

2.1

Analyzing Loans

YOU WILL NEED

- calculator
- financial application
- spreadsheet software

EXPLORE...

- Which loan option would you choose to borrow \$200? Why?
 - A bank loan at 5%, compounded quarterly, to be repaid in a single payment after 1 year.
 - A bank loan at 5%, compounded quarterly, with payments of \$51.57 every 3 months.
 - A loan from a friend to be repaid in one or more payments, totalling \$212, at any time within 1 year.

collateral

An asset that is held as security against the repayment of a loan.

amortization table

A table that lists regular payments of a loan and shows how much of each payment goes toward the interest charged and the principal borrowed, as the balance of the loan is reduced to zero.

GOAL

Solve problems that involve single payment loans and regular payment loans.

INVESTIGATE the Math

Lars borrowed \$12 000 from a bank at 5%, compounded monthly, to buy a new personal watercraft. The bank will use the watercraft as **collateral** for the loan. Lars negotiated regular loan payments of \$350 at the end of each month until the loan is paid off. Lars set up an **amortization table** to follow the progress of his loan.



Lars's Amortization Table

Payment Period (month)	Payment (\$)	Interest Paid (\$) $\left[\text{Balance} \cdot \left(\frac{0.05}{12} \right) \right]$	Principal Paid (\$) [Payment – Interest]	Balance (\$)
0				12 000.00
1	350	50.00	300.00	11 700.00
2	350	48.75	301.25	11 398.75

? How much will Lars still owe at the end of the first year?

- Complete Lars's amortization table for the first year.
- At the end of the first year,
 - how much has Lars paid altogether in loan payments?
 - how much interest has he paid altogether?
 - how much of the principal has he paid back?
- At the end of the first year, what is the balance of Lars's loan?

Reflecting

- Describe and explain what happens to the interest paid and principal paid over the term of the loan.
- Estimate how long it will take for Lars to pay off his loan. Explain your estimate.

APPLY the Math

EXAMPLE 1 Solving for the term and total interest of a loan with regular payments

As described on page 80, Lars borrowed \$12 000 at 5%, compounded monthly. After 1 year of payments, he still had a balance owing.

- In which month will Lars have at least half of the loan paid off?
- How long will it take Lars to pay off the loan?
- How much interest will Lars have paid by the time he has paid off the loan?

Pat's Solution: Using a spreadsheet

- The loan will be at least half paid off when the balance is reduced to \$6000 or less.

	A	B	C	D	E
1	Payment Period (month)	Payment (\$)	Interest Paid (\$)	Principal Paid (\$)	Balance (\$)
2	0				12000
3	1	350	50	300	11700
4	2	350	48.75	301.25	11398.75
20	18	350	28.0266922	321.9733078	6404.43281
21	19	350	26.6851367	323.3148633	6081.11795
22	20	350	25.3379915	324.6620085	5756.45594

I used a spreadsheet and set it up like an amortization table because I wanted to determine values in the middle of the term so I could figure out when half of the loan will be paid off. Also, there were a lot of calculations to do.

I started with a balance of \$12 000, since this is the amount owed.

I used the formula $P \cdot i$ or $\left(\text{Balance} \cdot \frac{0.05}{12}\right)$ to create the spreadsheet formula for the Interest Paid column.

I created a formula for the Principal Paid column that subtracted the interest paid from the payment.

I created a formula for the Balance column that subtracted the principal paid in each row from the previous balance.

I filled down the spreadsheet until the balance went below \$6000. Then I determined the sum of each column.

The loan will be at least half paid off in 20 months.



- b) The loan will be paid off when the balance is reduced to \$0.

	A	B	C	D	E
1	Payment Period (month)	Payment (\$)	Interest Paid (\$)	Principal Paid (\$)	Balance (\$)
2	0				12000
3	1	350	50	300	11700
...
22	20	350	25.33799	324.6620085	5756.45594
...
38	36	350	3.004147	346.9958533	373.999344
39	37	350	1.558331	348.4416694	25.5576746
40	38	25.66	0.10649	25.55350969	0.0041649
41		12975.66	975.6642	11999.99584	

I filled down the spreadsheet until the balance was less than \$350.

