Chapter 9 REVIEW

1. Solve the following inequalities and give the solutions as an interval.

a) $2x - 3 > 5x + 1$

$2x - 5x > 3 + 1$

$-3x > 4$

$x > -\frac{4}{3}$

Sol: $(-\infty ; -\frac{4}{3})$

b) $3 - 2(x - 5) \leq 2x + 3$

$3 - 2x + 10 \leq 2x + 3$

$-4x \leq -13 + 3$

$-4x \leq -10$

$x \geq \frac{5}{2}$

Sol: $[\frac{5}{2} ; +\infty)$

c) $\frac{2}{3}(5x + 1) < \frac{1}{2}(3x - 6)$

$\frac{10}{3}x + \frac{2}{3} < \frac{3}{2}x - 3$

$20x + 4 < 9x - 18$

$11x < -22$

$x < -2$

sol: $(-\infty ; -2)$

d) $\frac{2x+5}{3} \geq \frac{4x-1}{5}$

$5(2x+5) \geq 3(4x-1)$

$10x + 25 \geq 12x - 3$

$-2x \geq -28$

$x \leq 14$

sol. $(-\infty ; 14]$

$2x - 6x > 8 - 1$

$-4x > 7$

$x < -\frac{7}{4}$

Sol: $]-\infty ; -\frac{7}{4}[$

$5 - 2x + 10 \geq 2x - 3$

$-4x \geq -18$ [illegible]

$x \geq \frac{9}{2}$

Sol: $[\frac{9}{2} ; +\infty[$

$20x - 4 \leq 9x + 18$

$11x \leq 22$

$x \leq 2$

Sol: $]-\infty ; 2]$

[illegible]

$2x \leq 28$

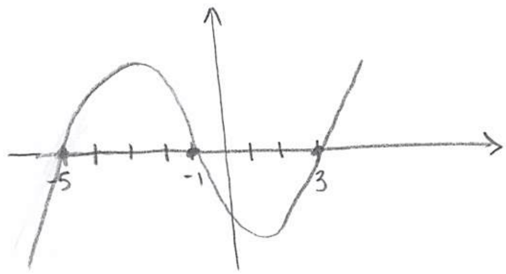
$x \leq 14$

Sol: $]-\infty ; 14]$

2. Determine the signs of the following expressions (and imagine what the graph can look like).

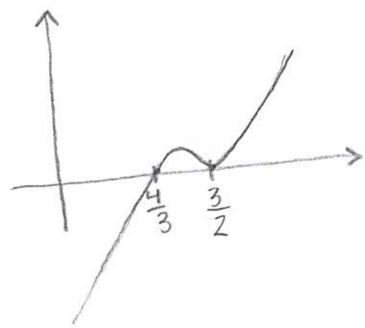
a) $f(x) = (x-3)(5+x)(x+1)$

x	$-\infty$	-5		-1		3	$+\infty$
$x-3$	$-$		$-$		$-$	0	$+$
$5+x$	$-$	0	$+$		$+$		$+$
$x+1$	$-$		$-$	0	$+$		$+$
$f(x)$	$-$	0	$+$	0	$-$	0	$+$



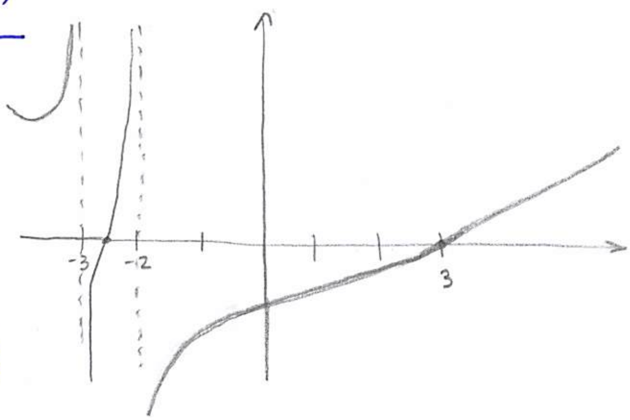
b) $g(x) = (2x-3)^2(3x-4)^3$

x	$-\infty$	$4/3$		$3/2$	$+\infty$
$(2x-3)^2$	$+$		$+$	0	$+$
$(3x-4)^2$	$-$	0	$+$		$+$
$g(x)$	$-$	0	$+$	0	$+$



