6.1 – Rational Expressions

Definition :
A **Rational Expression** is an algebraic expression than can be written as a fraction of polynomials.

Ex : $\frac{3}{x+1}$; $\frac{x^2-1}{3x-2}$; $4x^2+2x+1$; $3-\frac{2}{x}$; ...

When dealing with rational expressions, we need to determine its **domain**, i.e. determining the values of x where the expression exists. The expression won't exist for values of x where leading to a denominator that equals 0.

Examples : a) $\frac{5x+1}{3x-4}$ This expression doesn't exist if $3x-4=0$

Restrictions: $3x-4 \neq 0$

$3x \neq 4$

$x \neq \frac{4}{3}$

Domain : $\left\{x \in \mathbb{R}, x \neq \frac{4}{3}\right\}$ or $\mathbb{R} \setminus \left\{\frac{4}{3}\right\}$

b) $\frac{3}{(x+1)(x-2)}$

Restrictions: $(x-1)(x-2) \neq 0$

$x-1 \neq 0$ and $x-2 \neq 0$

$x \neq 1$ and $x \neq 2$

Domain : $\{x \in \mathbb{R}, x \neq 1; x \neq 2\}$ or $\mathbb{R} \setminus \{1; 2\}$

c) $\frac{2x-1}{x^2-x-12}$

Restrictions : $x^2-x-12 \neq 0$

By factoring or using the quadratic formula, we get :

$x \neq -3$ and $x \neq 4$

Domain: $\{x \in \mathbb{R}, x \neq -3; x \neq 4\}$ ou $\mathbb{R} \setminus \{-3; 4\}$

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If there is a common factor on the numerator and the denominator of a rational expression, you can **simplify** it. You'll get an **equivalent** expression, but it won't necessarily have the same domain. We often say they are equal, but technically they aren't for all values of x...

Examples : $\frac{3(x+1)}{(x-5)(x+1)}$ and $\frac{3}{x-5}$ are equivalent. with $D_1 = \mathbb{R} \setminus \{-1; 5\}$ and $D_2 = \mathbb{R} \setminus \{5\}$

i.e. $\frac{3x+3}{x^2-4x+5}$ and $\frac{3}{x-5}$ are equivalent.

Remember that a Rational Expression can only be **simplified** if it is **FACTORED**.
And you need to determine the **restrictions** on the variable **before simplifying** it.

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Example 1 : $\frac{x^2-2x-3}{x^2-6x+9}$

Restrictions : $x^2-6x+9 \neq 0$

$(x-3)^2 \neq 0$

$x \neq 3$

Simplification : $\frac{x^2-2x-3}{x^2-6x+9} = \frac{(x-3)(x+1)}{(x-3)^2} = \boxed{\frac{x+1}{x-3}}$

Example 2 : $\frac{2x^2-x-1}{x^2-1}$

Restrictions : $x^2-1 \neq 0$

$(x+1)(x-1) \neq 0$

$x \neq \pm 1$

or $x^2 \neq 1$

$x \neq \pm 1$

Simplification : $\frac{2x^2-x-1}{x^2-1} = \frac{(2x+1)(x-1)}{(x+1)(x-1)} = \boxed{\frac{2x+1}{x+1}}$

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