I entered the final payment as the balance plus interest:

$$\$25.557... \cdot \left(\frac{0.05}{12}\right) \text{ or } \$25.66.$$

For the final payment in the Interest Paid column, I entered the interest

$$\text{portion: } \$25.557... \cdot \left(\frac{0.05}{12}\right) \text{ or } \$0.106....$$

Then I filled down the rest of the columns to complete the final payment row and determined the sum of each column.

The loan will be paid off in 38 months.

- c) Lars will have paid \$975.66 in interest by the time he has paid off the loan.

The balance was reduced to a rounded value of \$0 after 38 payments or months. At this time, the total principal paid was a rounded value of \$12 000.

Lars's Solution: Using a financial application

- a) The present value is \$12 000.
 The regular payment amount is \$350.
 The payment frequency is 12 times per year.
The number of payments is unknown.
 The payments are made at the end of the payment periods.
 The annual interest rate is 5%.
 The compounding frequency is 12 times per year.
 The future value is \$6000.

I entered these values into the financial application on my graphing calculator to solve for the number of monthly payments.

Half of \$12 000 is \$6000. I used this for the future value.

The number of monthly payments is 19.250..., or 20, which means that I will have half of the loan paid off in 20 months.

19.250... monthly payments means payments of \$350 for 19 months and a much smaller payment in the 20th month.



- b) The present value is \$12 000.
 The regular payment amount is \$350.
 The payment frequency is 12 times per year.
The number of payments is unknown.
 The payments are made at the end of the payment periods.
 The annual interest rate is 5%.
 The compounding frequency is 12 times per year.
 The future value is \$0.

I used \$0 for the future value because this will be the value of my loan when it is paid off.

The number of monthly payments is 37.073..., which means that I will have the loan paid off in 38 months.

37.073... monthly payments means payments of \$350 for 37 months and a much smaller payment in the 38th month.

- c) I will have paid \$975.664... or \$975.66 in interest by the time I have paid off the loan.

Since the number of payments is a decimal amount, I used the sum of interest financial application to determine the total interest charged over 38 payment periods.

Your Turn

Suppose that Lars had decided to make \$400 monthly payments under the same loan conditions. What effect would the greater payments have had on the time to repay the loan and the amount of interest charged?

EXAMPLE 2

Solving for the future value of a loan with a single loan payment

Trina's employer loaned her \$10 000 at a fixed interest rate of 6%, compounded annually, to pay for college tuition and textbooks. The loan is to be repaid in a single payment on the maturity date, which is at the end of 5 years.

- How much will Trina need to pay her employer on the maturity date? What is the accumulated interest on the loan?
- Graph the total interest paid over 5 years. Describe and explain the shape of the graph.
- Suppose the interest was compounded monthly instead. Graph the total interest paid over 5 years. Compare it with your annual compounding graph from part b).



Lainie's Solution

a) $A = P(1 + i)^n$

$n = 5$

$i = 0.06$

$P = 10\,000$

$A = 10\,000(1 + 0.06)^5$

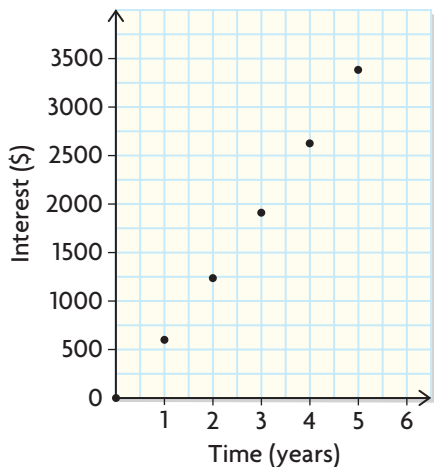
$A = 13\,382.255\dots$

Trina will need to pay her employer \$13 382.26 on the maturity date, including \$3382.26 in accumulated interest.

b) **\$10 000 Loan at 6%,
Compounded Annually**

	A	B	C
1	End of Year	Amount (\$)	Interest (\$)
2	0	10000	0
3	1	10600	600
4	2	11236	1236
5	3	11910.16	1910.16
6	4	12624.77	2624.77
7	5	13382.26	3382.256

**\$10 000 Loan at 6%,
Compounded Annually**



The data is discrete because the interest for each compounding period isn't charged until the end of each year. The relation is not linear because the interest increases each year by more than it increased the previous year.

