2.3 – The Sine Law

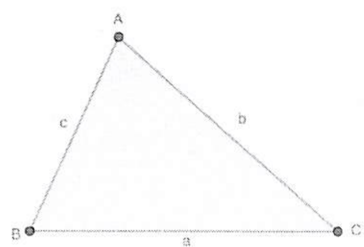
Reminders :

- An **oblique triangle** is a triangle that has no known right angle.
- **Solving a triangle** means determining the measures of all of its side and angles.

Up until now, we only have used trig ratios in right triangles (SOH CAH TOA).

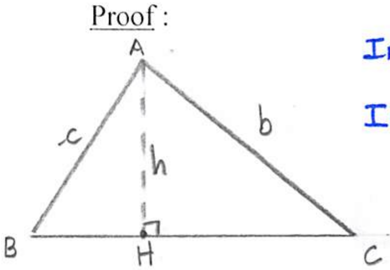
The **Sine Law** can be applied on any type of triangle.

Using standard notations, the **Sine Law** states:



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Proof :



In right triangle ABH, we have $\sin B = \frac{h}{c}$ i.e $h = c \times \sin B$

In right triangle $\triangle ACH$, we have $\sin C = \frac{h}{b}$ i.e $h = b \times \sin C$

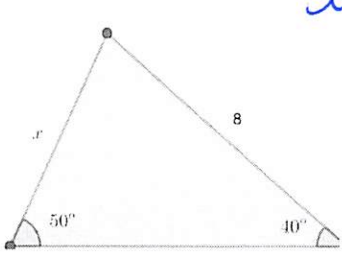
Therefore: $c \sin B = b \sin C$

i.e $\frac{\sin B}{b} = \frac{\sin C}{c}$

If we repeat the same process using the height from vertex B, we get: $\frac{\sin A}{a} = \frac{\sin C}{c}$. As a conclusion: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Applications :

- **Determining a side length :**



Using sine Law, we get: $\frac{\sin 50}{8} = \frac{\sin 40}{x}$

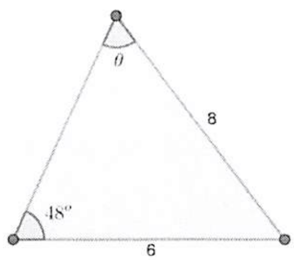
i.e $x = \frac{8 \sin 40}{\sin 50}$ (cross multiplication)

$\boxed{x \simeq 6.7}$

Your turn :p 103

- **Determining an angle :**

a) Determining an acute angle :



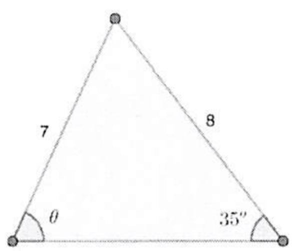
$$\frac{\sin 48}{8} = \frac{\sin\theta}{6} \quad \therefore \quad \sin\theta = \frac{6\sin 48}{8}$$

Therefore : $\theta_R = \sin^{-1}\left(\frac{6\sin 48}{8}\right)$

Since the angle is acute, we have

$$\theta = \theta_R \simeq 34°$$

Your turn :

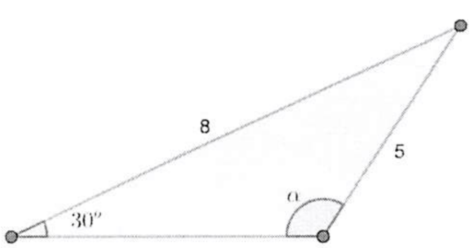


$$\frac{\sin\theta}{8} = \frac{\sin 35}{7} \quad \therefore \quad \sin\theta = \frac{8\sin 35}{7}$$

Therefore : $\theta_R = \sin^{-1}\left(\frac{8\sin 35}{7}\right)$

Acute angle $\Rightarrow \theta = \theta_R \simeq 41°$

b) Determining an obtuse angle :



$$\frac{\sin 30}{5} = \frac{\sin\alpha}{8} \quad \therefore \quad \sin\alpha = \frac{8\sin 30}{5}$$

