monthly, and has a current balance of \$5600
- The purchase, at the end of each year, of a 10-year \$500 CSB that earns 3.6%, compounded annually
- A savings account that earns 1.6%, compounded monthly, where she saves \$200 every month

In 10 years, whose portfolio will have the greater rate of return on investment?



Shelly's Solution

What Jason invested:

$$\text{Total investment} = 2000 + 55(52)(10) + 4000$$

$$\text{Total investment} = 34\,600$$

Jason invested \$34 600.

What Malique invested:

$$\text{Total investment} = 5600 + 10(500) + 200(12)(10)$$

$$\text{Total investment} = 34\,600$$

Malique invested \$34 600.

Jason's portfolio:

The GIC has a future value of \$3030.71.

The savings account has a future value of \$31 329.72.

The bond has a value of \$4856.65 after 5 years and, after reinvesting, a future value of \$5955.45.

$$\text{Portfolio value} = 3030.71 + 31\,329.72 + 5955.45$$

$$\text{Portfolio value} = 40\,315.88$$

Jason's portfolio has a future value of \$40 315.88.

$$\text{Rate of return} = \frac{40\,315.88 - 34\,600}{34\,600}$$

$$\text{Rate of return} = 0.165\dots$$

Jason's portfolio will have a rate of return of about 17%.

Malique's portfolio:

The TFSA has a future value of \$6976.62.

The CSBs have a future value of \$5892.88.

The savings account has a future value of \$26 007.87.

$$\text{Portfolio value} = 6976.62 + 5892.88 + 26\,007.87$$

$$\text{Portfolio value} = 38\,877.37$$

Malique's portfolio has a future value of \$38 877.37.

$$\text{Rate of return} = \frac{38\,877.37 - 34\,600}{34\,600}$$

$$\text{Rate of return} = 0.123\dots$$

Malique's portfolio will have a rate of return of about 12%.

Jason's portfolio will have a higher rate of return than Malique's portfolio.

The rate of return for an investment portfolio, at any point in time, is the ratio of how much it has gained in value to how much was originally invested.

I began by determining how much was invested by Jason and Malique, including the principal amounts of the single payment investments and the total of the regular payments.

I used the financial application on my graphing calculator to determine the future value of each investment portfolio:

- For the single payment investments, I entered the term (in years), the present value, the annual interest rate, and the compounding frequency.
- For the investments involving regular payments, I entered the number of payments, the regular payment amount, the payment frequency, the time in the compounding period when the payments are made (end or beginning), the annual interest rate, and the compounding frequency.

To determine the rate of return, I subtracted the amount invested from the future value and then divided by the amount invested.



Your Turn

What recommendations would you make to help Malique increase her rate of return?

In Summary

Key Ideas

- Rate of return is a useful measure for comparing investment portfolios.
- An investment portfolio can be built from different types of investments, such as single payment investments (for example, CSBs and GICs) and investments involving regular payments. Some of these investments, such as CSBs, lock in money for specified periods of time, thus limiting access to the money, but offer higher interest rates. Other investments, such as savings accounts, are accessible at any time but offer lower interest rates. Investments that involve greater principal amounts invested or greater regular payment amounts when contracted tend to offer higher interest rates.
- The factors that contribute to a larger return on an investment are time, interest rate, and compounding frequency. The longer that a sum of money is able to earn interest at a higher rate compounded more often, the more interest will be earned. For investments involving regular payments, the payment frequency is also a factor.

Need to Know

- Financial applications on calculators or spreadsheets and online financial tools at banking websites are valuable tools for analyzing and comparing investment portfolios.



CHECK Your Understanding

1. Stan plays in a band. Next year, he wants to have enough money to buy a new guitar. The new guitar costs \$1750, including taxes and shipping. Stan works part time and can afford to save \$15 every week. As well, he has \$300 left from his summer job. He needs an investment portfolio so that he can save money to buy the guitar in a year.
 - a) Why might Stan include a GIC and a high-interest savings account in his portfolio?
 - b) If the GIC earns 5%, compounded annually, and the savings account earns 2.9%, compounded weekly, will he have enough money in a year? If not, how much does he have to save each week?

2. In 2 years, Robin and Leslie are getting married. They anticipate that the wedding will cost about \$10 000.
- They plan to save \$50 a week in an investment account that earns 4.1%, compounded weekly.
 - Their parents have offered to contribute and would like to know how much to give them. They plan to invest their parents' contribution in a CSB that earns 6%, compounded annually.

How much should their parents give them?

PRACTISING

3. Hugh has created the following investment portfolio:
- At the end of each year, for the past 10 years, he has purchased a \$1000 CSB, with an average annual interest rate of 3.4%, compounded annually.
 - He has a trust account that was set up when he was born, 42 years ago, with a single deposit of \$3000. The trust fund earns an average annual interest rate of 4.3%, compounded quarterly.
 - He has a \$10 000 GIC, with a 10-year term, that he purchased 10 years ago and earned 3.95%, compounded semi-annually.

