6.3 – Adding and Subtracting Rational Expressions

Like for numerical fractions, to add or subtract fractions, they need to be on a **common denominator**…

In order to find the LEAST common denominator, we will have to **FACTOR** all the denominators to see which factors are missing in each fraction.

Examples : $A = \frac{3}{x+1} + \frac{5x}{2x-3}$

Restrictions : $x+1 \neq 0$ and $2x-3 \neq 0$ $\quad D = \mathbb{R} \setminus \{-1, \frac{3}{2}\}$

Simplification : $A = \frac{3(2x-3)}{(x+1)(2x-3)} + \frac{5x(x+1)}{(x+1)(2x-3)}$

$= \frac{3(2x-3)+5x(x+1)}{(x+1)(2x-3)}$

$= \frac{6x-9+5x^2+5x}{(x+1)(2x-3)} = \frac{5x^2+11x-9}{(x+1)(2x-3)}$

can't be simplified because can't be factored w integers ($\Delta = 301$)

$B = \frac{x^2-20}{x^2-4} + \frac{x-2}{x+2}$

Restrictions : $x^2-4 \neq 0$ and $x+2 \neq 0$

$(x+2)(x-2) \neq 0 \qquad x \neq -2$

$x \neq \pm 2$ $\quad D = \mathbb{R} \setminus \{\pm 2\}$

Simplification : $B = \frac{x^2-20}{(x+2)(x-2)} + \frac{(x-2)(x-2)}{(x+2)(x-2)}$

$= \frac{x^2-20+x^2-4x+4}{(x+2)(x-2)}$

$= \frac{2x^2-4x-16}{(x+2)(x-2)} = \frac{2(x-4)(x+2)}{(x+2)(x-2)} = \frac{2(x-4)}{x-2}$

always try to factor to simplify…

ATTENTION : We will sometimes have to add brackets when subtracting several terms…

Ex : $C = \frac{3}{x+5} - \frac{x-4}{x+5}$ $\qquad (x \neq 5)$

$= \frac{3-(x-4)}{x+5}$

$= \frac{3-x+4}{x+5}$ $\qquad C = \frac{-x+7}{x+5}$

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$D = \frac{5x}{x+1} - \frac{7x+1}{x-1}$

Restrictions : $x+1 \neq 0$ and $x-1 \neq 0$ $\quad D = \mathbb{R} \setminus \{\pm 1\}$

Simplification : $D = \frac{5x(x-1)}{(x+1)(x-1)} - \frac{(7x+1)(x+1)}{(x+1)(x-1)}$

$= \frac{5x(x-1) - (7x+1)(x+1)}{(x+1)(x-1)}$ ⚠ "−" before (...)

$= \frac{5x^2 - 5x - (7x^2 + 7x + x + 1)}{(x+1)(x-1)}$ can't be simplified ($\Delta = 161$)

$= \frac{5x^2 - 5x - 7x^2 - 8x - 1}{(x+1)(x-1)} = \frac{-2x^2 - 13x - 1}{(x+1)(x-1)}$

Fractions inside fractions…

$E = \frac{1+\frac{1}{x}}{x-\frac{1}{x}}$

Restrictions : $x \neq 0$ and $x - \frac{1}{x} \neq 0$

$\frac{x^2}{x} - \frac{1}{x} \neq 0$ i.e $\frac{x^2-1}{x} \neq 0$ i.e $x \neq \pm 1$

$D = \mathbb{R} \setminus \{-1, 0, 1\}$

Simplification :

$E = \frac{\frac{x}{x} + \frac{1}{x}}{\frac{x^2}{x} - \frac{1}{x}}$

$= \frac{\frac{x+1}{x}}{\frac{x^2-1}{x}}$

$= \frac{x+1}{x} \times \frac{x}{x^2-1}$

$E = \frac{(x+1)\,x}{x(x+1)(x-1)}$

$= \frac{1}{x-1}$

ATTENTION :

We will have to present our work very clearly. The main fraction line has to be clearly positioned. ex : $\frac{\frac{2}{3}}{4} \neq \frac{\frac{2}{3}}{4}$

$\frac{2}{\frac{3}{4}} = 2 \times \frac{4}{3} = \frac{8}{3}$

$\frac{\frac{2}{3}}{4} = \frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$

Your turn p 335

Hwk : p 336 # 3, 6 – 12, 15ab, 16, 18, 19a, 20 – 23.

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