Name: ______________________________ Precalc 11

5.3 – RADICAL EQUATIONS

Restrictions on the variable:

By definition, it is impossible to talk about the square root of a negative number.
For example, we can't write: $\sqrt{-4}$ because it doesn't make any sense for real numbers.
The square root of a negative number is a different type of number (imaginary number) which you haven't learned about yet…

Remember: For even index, the radicand has to be positive or zero.
For odd radicals, there is no problem. All radicands are possible.

Examples:
a) $\sqrt{-9}$ is not a real number. And we won't write it!
b) $\sqrt[3]{-8}$, $\sqrt{5}$ and $\sqrt[5]{2}$ are real numbers.

Application: Determine the restrictions on the variable for the following radicals.

1. $\sqrt{2x-3}$
   → $\sqrt{2x-3}$ exists if and only if $2x-3 \geq 0$.
   $2x \geq 3$
   $x \geq \frac{3}{2}$
2. $\sqrt[3]{2x-3}$
   → no restrictions for a cube root!
3. $\sqrt[4]{16-x}$
   → $16 - x \geq 0$
   $x \leq 16$
4. $\sqrt{x^2-9}$
   → $x^2 - 9 \geq 0$ -3 3
   (Be careful: quadratic inequality can only be solved with a graph or sign analysis!)
   $x \leq -3$ or $x \geq 3$

5. $\sqrt{2x+1} - 3\sqrt{x-4}$
↳ $2x+1 \geq 0$ & $x-4 \geq 0$
$x \geq -\frac{1}{2}$ $x \geq 4$
$-1/2$ 4
$D = \{x \in \mathbb{R} \mid x \geq 4\}$

Your turn: Determine the restrictions on the variable for the following radicals.

a) $\sqrt[4]{x+5}$ $x+5 \geq 0$ $\boxed{x \geq -5}$

b) $\sqrt{2-5x}$ $2-5x \geq 0$ $5x \leq 2$ $\boxed{x \leq \frac{2}{5}}$

c) $\sqrt{x^2-x-6}$ $x^2-x-6 \geq 0$ -2 3 $\boxed{x \leq -2 \text{ or } x \geq 3}$

d) $\sqrt[3]{x+1}$ NO restriction

e) $\sqrt{x+2} + \sqrt{x-3}$ $x \geq -2$ & $x \geq 3$ $\boxed{x \geq 3}$

f) $\sqrt{x-3} + \sqrt{8-x}$ $x \geq 3$ & $x \leq 8$ $\boxed{3 \leq x \leq 8}$

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Solving Radical Equations:

It comes in 3 different steps:

1. **RESTRICTIONS:** We determine the restrictions on the variable (which means for which values of x each expression involved in the equation makes sense). We make sure that no denominator will equal zero and that no radicand will be negative.
2. **ISOLATE THE RADICAL:** In order for it to disappear when we will square (or cube …) both sides of the equation.
3. **TEST POTENTIAL SOLUTIONS:** Because when we squared both sides, some solutions have potentially been added, and we need to cancel them.

Examples :

1) $5+\sqrt{2x-1}=12$

- Restrictions : $2x-1\geq 0$ $x\geq \frac{1}{2}$
- Resolution : $\sqrt{2x-1}=7$

$2x-1=49$

$x=25$

- Tests : $5+\sqrt{2x-1}=12$

$5+\sqrt{49}$	12
12	✓

Solution :

$\{25\}$

2) $x-\sqrt{5-x}=-7$

- Restrictions : $5-x\geq 0$ $x\leq 5$
- Resolution : $\sqrt{5-x}=x+7$

$5-x=x^2+14x+49$

$0=x^2+15x+44$

$\Delta=49$

$x=-11$ or $x=-4$ $\longrightarrow$

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- Tests : $x - \sqrt{5-x} = -7$

$x - \sqrt{5-x}$	-7
$-11 - \sqrt{5+11}$	-7
$-11 - \sqrt{16}$	
-15	x

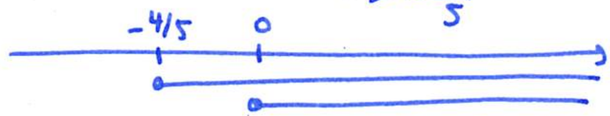
$x - \sqrt{5-x}$	-7
$-4 - \sqrt{5+4}$	-7
$-4 - 3$	
-7	✓

Solution : $\{-4\}$

3) $7 + \sqrt{3x} = \sqrt{5x+4} + 5$

- Restrictions : $3x \geq 0$ and $5x + 4 \geq 0$

$x \geq 0$ $\quad$ $x \geq -\frac{4}{5}$



$x \geq 0$

- Resolution : $\sqrt{5x+4} = \sqrt{3x} + 2$

$5x + 4 = 3x + 4\sqrt{3x} + 4$

$4\sqrt{3x} = 2x$

$2\sqrt{3x} = x$

$4 \times 3x = x^2$

$x^2 - 12x = 0$

$x(x-12) = 0$

$x = 0$ ou $x = 12$

- Tests : $7 + \sqrt{3x} = \sqrt{5x+4} + 5$

$7 + \sqrt{3x}$	$\sqrt{5x+4} + 5$
$7 + \sqrt{0}$	$\sqrt{4} + 5$
7	7 ✓

$7 + \sqrt{3x}$	$\sqrt{5x+4} + 5$
$7 + \sqrt{36}$	$\sqrt{64} + 5$
13	13 ✓

Solutions: $\{0; 12\}$

Hwk : p 300 # 1, 4, 6 – 10, 12 – 14, 17, 18, 21