Quadratic Functions in Standard Form

A **quadratic function** is a function that is a degree 2 polynomial.

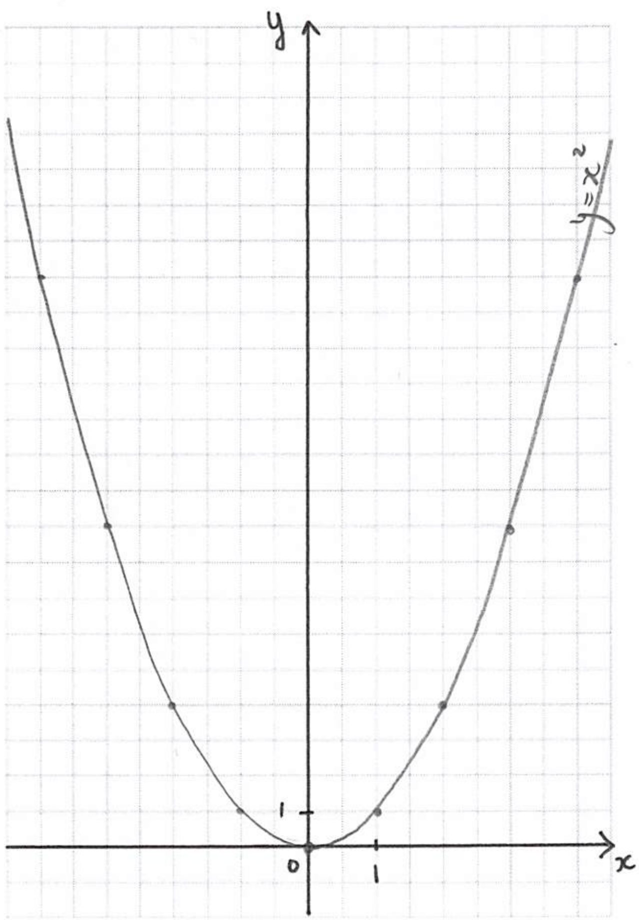
Ex : $f(x) = 3x^2 - 5x + 1$
$g(x) = -\frac{1}{2}x^2 + 5$

I – The Reference quadratic function : $y = x^2$

Table of values :

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

Graph :



Axis of symmetry : $x = 0$ (the y - axis)

Vertex : (0,0)

Opening upwards

Domain : $\{x \in \mathbb{R}\}$

Range : $\{y \in \mathbb{R}, y \geq 0\}$

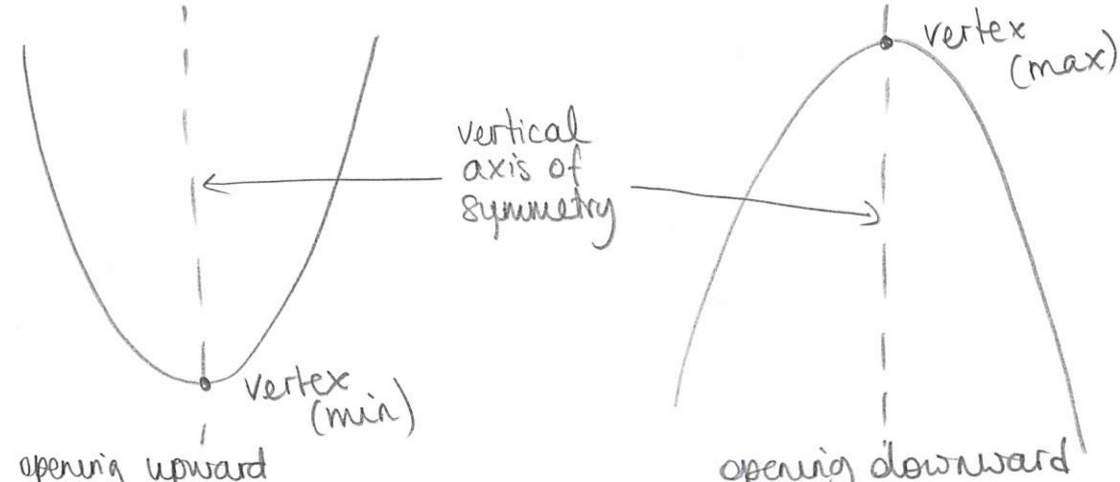
This type of graph is called a **PARABOLA**.

