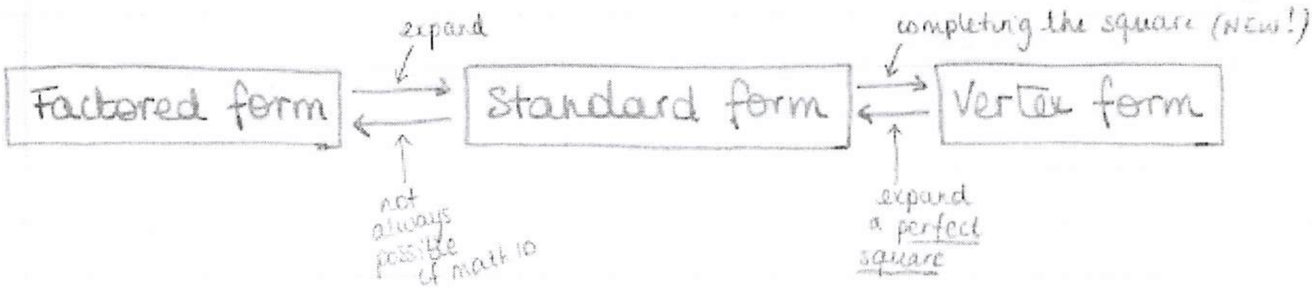
Changer la forme d'une fonction quadratique



I – Changing into Standard Form : EXPANDING

Example 1 : $y = (x-3)(x+2)$

$y = x^2 + 2x - 3x - 6$

$y = x^2 - x - 6$

Example 2 : $y = -3(x-5)(x+1)$

$y = -3(x^2 + x - 5x - 5)$

$y = -3x^2 + 12x + 15$

Your Turn : $y = (x-7)(x-1)$ → $a = 1, b = -8, c = 7$

$y = -2(x+1)(x+4)$ → $a = -2, b = -10, c = -8$

Example 3 : $y = (x-1)^2 + 3$

$y = x^2 - 2x + 1 + 3$

$y = x^2 - 2x + 4$

⚠ $(x-1)^2 = (x-1)(x-1)$

$= x^2 - x - x + 1$

$= x^2 - 2x + 1$ ($-2x$: double product)

⚠ $(x-1)^2 \neq x^2 - 1$

Example 4 : $y = -2(x+3)^2 - 5$

$y = -2(x^2 + 6x + 9) - 5$

$y = -2x^2 - 12x - 18 - 5$

$y = -2x^2 - 12x - 23$

Example 5 : $y = -9\left(x - \frac{1}{3}\right)^2 + 4$

$y = -9\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) + 4$

$y = -9x^2 + 6x - 1 + 4$

$y = -9x^2 + 6x + 3$

II – Changing into Factored Form: FACTORING

Always start by looking for common factors.
Think about differences of squares, perfect squares, otherwise, factor the long way…

Example 1 : $y = 3x^2 - 9x$

$y = 3x(x-3)$

Example 2 : a) $y = 3x^2 - 75$

$y = 3(x^2 - 25)$
$y = 3(x+5)(x-5)$

b) $y = 4x^2 - 1$

$y = (2x+1)(2x-1)$

Example 3 : $y = x^2 + 5x + 6$

$\otimes 6$, $\oplus 5$ } 2 & 3

$y = (x+2)(x+3)$

Your turn :
$y = x^2 - x - 6 \rightarrow y = (x-3)(x+2)$
$y = 8x^2 - 2 \rightarrow y = 2(2x+1)(2x-1)$
$y = x^2 + 13x + 12 \rightarrow y = (x+12)(x+1)$

Example 4 : $y = 3x^2 + 8x + 4$

$\otimes 12$, $\oplus 8$ } 2 & 6

$y = 3x^2 + 2x + 6x + 4$
$y = x(3x+2) + 2(3x+2)$
$y = (x+2)(3x+2)$

Example 5 : $y = -6x^2 - x + 2$

$\otimes -12$, $\oplus -1$ } −4 & 3

$y = -6x^2 - 4x + 3x + 2$
$y = -2x(3x+2) + 1(3x+2)$
$y = (-2x+1)(3x+2)$

Example 6 : $y = 4x^2 + 12x + 9$

$(2x)^2$ ↗ $2 \cdot 2x \cdot 3$ ↖ 3^2

$y = (2x+3)^2$

Your turn :
$y = 2x^2 + 3x + 1 \rightarrow y = (x+1)(2x+1)$
$y = 5x^2 - 3x - 2 \rightarrow y = (x-1)(5x+2)$
$y = x^2 - 10x + 25 \rightarrow y = (x-5)^2$

III – Changing into Vertex Form : COMPLETING THE SQUARE

You have to regroup all the terms in x and x^2 in a "forced" perfect square and compensate…

Example 1 : $y = x^2 - 4x + 2$

compensate what you added

$$y = x^2 - 4x + 4 - 4 + 2$$

perfect square

$$y = (x-2)^2 - 2$$

Example 2 : $y = 3x^2 - 18x + 20$

$$y = 3(x^2 - 6x) + 20$$
$$y = 3(x^2 - 6x + 9 - 9) + 20$$
$$y = 3(x-3)^2 - 27 + 20$$
$$y = 3(x-3)^2 - 7$$

Your turn :
$y = x^2 + 6x - 5 \rightarrow y = (x+3)^2 - 14$
$y = -2x^2 + 8x - 3 \rightarrow y = -2(x-2)^2 + 5$
$y = 3x^2 - 3x + 1 \rightarrow y = 3(x - \frac{1}{2})^2 + \frac{1}{4}$

It becomes complicated very fast, especially when fractions start appearing…
We are going to actually use a shortcut:

- Coefficient a is the same one in every form
- If we know how to find the coordinates of the vertex, we just need to place p and q in the formula…

Examples : $y = x^2 + 6x - 5$ | $y = -2x^2 + 8x - 3$ | $y = 3x^2 - 3x + 1$

$y = x^2 + 6x - 5$:
$a = 1$
$\frac{-b}{2a} = -3$
vertex $(-3, -14)$
$y = (x+3)^2 - 14$

$y = -2x^2 + 8x - 3$:
$a = -2$
$\frac{-b}{2a} = 2$
vertex $(2, 5)$
$y = -2(x-2)^2 + 5$

$y = 3x^2 - 3x + 1$:
$a = 3$
$\frac{-b}{2a} = \frac{1}{2}$
vertex $(\frac{1}{2}, \frac{1}{4})$
$y = 3(x - \frac{1}{2})^2 + \frac{1}{4}$

Hwk : p 192 # 1, 2ac, 3ac, 8, 10, 14, 17 – 19, 22, 29, 31 + handout