I knew that I could think of this as a loan (from Trina's perspective) or as an investment (from the employer's perspective). The future value of the investment is also the future value of the loan, which is the amount that Trina will need to repay.

I used the formula $A = P(1 + i)^n$ to determine the future value or amount.

I subtracted the principal from the amount to determine the accumulated interest.

I created a table in a spreadsheet to graph the interest charged each year, from 0 to 5 years.

I used the formula $A = P(1 + i)^n$ to create a spreadsheet formula for the Amount column. I used \$10 000 for P , 0.06 for i , and the values in the End of Year column for n .

I created a formula in the Interest column that subtracted \$10 000 from the Amount in each row.

I graphed the first column as the independent variable and the third column as the dependent variable.

c)

**\$10 000 Loan at 6%,
Compounded Monthly**

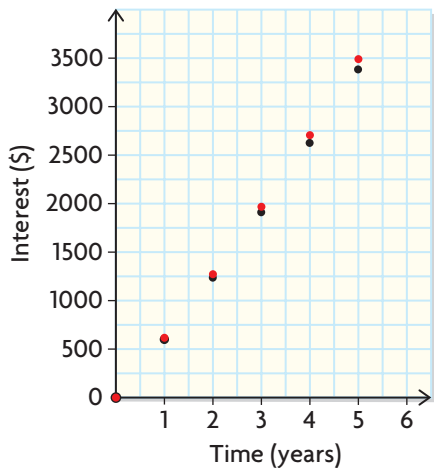
	E	F	G
1	End of Year	Amount (\$)	Interest (\$)
2	0	10000	0
3	1	10616.78	616.7781
4	2	11271.6	1271.598
5	3	11966.81	1966.805
6	4	12704.89	2704.892
7	5	13488.5	3488.502

I predicted that the graph will show the interest accumulating faster due to more frequent compounding.

I decided to determine and plot the points for only whole years and graph the relation on the same axes as the annual compounding graph, using a different colour. I did this by adding to my spreadsheet.

I used the formula $A = P(1 + i)^n$ again to create a spreadsheet formula for the Amount column, but this time I used $\frac{0.06}{12}$ for i and 12 times the value in the End of Year column for n (since the compounding was 12 times a year).

**\$10 000 Loan at 6%,
Compounded Annually vs. Monthly**



I created a graph from the spreadsheet.

In the first few years, there was little or no noticeable difference between the red and black points. As time increased, however, the difference between the points increased.

By the end of year 5, the monthly compounded loan has accumulated \$106.25 more in interest than the annually compounded loan.

Your Turn

Suppose that the interest rate was 10%. How would the graphs of monthly versus annual compounding compare with the graphs above? Why?

EXAMPLE 3**Solving for the present value and interest of a loan with a single payment**

Annette wants a home improvement loan to renovate her kitchen. Her bank will charge her 3.6%, compounded quarterly. She already has a 10-year GIC that will mature in 5 years. When her GIC reaches maturity, Annette wants to use the money to repay the home improvement loan with one payment. She wants the amount of the payment to be no more than \$20 000.



- How much can she borrow?
- How much interest will she pay?

Drew's Solution: Using a formula

$$\text{a) } P = \frac{A}{(1 + i)^n}$$

I knew that I needed to determine how much Annette can borrow (the principal or present value of the loan) for a future value of \$20 000.

Since the interest charged is compound interest and there is only one loan payment involved, I knew that I could use the compound interest formula to determine the present value and solve for P .

$$\begin{aligned} A &= 20\,000 \\ i &= \frac{0.036}{4} \text{ or } 0.009 \\ n &= 5(4) \text{ or } 20 \end{aligned}$$

The future value of the loan is \$20 000.

The interest is 3.6%, compounded quarterly, so I knew that I needed to divide the annual interest rate by 4.

