

## 6.4 – Rational Equations

When we want to solve a rational equation (equation made of rational expressions), the easiest is if all the terms are written on the **same denominator**...

Ex:  $\frac{2}{7} + \frac{x}{7} = \frac{5}{7}$  Here the solution is clearly  $x = 3$ .

When all the expressions have the same denominator, we only need to make the equality between the numerators be true... Be careful about the forbidden values of the variable from the **restrictions** on the **ORIGINAL** equation...

Examples: 1)  $\frac{2}{x^2-4} + \frac{10}{6x+12} = \frac{1}{x-2}$

Restrictions:  $x^2 - 4 \neq 0$        $6x + 12 \neq 0$        $x - 2 \neq 0$   
 $x \neq \pm 2$                        $x \neq -2$                        $x \neq 2$

$D = \mathbb{R} \setminus \{\pm 2\}$

Resolution:  $\frac{2}{(x+2)(x-2)} + \frac{\cancel{10}^5}{3\cancel{6}(x+2)} = \frac{1}{x-2}$   
 $\frac{2 \times 3}{3(x+2)(x-2)} + \frac{5(x-2)}{3(x+2)(x-2)} = \frac{3(x+2)}{3(x+2)(x-2)}$

$$6 + 5(x-2) = 3(x+2)$$

$$6 + 5x - 10 = 3x + 6$$

$$2x = 10$$

$$\boxed{x = 5}$$

Restrictions checked!

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2)  $\frac{4x-1}{x+2} - \frac{x+1}{x-2} = \frac{x^2-4x+24}{(x+2)(x-2)}$

Restrictions:  $x+2 \neq 0$        $x-2 \neq 0$        $D = \mathbb{R} \setminus \{\pm 2\}$

Resolution:  $\frac{(4x-1)(x-2)}{(x+2)(x-2)} - \frac{(x+1)(x+2)}{(x+2)(x-2)} = \frac{x^2-4x+24}{(x+2)(x-2)}$

$$4x^2 - 8x - x + 2 - (x^2 + 2x + x + 2) = x^2 - 4x + 24$$

$$4x^2 - 9x + 2 - x^2 - 3x - 2 = x^2 - 4x + 24$$

$$2x^2 - 8x - 24 = 0$$

$$2(x^2 - 4x - 12) = 0$$

$$2(x+2)(x-6) = 0$$

$x = -2$   
 Restr x

$$\boxed{x = 6}$$

Your Turn 343

**Applications :**

1) Shared Work

Marco wants to plant potatoes in his field. Alone, it would take him 4h.  
 But Tina is willing to help him. Alone, it would take her 3h.  
 How long is it going to take them if they work together?

Fraction of work done in 1h:

Marco :  $\frac{1}{4}$

Tina :  $\frac{1}{3}$

Together :  $\frac{1}{x}$

$$\frac{1}{4} + \frac{1}{3} = \frac{1}{x}$$

$$\frac{3x}{12x} + \frac{4x}{12x} = \frac{12}{12x}$$

$$7x = 12$$

Let  $x$  be the time it will take together

$$0 < x < 3$$

$$x = \frac{12}{7} \text{ h}$$

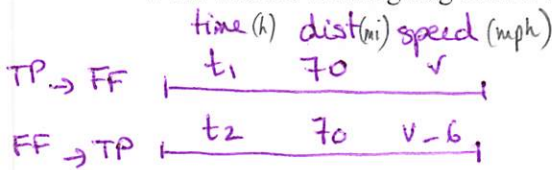
$$x \approx 1\text{h}43\text{min}$$

2) Time/Distance/Speed

Last year, a dog race was non-stop from The Pas to FlinFlon and back (in Manitoba).  
 The total distance was 140 mi. Conditions were excellent on the way to FlinFlon. However, bad weather caused the winner's average speed to decrease by 6 mph on the return trip.  
 The total time for the trip was 8h30min.

What was the winning dog team's average speed on the way to FlinFlon?

$$v = \frac{d}{t} \quad \boxed{t = \frac{d}{v}} \quad d = vt$$



$$t_1 = \frac{70}{v}$$

$$t_2 = \frac{70}{v-6}$$

$$t_1 + t_2 = 8.5$$

$$\boxed{v > 6}$$

↑  
to be able to reduce it by 6mph...

$$\frac{70}{v} + \frac{70}{v-6} = \frac{17}{2}$$

$$\frac{2 \times 70(v-6)}{2v(v-6)} + \frac{2 \times 70v}{2v(v-6)} = \frac{17v(v-6)}{2v(v-6)}$$

$$v = \frac{382 \pm \sqrt{88804}}{34}$$

$$140v - 840 + 140v = 17v^2 - 102v$$

$$17v^2 - 382v + 840 = 0$$

$$\Delta = 88804$$

$$v \approx 2.5 \text{ mph}$$

Restr<sub>x</sub>

$$\boxed{v = 20 \text{ mph}}$$

Hwk: Day 1 p 348 # 2, 3, 5, 8, 10, 20, 21, 26, 27.

Day 2 p 348 # 12 - 14, 16, 17 + Extra Practice handout