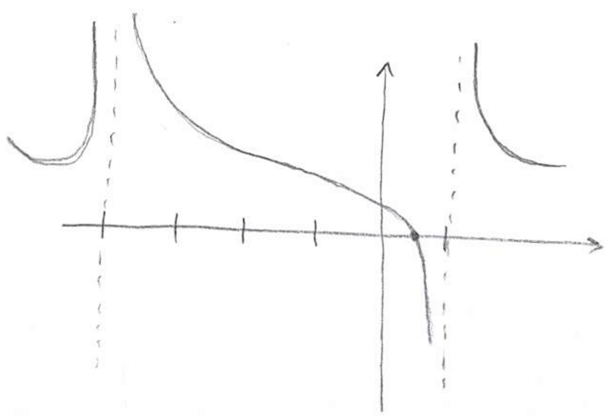
c) $h(x) = \frac{2x^2-x-15}{x^2+5x+6} = \frac{(2x+5)(x-3)}{(x+2)(x+3)}$

x	$-\infty$	-3		$-5/2$		-2		3	$+\infty$
$2x+5$	$-$		$-$	0	$+$		$+$		$+$
$x-3$	$-$		$-$		$-$		$-$	0	$+$
$x+2$	$-$		$-$		$-$	0	$+$		$+$
$x+3$	$-$	0	$+$		$+$		$+$		$+$
$h(x)$	$+$	$\|\|$	$-$	0	$+$	$\|\|$	$-$	0	$+$



d) $k(x) = \frac{2x^2+7x-4}{x^2+3x-4}$ $\frac{(2x-1)(x+4)}{(x+4)(x-1)}$

x	$-\infty$	-4		$1/2$		1	$+\infty$
$2x-1$	$-$		$-$	0	$+$		$+$
$x+4$	$-$	0	$+$		$+$		$+$
$x+4$	$-$	0	$+$		$+$		$+$
$x-1$	$-$		$-$		$-$	0	$+$
$K(x)$	$+$	$\|\|$	$+$	0	$-$	$\|\|$	$+$



3. Solve the following inequalities and write your answer as an interval.

a) $(2-x)(x+5) > 0$

x	$-\infty$		-5		2		$+\infty$
$2-x$		$+$		$+$	0	$-$	
$x+5$		$-$	0	$+$		$+$	
$(2-x)(x+5)$		$-$	0	$+$	0	$-$	

Sol: $(-5;2)$

b) $(2x-5)(x+3)(2x-4) \leq 0$

x	$-\infty$		-3		2		$\frac{5}{2}$		$+\infty$
$2x-5$		$-$		$-$		$-$	0	$+$	
$x+3$		$-$	0	$+$		$+$		$+$	
$2x-4$		$-$		$-$	0	$+$		$+$	
$(2x-5)(x+3)(2x-4)$		$-$	0	$+$	0	$-$	0	$+$	

Sol: $(-\infty;-3] \cup [2;\frac{5}{2}]$

c) $\frac{x^2+x-6}{x+1} < 0$ $\quad x^2+x-6=(x+3)(x-2)$

x	$-\infty$		-3		-1		2		$+\infty$
$x+3$		$-$	0	$+$		$+$		$+$	
$x-2$		$-$		$-$		$-$	0	$+$	
$x+1$		$-$		$-$	0	$+$		$+$	
$\frac{x^2+x-6}{x+1}$		$-$	0	$+$	0	$-$	0	$+$	

Sol: $(-\infty;-3) \cup (-1;2)$

d) $\frac{x^2-4x+4}{x^2+2x-3} \geq 0$ $\quad \frac{(x-2)^2}{(x+3)(x-1)}$

x	$-\infty$		-3		1		2		$+\infty$
$(x-2)^2$		$+$		$+$		$+$	0	$+$	
$x+3$		$-$	0	$+$		$+$		$+$	
$x-1$		$-$		$-$	0	$+$		$+$	
$\frac{x^2-4x+4}{x^2+2x-3}$		$+$	$\Vert$	$-$	$\Vert$	$+$	0	$+$	

Sol: $(-\infty;-3) \cup (1;+\infty)$

4. Test the given values for the following inequalities :

a) $x = -3$ for $-2x^3 + 2x - 5 > 0$

$-2(-3)^3 + 2(-3) - 5 \mid 0$

$54 - 6 - 5$

$43 > 0$

YES!

b) $x = 2$ for $x^4 - 5x^2 + 4 \leq 0$

$2^4 - 5(2)^2 + 4 \mid 0$

$16 - 20 + 4$

$0 = 0$

YES!

c) $x = 2$ for $\frac{-2x^3+2x-5}{x^2-5} > 0$

$\frac{-2(2)^3+2(2)-5}{2^2-5} \mid 0$

$\frac{-16+4-5}{-1}$

$17 > 0$

YES!

d) $x = -3$ for $\frac{3\sqrt{5-x}}{2x-4} > 0$

$\frac{3\sqrt{5+3}}{-6-4} \mid 0$

$\frac{-3\sqrt{8}}{10} < 0$

No!

e) $x = -1$ for $-x^4 - 5x + 12 \geq 0$

$-(-1)^4 - 5(-1) + 12 \mid 0$

$-1 + 5 + 12$

$16 > 0$

YES!

f) $x = -2$ for $\frac{(x+3)(x-2)}{x^2+x+1} \leq 0$

$\frac{(-2+3)(-2-2)}{(-2)^2+(-2)+1} \mid 0$

$\frac{-4}{3} < 0$

YES!