II – Quadratic Functions in Standard Form : $y = ax^2 + bx + c$

Ex : $y = 3x^2 + x - 6$ → $a = 3$, $b = 1$, $c = -6$
$y = -x^2 + x + 6$ → $a = -1$, $b = 1$, $c = 6$

} look at their graphs on your calculator.

The graph of any quadratic function is a **parabola** which can open upwards or downwards:



To graph a quadratic function, you need to determine the coordinates of its vertex, the direction of opening and the "speed" of its opening (or "width of opening").

- **Direction of opening** : The sign of coefficient a tells us if the parabola opens upwards or downwards.

$\cup$ if $a > 0$ ex $y = 3x^2 - 5x + 1$

$\cap$ if $a < 0$ ex: $y = -2x^2 + x - 24$

- **Coordinates of the vertex** : The calculation $\boxed{\frac{-b}{2a}}$ gives the x-coordinate of the vertex $\left(\frac{-b}{2a}, \quad \right)$
Then, you just need to replace x by that value to find the corresponding y value.

Example : $y = x^2 - 4x + 2$

↳ $a = 1$ $b = -4$ $c = 2$

- the parabola opens upward
- $\frac{-b}{2a} = \frac{+4}{2} = 2$ If $x = 2$, then $y = 2^2 - 4(2) + 2$
$= -2$

vertex $(2, -2)$

Note : The y-intercept is the value of y when $x = 0$.

In Standard form, it always is the value of coefficient c.
Ex : for $y = x^2 - 4x + 2$ the y-intercept is 2.

To finish graphing the parabola, you can either create a table of values (by hand of with a graphing calculator) or use the reference function.

Example 1 : Using a table of values.

$y = x^2 - 4x + 2$

min

We already figured out that the parabola opens upwards, that its vertex is $(2, -2)$ and its y-intercept is 2.

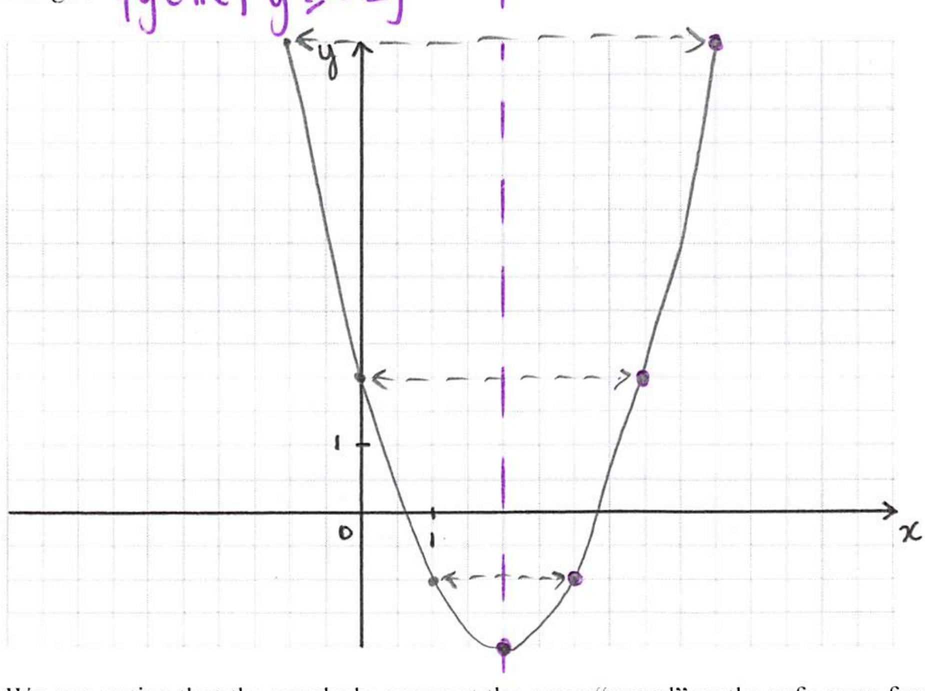
We also know that parabolas have a vertical axis of symmetry that goes through the vertex (here $x = 2$). Therefore, if we can find some points on one side of the vertex, we can position the "same" points on the other side easily. We will fill a table of values with x values only on one side of the vertex…

x	2	3	4	5
y	-2	-1	2	7

Domain : $\mathbb{R}$

Range : $\{y \in \mathbb{R} \mid y \geq -2\}$

axis of symmetry
$x=2$



We can notice that the parabola opens at the same "speed" as the reference function … that's because they have the same a coefficient.