The number of quarterly compounding periods in 5 years is 20.

$$P = \frac{20\,000}{(1 + 0.009)^{20}}$$

I substituted the values into the formula and determined the present value of the loan.

$$P = 16\,718.860\dots$$

Annette can borrow a maximum of \$16 718.860... or \$16 718.86.

$$\text{b) } I = A - P$$

To determine the amount of interest she will pay, I subtracted the present value from the future value.

$$\begin{aligned} I &= 20\,000 - 16\,718.86 \\ I &= 3281.14 \end{aligned}$$

Annette will pay \$3281.14 in interest.



Kikoak's Solution: Using technology

- a) *The present value is unknown.*
The annual interest rate is 3.6%.
The compounding frequency is 4 times per year.
The term is 5 years.
The future value is \$20 000.

Annette can borrow a maximum of \$16 718.860... or \$16 718.86.

I needed to determine how much Annette can borrow (the present value of the loan) for a future value of \$20 000.

I used the financial application on my calculator. I entered the known values to solve for the present value.

b) $I = A - P$

$$I = 20\,000 - 16\,718.86$$

$$I = 3281.14$$

To determine the amount of interest that she will pay, I subtracted the present value from the future value.

Annette will pay \$3281.14 in interest.

Your Turn

- a) How would you have solved the problem if the interest had been simple interest?
- b) Predict whether the interest amount would have been greater or less if the interest had been simple interest. Explain your prediction, and then determine the interest amount to verify your prediction.

EXAMPLE 4

Solving for the payment and interest of a loan with regular payments

Jose is negotiating with his bank for a **mortgage** on a house. He has been told that he needs to make a 10% down payment on the purchase price of \$225 000. Then the bank will offer a mortgage loan for the balance at 3.75%, compounded semi-annually, with a term of 20 years and with monthly mortgage payments.

mortgage

A loan usually for the purchase of real estate, with the real estate purchased used as collateral to secure the loan.

- a) How much will each payment be?
- b) How much interest will Jose end up paying by the time he has paid off the loan, in 20 years?
- c) How much will he pay altogether?



Lu's Solution

- a) Principal of the loan:

$$225\,000 - (0.1)(225\,000) \text{ or } \$202\,500$$

I determined the principal of the loan by subtracting the down payment from the purchase price of the house.

The present value is \$202 500.

The regular payment amount is unknown.

The payments are made 12 times per year.

The number of payments is $12(20)$ or 240.

I entered these values into the financial application on my graphing calculator and solved for the regular payment amount.

I assumed an annual interest rate of 3.75% for the entire term.

The payments are made at the end of the payment periods.

The annual interest rate is 3.75%.

The compounding frequency is 2 times per year.

The future value is \$0.

The regular payment amount will be 1197.548... or \$1197.55.

- b) Jose will end up paying 84 911.644... or \$84 911.64 in interest.

I used the sum of interest financial application to determine the total interest that he will pay over 240 payment periods.

- c) The total payment is the principal or present value plus the interest charged.

$$\$202\,500 + \$84\,911.644... = \$287\,411.644...$$

Jose will pay \$287 411.64 altogether.

Your Turn

For each payment frequency, how does the amount of the regular payment, the total payments, and the total amount of interest change compared to monthly payments?

- bi-monthly payments (every 2 months)
- bi-weekly payments (every 2 weeks)

EXAMPLE 5**Relating payment and compounding frequency to interest charged**

Bill has been offered the following two loan options for borrowing \$8000.

What advice would you give?

Option A: He can borrow at 4.06% interest, compounded annually, and pay off the loan in payments of \$1800.05 at the end of each year.

Option B: He can borrow at 4.06% interest, compounded weekly, and pay off the loan in payments of \$34.62 at the end of each week.

Bronwyn's Solution

Option	A	B
Present Value (\$)	8000	8000
Interest Rate (%)	4.06	4.06
Payments (\$)	1800.05	34.62
Payments/Year	1	52
Compounding/Year	1	52
Term (Years)	?	?

I created a chart to compare the options and to figure out the information that was missing.