5. A theatre seats 2000 people and charges \$10 per ticket. At this price, all the tickets can be sold. A recent survey indicates that for every \$1 increase in price, the number of tickets sold will decrease by 100. Determine the ticket prices that would result in revenue of at least \$15000 by solving an inequality.

let x be the nb of augmentations of the price

R the revenu

$R(x) = (10 + x)(2000 - 100x)$

$= -100x^2 + 1000x + 20\,000$

$-100x^2 + 1000x + 20\,000 \geq 15\,000$

$100x^2 - 1000x - 5000 \leq 0$

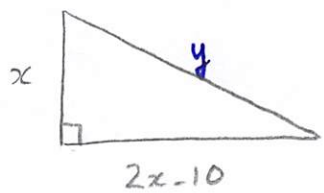
$x^2 - 10x - 50 \leq 0$

Zeros: $\Delta = 100 - 4(-50) = 300$

$x = \frac{10 \pm \sqrt{300}}{2}$ ↗ $x \approx -3.7$ ↘ $x \simeq 13.7$

⇒ between \$7 and \$23

6. In a right triangle, one leg is 10cm shorter than twice the length of the other leg. If the legth of the hypotenuse is at least 8cm, determine the possible lengths of the legs by solving an inequality.



$y^2 = x^2 + (2x-10)^2$

$y^2 = 5x^2 - 40x + 100$

$5x^2 - 40x + 100 \geq 64$

$5x^2 - 40x + 36 \geq 0$

Zeros: $\Delta = 880$

$x = \frac{40 \pm \sqrt{880}}{10}$ → $x_1 \simeq 1.03$, $x_2 \simeq 6.97$

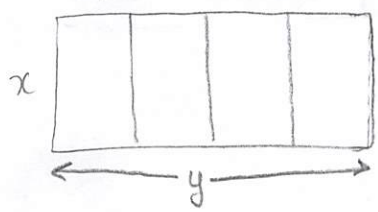
x	$-\infty$		x_1		x_2		$+\infty$
$5x^2-40x+36$		+	0	−	0	+	

⚠ The dimensions need to be positive, so $x > 5$

$\Rightarrow x \in \left[\frac{40+\sqrt{880}}{10} ; +\infty\right)$

$2x - 10 \in \left[\frac{-10+\sqrt{880}}{10} ; +\infty\right)$

7. A farmer wants to build a rectangular pen with some fencing material. He also wants to divide it into four equal compartments using the same fencing material (parallel to the width). If the farmer has only 1000m of fencing, what widths would make the area of the pen at most 15000 m^2?



$A = xy$ $\quad 5x + 2y = 1000$

$y = -\frac{5}{2}x + 500$

$A = -\frac{5}{2}x^2 + 500x$

$-\frac{5}{2}x^2 + 500x \leq 15\,000$

$-5x^2 + 1000x - 30\,000 \leq 0$

$x^2 - 200x + 6000 \geq 0$

Zeros: $\Delta = 16\,000$

$x = \frac{200 \pm \sqrt{16\,000}}{2}$ → $x_1 \simeq 36.75$, $x_2 \simeq 163.25$

x	0		x_1		x_2		200
A		+	0	−	0	+	

$x \in (0 ; x_1] \cup [x_2 ; 200)$

8. Determine 2 numbers whose sum is 30 and whose product is at least 200.

let x and y be the two numbers

$\begin{cases} x + y = 30 \longrightarrow y = 30 - x \\ xy \geq 200 \end{cases}$

↳ $x(30 - x) \geq 200$

$x^2 - 30x + 200 \leq 0$

$(x - 20)(x - 10) \leq 0$

x	$-\infty$		10		20		$+\infty$
$x^2-30x+200$		+	0	−	0	+	

$x \in [10 ; 20]$

the other number $30 - x \in [10 ; 20]$

9. Determine an inequality that would have the given solutions sets.

a) $-\frac{1}{2} < x < 4$

$(x-4)(2x+1) < 0$

$2x^2 - 7x - 4 < 0$

for example.

b) $x \leq -\frac{3}{2}$ et $x \geq \frac{1}{4}$

$(2x+3)(4x-1) \geq 0$

$8x^2 + 10x - 3 \geq 0$

c) $0 \leq x \leq \frac{4}{3}$

$x(3x-4) \leq 0$

$3x^2 - 4x \leq 0$