Your turn : $y = 2x^2 + 6x - 1$

vertex $\left(-\frac{3}{2}, -\frac{11}{2}\right)$

x	$-3/2$	-1	0	1	2
y	-5.5	-5	-1	7	19

y-int.

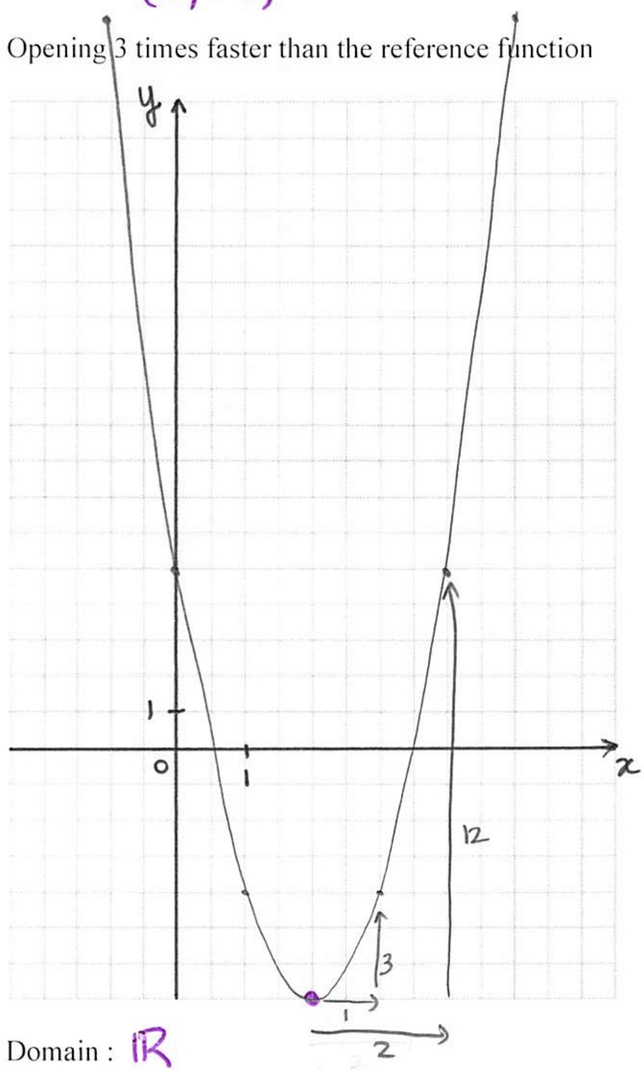
Example 2 : Using the « speed of opening » of the reference function.

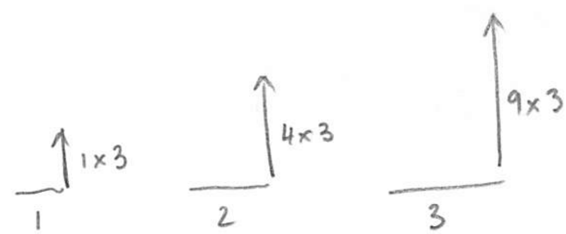
$y = 3x^2 - 12x + 5$

Opening upwards

Vertex : $(2, -7)$

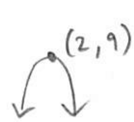
Opening 3 times faster than the reference function





Domain : $\mathbb{R}$

Range : $\{y \in \mathbb{R}, y \geq -7\}$



Your turn : $y = -x^2 + 4x + 5$ verif. w calc.

Note : In « real life » situations, domain might have to be restricted for all values to make sense…

Hwk : p 174 # 1 – 12, 15, 23