Therefore : $\alpha_R = \sin^{-1}\left(\frac{8\sin 30}{5}\right)$

Obtuse angle $\Rightarrow \alpha = 180 - \alpha_R$

$\alpha \simeq 127°$

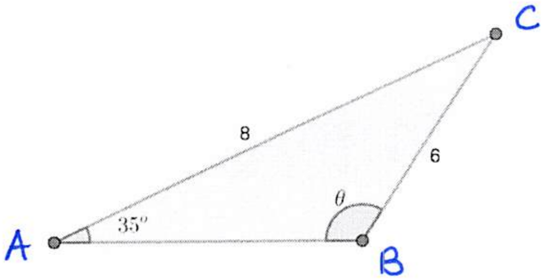
Hwk : p 108 # 1 – 4, 10, 11, 24

c) The Ambiguous Case

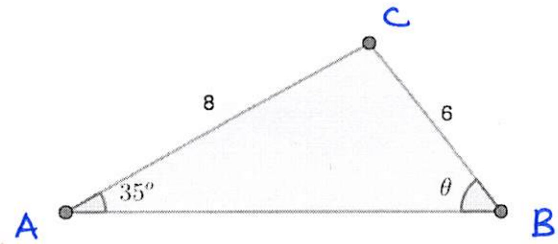
Sometimes, the info we get is not enough to know if we're looking for an acute or an obtuse angle (either because we can't see the angle of it is not drawn to scale…). In this case, we need to solve both possibilities and give 2 different answers.

Example : Consider triangle $\triangle ABC$ with standard notations such that $\angle A = 35°$, $a = 6$cm and $b = 8$cm. Determine $\angle B$.

→ Here, we don't know which of the 2 options (drawn on the next page)Ici, on ne peut pas savoir à laquelle des options ci-aprèwe are dealing with. We don't know if $\angle B$ is acute or obtuse…



or



In both cases, we have: $\frac{\sin 35}{6} = \frac{\sin \theta}{8}$

$\therefore \sin\theta = \frac{8 \sin 35}{6}$

$\theta_R = \sin^{-1}\left(\frac{8\sin 35}{6}\right)$

$\theta = 180 - \theta_R$
$\theta \simeq 130°$

$\theta = \theta_R$
$\theta \simeq 50°$

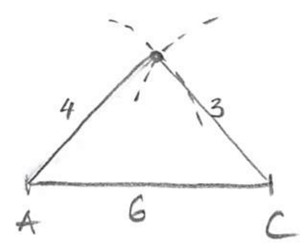
- **<u>Determining the number of possible triangles, knowing side lengths and angles:</u>**

If we are given less than 3 measurements, there are usually an infinity of possible triangles.

<u>a) If we are given 3 side lengths:</u>
There is exactly 1 possible triangle as long as the sum of 2 side lengths is always greater than the 3rd one.
We can draw the triangle using a compass.
<u>Example:</u> $\triangle$ABC such that $a = 3$cm, $b = 4$cm and $b = 6$cm.

① draw 1 side

② use compass to find where the 2 other sides intersect



<u>b) If we are given 2 side lengths and 1 angle:</u>

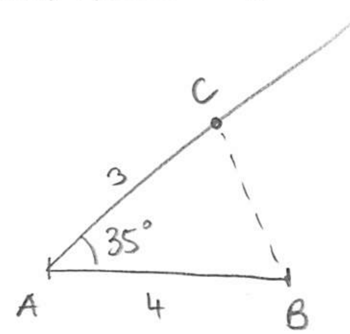
- If the given angle is between the 2 side lengths, then there is only 1 possible triangle.

<u>Example :</u> $\triangle$ABC such that $\angle A = 35°$, $b = 3$cm and $c = 4$cm.

