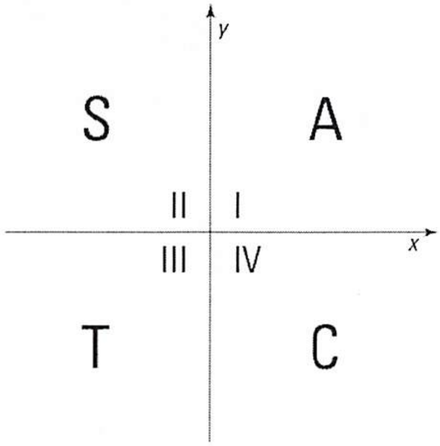
2.2 – TRIGONOMETRIC RATIOS OF ANY ANGLE

In grade 10, we have defined trigonometric ratios for acute angles (between 0º and 90º) in a **right triangle** using SOH CAH TOA.

This year, we will extend the definition to any angle (not necessarily part of a right triangle).

You need to remember that the **sign of a ratio depends on the quadrant** of the terminal arm, and the **numerical value (without the sign) depends only on the reference angle**.

If we remember that the cos and sin values can be found on the unit circle with the coordinates of the points on the circle, and that $tan\ \theta = \frac{\sin\theta}{\cos\theta}$, then we get:



We remember which ratio is positive with the acronym ASTC:

"All Students Take Calculus"

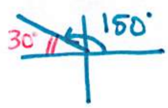
Applications: Determining a Ratio

*I suggest to **always sketch** the situation*

Examples: Finding a Ratio when the reference angles are SPECIAL:

Use the quadrant and the reference angle

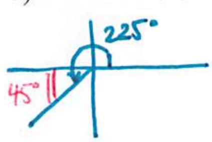
a) determine cos 150°:



$\cos 150° = -\frac{\sqrt{3}}{2}$ ←value of cos for the ref angle

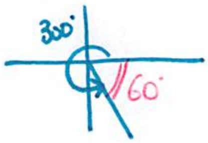
quad II

b) determine sin 225°:



$\sin 225° = -\frac{\sqrt{2}}{2}$ or $-\frac{1}{\sqrt{2}}$

c) determine the 3 trigonometric ratios for 300º:



$\cos 300° = \frac{1}{2}$ $\sin 300° = -\frac{\sqrt{3}}{2}$ $\tan 300° = -\sqrt{3}$

Examples: Finding a Ratio when the reference angles are NOT SPECIAL

You just need to type it in your calculator

Determine an approximation of cos 137° to the nearest hundredth:

$\cos 137° \simeq -0.73$

Note that the sign of the answer makes sense…

Examples: Finding an exact Ratio when we don't know the angle:

You will need to use ***SOH CAH TOA*** *in a right triangle involving the reference angle*

⚠

a) P(-8, 15) is on the terminal arm of an angle θ in standard position.
Determine the exact values of the 3 trigonometric ratios $\sin\theta$, $\cos\theta$ *and* $\tan\theta$.

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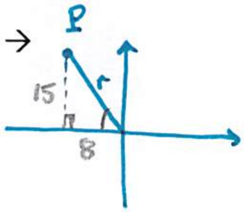

$\cos\theta = -\frac{8}{17}$

$\sin\theta = \frac{15}{17}$

$\tan\theta = -\frac{15}{8}$

Pythagorean theorem

$r^2 = 15^2 + 8^2$

$r^2 = 289$

$r = 17$

Note that if you try to find the angle, you won't get an exact value… (see later)

b) An angle θ is in quadrant III and we know that $\cos\theta = -\frac{3}{4}$.
Determine the exact values of $\sin\theta$ *and* $\tan\theta$

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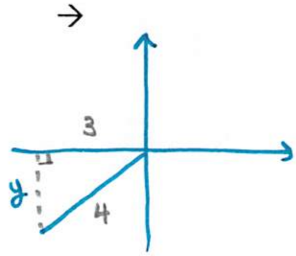


$\sin\theta = -\frac{\sqrt{7}}{4}$

$\tan\theta = \frac{\sqrt{7}}{3}$

Pythagorean theorem

$y^2 = 4^2 - 3^2$

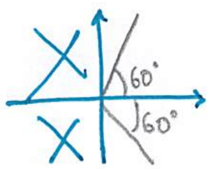
$y^2 = 7$

$y = \sqrt{7}$

Applications: Determining Angles given a Ratio – Solving Equations

Examples: when the Ratios are SPECIAL

a) Solve $\cos\theta = \frac{1}{2}$ for $0° \le \theta \le 360°$

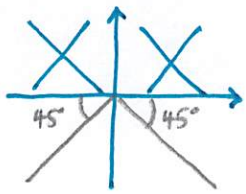


quad I or IV

$\theta_R = 60°$

$\Rightarrow \theta = 60°$ or $300°$

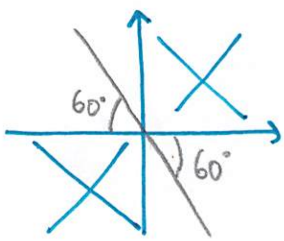
b) Solve $\sin\theta = -\frac{\sqrt{2}}{2}$ for $0° \le \theta \le 360°$



$\theta_R = 45°$

$\Rightarrow \theta = 225°$ or $315°$

c) Solve $\tan\theta = -\sqrt{3}$ for $0° \le \theta \le 360°$



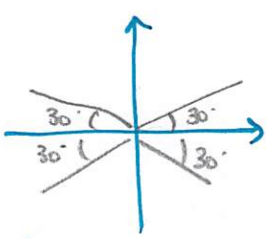
$\theta_R = 60°$ (because $\sqrt{3}$ comes from $\frac{\sqrt{3}/2}{1/2} = \frac{\sin\theta}{\cos\theta}$)

$\Rightarrow \theta = 120°$ or $300°$

d) Solve $3\tan^2\theta = 1$ for $0° \le \theta \le 360°$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$



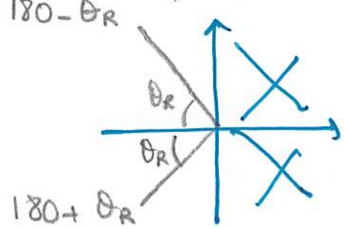
$\theta_R = 30°$ (because $\frac{1}{\sqrt{3}}$ comes from $\frac{1/2}{\sqrt{3}/2} = \frac{\sin\theta}{\cos\theta}$)

$\Rightarrow \theta = 30°$ or $150°$ or $210°$ or $330°$

All quadrants

Examples: when the Ratios are NOT special

a) Solve $\cos\theta = -0.3$ for $0° \le \theta \le 360°$

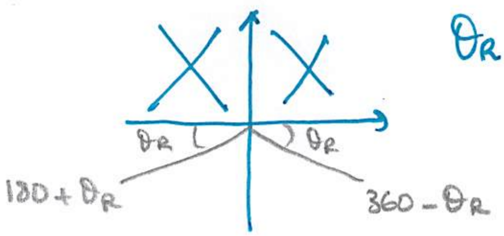


$\theta_R = \cos^{-1}(0.3)$

$\simeq 72.5°$

$\Rightarrow$ 107.5° or 252.5°

b) Solve $\sin\theta = -\frac{1}{3}$ for $0° \le \theta \le 360°$



$\theta_R = \sin^{-1}\left(\frac{1}{3}\right)$

$\simeq 19.5°$

$\Rightarrow \theta \approx 199.5°$ or $340.5°$

Hwk: p 96 # 1 – 13, 15, 18, 19, 22, 29.