It was difficult to predict which option was better. Option B had more frequent payments to decrease the amount of interest, but more frequent compounding to increase the interest.

I decided to compare the amount of interest charged for each option.

Option A:

The present value is \$8000.

The regular payment amount is \$1800.05.

The payment frequency is 1 time per year.

The number of payments is unknown.

The payments are made at the end of the payment periods.

The annual interest rate is 4.06%.

The compounding frequency is 1 time per year.

The future value is \$0.

To determine the total interest for option A, I determined the number of payments using the financial application on my calculator.

I entered the known values, and then solved for the number of payments.

The number of payments is 4.999... or 5.

The total interest paid is 1000.233...

I used the sum of interest financial application to determine the total interest that would be paid over 5 payment periods.

Option A will cost \$1000.23 in interest and will take 5 years to pay off.



Option B:

The present value is \$8000.

The regular payment amount is \$34.62.

The payment frequency is 52 times per year.

The number of payments is unknown.

The payments are made at the end of the payment periods.

The annual interest rate is 4.06%.

The compounding frequency is 52 times per year.

The future value is \$0.

I used the financial application again to determine the number of payments.

The number of payments is 254.929... or 255.

The total interest paid is 825.673....

I used the sum of interest financial application to determine the total interest that would be paid over 255 payment periods.

Option B will cost \$825.67 in interest and will take 255 weeks, or 4 years 47 weeks, to pay off.

My advice is to choose option B.

The amount of interest is lower and, as an added bonus, the loan will take less time to pay off.

Even though option B has more frequent interest compounding, it also has more frequent payments, which results in the loan being paid off sooner and less interest being charged.

Your Turn

Predict how the interest amounts for the following loan options would compare with the interest amounts for options A and B described on page 89. Then verify your predictions by calculating the interest amounts for options C and D.

Option C: \$8000 at 4.06% interest, compounded annually, with the entire loan paid off in a single payment in 5 years.

Option D: \$8000 at 4.06% interest, compounded semi-annually, with payments of \$900.03 every 6 months.

In Summary

Key Ideas

- The large majority of commercial loans are compound interest loans, although simple interest loans are also available.
- The cost of a loan is the interest charged over the term of the loan.
- A loan can involve regular loan payments over the term of the loan or a single payment at the end of the term.
- The same formulas that are used for investment situations are also used for loans with a single payment at the end of the term:
 - For a loan that charges simple interest, $A = P + Prt$ or $A = P(1 + rt)$
 - For a loan that charges compound interest, $A = P(1 + i)^n$
- Technology can be used to determine unknown variables in compound interest loan situations for both single payment loans and regular payment loans.

Need to Know

- The interest that is charged on a loan will be less under any or all of these conditions:
 - The interest rate is decreased.
 - The interest compounding frequency is decreased.
 - Regular payments are made.
 - The regular payment amount is increased.
 - The payment frequency is increased.
 - The term is decreased.
- An amortization table is a payment schedule for a loan with regular payments. It shows what happens in each payment period. It shows the amount of each payment, the interest and the principal portion of each payment, and the balance of the loan. An amortization table can be created with spreadsheet software.

Payment Period	Payment (\$)	Interest Paid (\$)	Principal Paid (\$)	Balance (\$)
0				
1				
2				

- With each payment period, the interest paid decreases while the principal paid increases. This occurs because each payment decreases the balance of the loan, so the interest on the remainder of the balance will be less on the next payment. Also, because the payment amount stays the same, more of the payment goes toward paying off the principal, since less is being paid toward the interest.
- Technology can be used to investigate and analyze “what if” situations that involve borrowing money.

