## FREQUENTLY ASKED Questions

Q: How does the future value of a single payment investment compare with the future value of an investment involving regular payments?

A: How they are alike:
Each payment in a regular payment investment is like a single payment investment. The future value of the regular payment investment is the sum of the future values of all the payments. For example, the timeline below shows an investment involving \$100 payments made at the end of each year at $5 \%$ interest, compounded annually, for 4 years.


How they are different:
If the four $\$ 100$ payments had been invested as a single payment, under the same conditions, the future value would be greater than $\$ 431.01$. This is because the entire amount invested in a single payment investment earns compound interest for the entire term. In contrast, if an investment involves regular payments, only the first payment earns interest for the full term. Each payment after that earns interest over less time.

Study Aid

- See Lesson 1.5, Examples 1 to 5 .
- Try Chapter Review Questions 8 to 10.


## Study Aid

- See Lesson 1.6, Examples 1 to 3.
- Try Chapter Review Questions 11 and 12.


## Q: How can you evaluate or compare investment portfolios that are made up of multiple investments?

A: You can use their rates of return. The rate of return is the ratio of interest earned to the amount invested. It is particularly useful when comparing investments in which the principal or the term is different. For example, an investor is considering the following two portfolio options. Portfolio B has a greater amount invested but for a shorter term than portfolio A. Which portfolio will have the greater rate of return?

## Portfolio A

$\$ 25000$ invested for 4 years:
i) $\$ 20000$ in a tax-free savings account, earning an average annual interest rate of $1.3 \%$, compounded annually each year for 4 years
ii) A $\$ 5000$ Canada Savings Bond at $5 \%$, compounded annually

## Portfolio B

$\$ 30000$ invested for 3 years:
i) $\$ 10000$ earning an average annual interest rate of $3 \%$, compounded monthly
ii) A \$20 000 GIC earning $2.5 \%$ simple interest

Determining rate of return
i) Future value: 21060.46

Interest earned: $21060.46-20000$ or 1060.46
ii) Future value: 6077.53

Interest earned: $6077.53-5000$ or 1077.53
Total interest earned: $1060.46+1077.53$ or 2137.99
Total amount invested: 25000
Rate of return: $\frac{2137.99}{25000}$ or $8.55 \%$
Determining rate of return
i) Future value: 10940.51

Interest earned: 10940.51 - 10000 or 940.51
ii) Future value: 21500

Interest earned: $21500-20000$ or 1500
Total interest earned: $940.51+1500$ or 2440.51
Total amount invested: 30000
Rate of return: $\frac{2440.51}{30000}$ or $8.14 \%$

Portfolio A has a greater rate of return than portfolio B.

## PRACTISING

## Lesson 1.1

1. When Trish was 12 , her grandmother bought her a $\$ 1500$ CSB that earned simple interest.
a) When Trish turned 18, her CSB was worth $\$ 2850$. At what rate did the CSB earn interest?
b) If the interest was paid annually and Trish redeemed the CSB when she was 18 and a half years old, how much would she get? Explain.
2. Steve is celebrating his 18 th birthday.

- On his 5th birthday, his grandmother bought him a $\$ 10000$ GIC that earns $6.3 \%$ simple interest.
- On the same birthday, his grandfather bought him a $\$ 7000$ CSB that earns $11.4 \%$ simple interest.

a) What is the value of each investment now?
b) Graph both investments on the same grid to show how the values of the investments change over time.
c) What conclusion can you draw from comparing the graphs?


## Lesson 1.2

3. Examine these two investments, and then answer the questions below.

- Sonia invested in a $\$ 2000$ GIC that earns $6.2 \%$ simple interest, paid annually, for 5 years.
- Trent bought a $\$ 2000$ GIC that earns $5.3 \%$, compounded monthly, for a 5-year term.
a) Predict which investment will have the greater rate of return. Explain.
b) Verify your prediction.
c) Explain the difference in the interest earned on the two investments.