① draw 1 side

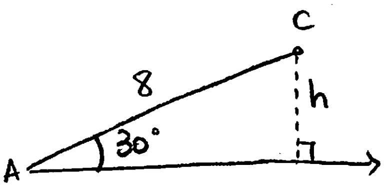
② draw the angle (protractor)

③ measure the other side



- If NOT, ATTENTION there can be 0, 1 or 2 possible triangles !!
  We will need to determine the height of the triangle to see if the given length is long enough.

Example : $\triangle ABC$ such that $\angle A = 30°$, $a = 3$cm and $b = 8$cm.



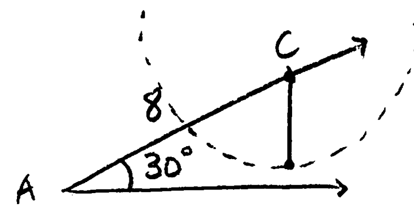
$\sin 30 = \frac{h}{8}$

$h = 8 \sin 30°$

$h = 4$ ← min length required for triangle to exist

① draw angle

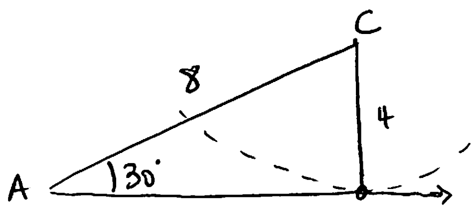
② draw the adjacent side (TIP: the one going up ...)



too short to intersect !

⇒ 0 triangle !

Example : $\triangle ABC$ such that $\angle A = 30°$, $a = 4$cm and $b = 8$cm.

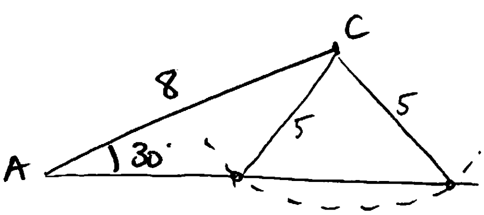


exactly h !

exactly 1 intersection

⇒ 1 (right) triangle

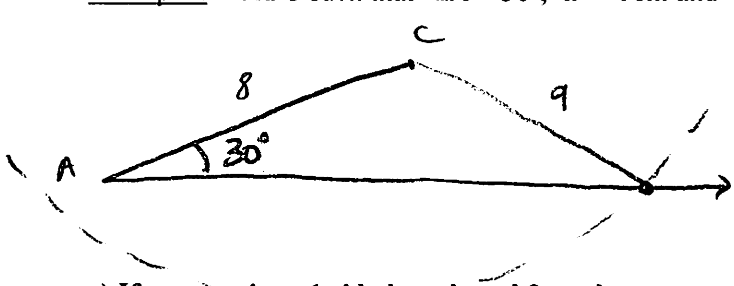
Example : $\triangle ABC$ such that $\angle A = 30°$, $a = 5$cm and $b = 8$cm.



2 possible triangles

(ambiguous case)

Example : $\triangle ABC$ such that $\angle A = 30°$, $a = 9$cm and $b = 8$cm.



only 1 intersection

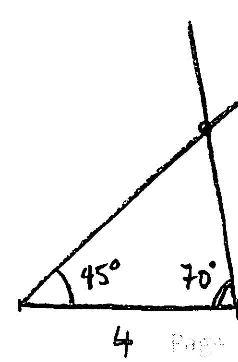
⇒ 1 oblique triangle.

c) If we are given 1 side length and 2 angles :

2 angles, is equivalent to 3 angles (because the sum of the 3 angles equals 180°).
There will be exactly 1 possible triangle. We will trace it using a protractor.

Example : $\triangle ABC$ such that $\angle A = 45°$, $\angle B = 65°$ and $b = 4$cm.

↳ we already know that $\angle C = 70°$



1 possible point for B

Hwk : p 108 # 5, 8, 9, 17

⇒ 1 triangle.