CHECK Your Understanding



1. Alex borrowed \$2500 to help pay for his summer school tuition. His bank offered him a simple interest rate of 2.4%, with the entire amount to be paid in full in 1 year.
 - a) What amount did Alex need to pay back?
 - b) How much interest did Alex need to pay?
2. In November, Holly borrowed \$1200 at 11.2%, compounded monthly, to buy gifts for her family. Holly arranged to pay off the loan in 6 months, with a single payment.
 - a) What amount did Holly need to pay back?
 - b) What amount of interest did Holly pay?
3. Matt is laying new floors in three rooms of his house and needs a loan that he will not have to pay back for 18 months. The interest rate for the loan is 4.9%, compounded quarterly. On the maturity date, Matt wants to make a single payment of no more than \$12 000.
 - a) What is the most that Matt can borrow?
 - b) How much interest will Matt pay on his loan?
4. David mows lawns as a part-time job. He needs to buy a new lawn tractor, which will cost \$6583. The bank offers him a loan at 12.4%, compounded monthly, with payments of \$250 at the end of each month.
 - a) How long will David need to make payments?
 - b) How much interest will he pay?

PRACTISING



5. Perry's bank has approved a personal loan of \$14 000 at 7.5%, compounded quarterly, so that Perry can pave his driveway. Perry wants to repay the loan at the end of 4 years, with a single payment.
 - a) How much will Perry need to pay?
 - b) For each situation below, predict whether Perry would end up paying more or less than the amount in part a). Explain your prediction. Then verify your prediction by calculating how much more or less.
 - i) He took twice the time to repay the loan.
 - ii) He paid off the loan in half the time.
6. Louis, an art dealer, wants to borrow money at 5.6%, compounded monthly, to purchase a soapstone sculpture. He believes that he can sell the sculpture for a profit within a year. Louis wants to make a single payment of no more than \$12 000.
 - a) What is the most that Louis can borrow if he repays the loan at the end of a year?
 - b) How much interest will he pay?

7. Sara and Sylvie have found a small house in the St. Norbert neighbourhood of Winnipeg. They can buy the house for \$179 900. After negotiating with their bank, they have been offered a mortgage for 90% of the cost at 4.5% compounded semi-annually, with regular weekly payments for 15 years.
 - a) How much will the down payment be?
 - b) How much will the principal of the mortgage be?
 - c) What will the regular payment amount be?
 - d) How long will it take before they have paid off half the loan?
 - e) How much interest will they pay in all?

8. Lissa, the owner of a health food store, was advanced \$15 000 by an investor. She signed a promissory note that stated the conditions of the loan: interest will accumulate at a rate of 2.6%, compounded quarterly, and payments of \$1200 will be made at the end of every 3-month period.
 - a) How long will it take Lissa to repay her investor?
 - b) How much interest will Lissa pay?

9. Vicky wants to customize her car so that she can enter some races. She negotiates a loan at 3.8%, compounded weekly, with regular payments of \$25 at the end of each week. She wants to repay the loan in 1 year.
 - a) What is the most she can borrow?
 - b) How much will she pay in interest?

10. Dylan, a hair stylist, has decided to open a home business but needs a loan to renovate. The cost of renovations will include materials, labour, and equipment. He wants to pay off the loan in 5 years by paying no more than \$80 at the end of every week. If the bank offers him an interest rate of 9.5%, compounded weekly, how much can Dylan borrow?

11. Paul wants to buy a new car for \$17 899. The dealership has offered him \$2000 for his old car and has agreed to finance a loan at 2.1%, compounded semi-annually, for 4 years.
 - a) What would Paul's payment be semi-annually?
 - b) Create an amortization table for the loan. When will he have paid off half of the loan?
 - c) How much interest will Paul end up paying altogether?

12. Bernice has a \$30 000 loan at 6.4%, compounded monthly, for 5 years.
 - a) Suppose that she pays off the loan in one payment at the end of the term. How much will she have to pay? How much of this amount will be interest?
 - b) Suppose that she decides to make regular monthly payments instead.
 - i) What will each payment be?
 - ii) What will be the balance on the loan after each of the 5 years?
 - iii) What total interest will she pay by the end of the 5-year term?





