**5.3 – RADICAL EQUATIONS**

**Restrictions on the variable:**

By definition, it is impossible to talk about the square root of a negative number.

For example, we can’t write: $\sqrt{-4}$ because it doesn’t make any sense for real numbers.

The square root of a negative number is a different type of number (imaginary number) which you haven’t learned about yet…

Remember: For even index, the radicand has to be positive or zero.

 For odd radicals, there is no problem. All radicands are possible.

Examples:

a) $\sqrt{-9}$ is not a real number. And we won’t write it!

b) $\sqrt[3]{-8}$ , $\sqrt{5}$ and $\sqrt[5]{2}$ are real numbers.

Application: Determine the restrictions on the variable for the following radicals.

1. $\sqrt{2x-3}$ 4. $\sqrt[3]{2x-3}$
🡪 $\sqrt{2x-3}$ exists if and only if $2x-3\geq 0$. 🡪 no restrictions for a cube root!
 $2x\geq 3$
 $x\geq \frac{3}{2}$
2. $\sqrt[4]{16-x}$ 5. $\sqrt{2x+1}-3\sqrt{x-4}$
🡪 $16-x\geq 0$
 $x\leq 16$
3. $\sqrt{x^{2}-9}$
🡪 $x^{2}-9\geq 0$
 (Be careful: quadratic inequality can only be solved with a graph or sign analysis!)
 $x\leq -3$ or $x\geq 3$

Your turn: Determine the restrictions on the variable for the following radicals.
a) $\sqrt[4]{x+5}$ d) $\sqrt[3]{x+1}$

b) $\sqrt{2-5x}$ e) $\sqrt{x^{2}-x-6}$

c) $\sqrt{x+2}+\sqrt{x-3}$ f) $\sqrt{x-3}+\sqrt{8-x}$

**Solving Radical Equations:**

It comes in 3 different steps:

1. **RESTRICTIONS:** We determine the restrictions on the variable (which means for which values of *x* each expression involved in the equation makes sense). We make sure that no denominator will equal zero and that no radicand will be negative.
2. **ISOLATE THE RADICAL:** In order for it to disappear when we will square (or cube …) both sides of the equation.
3. **TEST POTENTIAL SOLUTIONS:** Because when we squared both sides, some solutions have potentially been added, and we need to cancel them.

Examples :

1. $5+\sqrt{2x-1}=12$
* Restrictions :
* Resolution :
* Tests :

Solution :

1. $x-\sqrt{5-x}=-7$
* Restrictions :
* Resolution :
* Tests :

 Solution :

1. $7+\sqrt{3x}=\sqrt{5x+4}+5$
* Restrictions :
* Resolution :
* Tests :

 Solution :

Hwk : p 300 # 1, 4, 6 – 10, 12 – 14, 17, 18, 21