## Lesson 1.3

4. James and Johnny received equal inheritances of $\$ 2000$, which they invested for 5 years at $7.4 \%$. James's account compounded semi-annually, and Johnny's account compounded weekly.
a) Predict who will earn more interest. Verify your answer.
b) Compare their rates of return.
5. Kyle invested $\$ 900$ in an account that earned interest, compounded daily. After 1 year, her investment was worth $\$ 1000$. What was the annual rate of interest?

## Lesson 1.4

6. Phil and his daughter Lina opened accounts at different times. Each account earned 6.5\%, compounded semi-annually.

- Phil kept his account for 18 years and now has $\$ 125000$ in the account.
- Lina kept her account for 36 years and now has $\$ 125000$ in the account.
a) Who invested the greater principal? How much more did he or she invest?
b) If Lina had invested the same principal as Phil, what would be the future value of her account after 36 years?

7. Mel and his friend Mike each made an investment.

- Mel invested, 10 years ago, in a bond that matures this year. The bond currently has a value of \$13 140 and has been earning $6.5 \%$, compounded annually.
- Mike invested in a 10 -year GIC that also matures this year. Mike's investment is currently worth $\$ 13009$ and has been earning $9.6 \%$, compounded monthly.
a) Who invested the greater principal 10 years ago?
b) Graph both investments on the same grid.
c) Make a prediction about what will happen to the future values of the two investments. Explain.


## Lesson 1.5

8. Mary decided to invest $\$ 800$ per month for the next 6 years. She plans to start her own business making lunches for the elementary schools in her area, but she needs to save enough money for her start-up costs.

- Bank A has offered her $12.2 \%$, compounded monthly.
- Bank B has offered her $11.4 \%$, compounded monthly.
If Mary chooses bank $A$, how much more money will she end up with?


9. Two brothers, Josh and Jeff, held investments that earned $6 \%$, compounded annually. Both of them made regular payments into their investments until they were 65 .

- Josh started making yearly payments of $\$ 1000$ when he was 20 .
- Jeff did not start until he was 50 , but made annual deposits of $\$ 3000$.
a) What is the future value of each investment?
b) How much did each man invest altogether?
c) How much interest did each man earn?
d) What annual deposit would Jeff have needed to make if he had wanted his investment to have the same future value as Josh's investment at age 65?

10. Drew and John are both planning to buy a new car in 5 years. Each of them started an investment to save the money they will need.

- Drew deposited $\$ 100$ every 2 weeks and earned $4.8 \%$, compounded biweekly.
- John deposited $\$ 217$ every month and earned $4.8 \%$, compounded monthly.
a) Why is it difficult to predict who will be able to purchase the more expensive car in 5 years?
b) Determine who will be able to purchase the more expensive car.


## Lesson 1.6

11. When Chandra was 8 years old, an investment portfolio was started for her education.

- Her parents deposited $\$ 450$ every 3 months into a savings account that earns $4.5 \%$, compounded quarterly.
- Her grandparents invested $\$ 5000$ in a trust account that earns an average annual interest rate of $6 \%$, compounded annually.
a) Chandra plans to redeem her portfolio when she turns 18 to pay for university.
i) What will be the value of her portfolio?
ii) What was the portfolio's rate of return?
b) If she withdraws $\$ 10000$ each year for university and then invests what is left each time at $6.2 \%$, compounded annually, will she have enough for 4 years?

12. a) Predict which portfolio will have the greater rate of return over 10 years. Explain your prediction.
b) Verify your prediction. Was your prediction correct?

## Portfolio 1:

- A 10 -year $\$ 25000$ GIC, purchased 10 years ago, that earns $8.7 \%$, compounded annually
- A 10-year \$10 000 CSB , purchased 10 years ago, that earns $6.4 \%$ simple interest
- An investment involving deposits of $\$ 2500$ at the end of every year for 10 years, which earns $4.9 \%$, compounded annually


## Portfolio 2:

- A 10 -year $\$ 25000$ GIC, purchased 10 years ago, that earns $8.7 \%$, compounded monthly
- A 10-year \$10 000 CSB , purchased 10 years ago, that earns $6.4 \%$, compounded annually
- An investment involving deposits of $\$ 1250$ at the end of every 6 months for 10 years, which earns $4.9 \%$, compounded semi-annually