13. Violet wants to go to college to become a diesel mechanic. Violet estimates that she will need \$10 000 to pay for tuition and books and \$1500 monthly, for 8 months, to cover her expenses. Her bank has offered her a loan at 1.1%, compounded monthly.
- Suppose that Violet pays off her loan in a single payment a year after she finishes her course. How much interest will she pay?
 - Suppose that Violet makes monthly payments of \$500, starting the month after she finishes the course, until the loan is repaid.
 - How long will it take her to pay off the loan?
 - How much interest will she pay?
14. Frank wants to consolidate his debt of \$37 478 into one loan, with payments at the end of each month. The bank has offered a debt consolidation loan at 4.5%, compounded monthly, for a term of 6 years.
- What will Frank's monthly payments be?
 - What balance is owing at each point in the term?
 - 25%
 - 50%
 - 75%
 - 100%
 - How much interest will have been paid at each point in part b)?
15. For the upcoming season, Mike plans to buy a new biathlon rifle that costs \$2152.



- The sporting goods store has offered to finance the purchase at 16.5%, compounded monthly, for a term of 3 years with payments at the end of each month.
 - Mike could also borrow the money from a bank at 8.5%, compounded weekly, for a term of 2 years with weekly payments.
- How much would the rifle cost if he financed it through the store?
 - How much would the rifle cost if he financed it through the bank?
 - What is the difference in the amount of interest that Mike would pay for the two loans?
 - What features of the loan from the sporting goods store might encourage Mike to choose it over the bank loan?

16. Elise is the owner of Café Patisserie. She needs to upgrade her coffee-making equipment. She has two loan options:
- Her bank has offered her a loan of \$3000 at 4.7%, compounded monthly, with monthly payments of \$125.
 - Her investors have offered her the \$3000 at 5%, compounded monthly, with monthly payments of \$250.
- What is the term of each loan option?
 - How much interest would Elise need to pay for each loan option?
 - What is the total she would pay, including principal and interest, for each loan?
 - What would you advise Elise to do? Justify your recommendation.
17. Connor is negotiating the purchase of a new car and has two options:
- Option A: Borrow \$21 000 at 1.8%, compounded monthly, with a term of 4 years, and pay off the loan by making regular monthly payments.
- Option B: Pay \$5000 at the time of purchase. Borrow \$16 000 at 1.8%, compounded monthly, for a term of 3 years, and pay off the loan with regular monthly payments.
- For each option, what is the regular monthly payment?
 - For each option, what is the total amount of interest?
 - What would you advise Connor to do? Justify your recommendation.

Closing

18. What can you determine about the variables of a loan from its amortization table below?

	A	B	C	D	E
1	Payment Period	Payment (\$)	Interest Paid (\$)	Principal Paid (\$)	Balance (\$)
2	0				5000
3	1	689.93	112.5	577.43	4422.57
4	2	689.93	99.50783	590.4222	3832.15
5	3	689.93	86.22333	603.7067	3228.44
6	4	689.93	72.63993	617.2901	2611.15
7	5	689.93	58.7509	631.1791	1979.97
8	6	689.93	44.54937	645.3806	1334.59
9	7	689.93	30.02831	659.9017	674.69
10	8	689.87	15.18052	674.6895	0.00017
11		5519.38	519.3802	5000	

Extending

19. You and a partner are planning to open a business and need to borrow \$120 000 for start-up costs. The bank has offered you four options for the loan. Which option would you choose? Justify your choice.
- Option A: A term of 20 years at 4%, compounded annually, with annual loan payments
- Option B: A term of 20 years at 4%, compounded monthly, with monthly loan payments
- Option C: A term of 10 years at 8%, compounded annually, with annual loan payments
- Option D: A term of 10 years at 8%, compounded monthly, with monthly loan payments
20. Gabe wants the best loan possible to borrow \$25 000. His bank has offered him two options. Which is the better option? Explain.
- Option A: A loan at an interest rate of 3.5%, compounded monthly, that he could pay back in a single payment at the end of 5 years
- Option B: A loan at an interest rate of 7%, compounded monthly, for which he could make regular monthly payments for 5 years to pay it off
21. Sal borrowed \$50 000 at 3.5%, compounded monthly, and has been paying \$1000 each month for the past 2 years.
- What is the outstanding balance at the end of 2 years?
 - How long will it take Sal to pay off the loan?
 - The conditions of the loan allow Sal to increase his payment amount at any point in the term. Suppose that Sal increases his monthly payment to \$1200 at the end of 2 years. How much sooner will he repay the loan?