Hugh intends to redeem everything and then invest all the money in a 5-year bond that earns 5.1%, compounded annually. How much will Hugh's bond be worth in 5 years?

4. Paula just turned 18 and is about to start a 3-year college program. She lives with her family, but she still needs about \$2000 each year for expenses.
- Paula has been working part-time for the past 3 years and has deposited \$50 each month into an investment account that earns 2.7%, compounded monthly.
 - When she was born, her parents opened an RESP account that earns 3.2%, compounded monthly. Her parents have deposited \$10 each month into this account.
- a) How much money does Paula have when she starts first year?
- b) Paula decides to redeem her investments when she starts first year, and she withdraws \$2000 for her expenses. She then reinvests the rest of the money in a savings account that earns 3.5%, compounded daily. Will she have enough money for her expenses when she starts second year? Explain.
- c) If Paula withdraws another \$2000 for second year, will she have enough money for third year? If not, how much does she need to save over the summer between second and third year?





5. Gayla and Corey are both 35 years old but have been investing different amounts of money for different lengths of time.
- Gayla saved \$15 each month for 12 years at an average annual interest rate of 3.6%, compounded monthly, until she was 25. Then she reinvested the entire amount at 3.8%, compounded monthly, for 10 years.
 - Corey has been depositing \$18 each month, for the past 10 years, into a savings account that earns an average annual interest rate of 3.8%, compounded monthly.
- a) How much did each woman invest altogether?
b) Predict who has the greater savings. Explain your prediction, and then verify it.
6. Jayne's investment portfolio is described below.
- When Jayne was born, 40 years ago, her parents opened a trust account for her. They invested \$500 at the end of each year into the trust account until she was 20. Since then, there have been no more deposits, but the account has continued to earn interest at an average annual rate of 5%, compounded annually.
 - 10 years ago, Jayne purchased a 10-year \$10 000 GIC that earned 4.4%, compounded semi-annually.
 - 5 years ago, she started buying a 5-year \$1000 CSB at the beginning of each year. The first two CSBs earned 4.7%, compounded annually; the next two CSBs earned 4.8%, compounded annually; and the last CSB earned 4.9%, compounded annually.
- How much is Jayne's investment portfolio worth now? What is her rate of return?
7. When Ela was 20, he started to build an investment portfolio.
- He opened a savings account and invested \$50 a month until he was 40 and earned an average annual rate of 2.7%, compounded monthly.
 - When Ela was 40, he redeemed the savings account and invested the entire amount in a 10-year GIC that earned 4.2%, compounded monthly.
 - At maturity, he reinvested the entire value of the GIC in another 10-year GIC that earned 4.3%, compounded monthly.
 - When he was 40, he also purchased a 10-year \$500 bond that earned 3.9%, compounded annually.
 - He reinvested the money at the same interest rate when the bond matured.
- a) What is the value of Ela's portfolio when he turns 60?
b) What is Ela's rate of return?

8. In 4 years, Jo wants to travel somewhere exotic. She estimates that she will need about \$2000 for her flights and about \$300 a week for 12 weeks of travel. She has savings of \$1200 that she can invest immediately.
- Suppose that she invests her savings in a 4-year GIC that earns 5%, compounded annually. Will she have enough money in 4 years?
 - If not, estimate how much she will need to save each week in a savings account that earns 2.3%, compounded weekly. Explain your estimate, and then determine the exact amount.
9. Derek was laid off, after 20 years of service. His severance pay was \$18 638. Since Derek found another job immediately, he decided to invest his severance pay. Which of the two options would you advise him to choose for the next 10 years? Explain.
- A 10-year \$15 000 GIC at 4.1%, compounded annually, and a high-interest savings account at 3.9%, compounded weekly, for the remaining \$3638
 - A high-interest savings account at 3.9%, compounded daily, for all the severance pay



Closing

10. What factors do you need to keep in mind when creating or evaluating an investment portfolio? Explain.

Extending

11. Betty is retiring and has \$200 000 in savings. She wants to invest the \$200 000 in an investment that will earn interest at 5%, compounded monthly. She wants to receive equal monthly payments from the account for the next 10 years, starting next month. At the end of the 10 years, the account will have a zero balance.
- How much will her payment be each month?
 - How much would her monthly payment be if the interest rate was 7.5%, compounded monthly?
12. Stu's 75-year-old grandmother set up an investment portfolio when she was 20. She invested \$150 every year and earned an average annual interest rate of 8.8%, compounded annually. When she was 70 years old, she redeemed the investment and invested all the money in an account that earned 4.8%, compounded monthly. While the investment earns interest, she receives a monthly allowance from the investment. How much will Stu's grandmother receive altogether in payments if the investment will have a zero balance at the end of 10 years?