

Midterm Review – Chapters 3 and 4

1. For each of the following, determine the coordinates of the vertex, the axis of symmetry, the x- and y- intercepts, the domain, and the range.

a) $y = 2(x - 3)^2 - 4$

vertex: $(3, -4)$ y-int: 14 x-int: $2(x-3)^2 = 4$ $R = [-4, +\infty)$
 axis of sym: $x = 3$ $D = \mathbb{R}$ $(x-3)^2 = 2$
 $x-3 = \pm\sqrt{2}$
 $x = 3 \pm \sqrt{2}$

b) $y = -3(x - 2)(x + 4)$

x-int: 2 & -4 $D = \mathbb{R}$ vertex: $(-1; 27)$
 y-int: 24 $R = (-\infty; 27]$ axis of sym: $x = -1$

c) $y = -2x^2 + 8x - 6$

$= -2(x^2 - 4x + 3)$ x-int: 1 & 3 $D = \mathbb{R}$ vertex: $(2; 2)$
 $= -2(x-3)(x-1)$ y-int: -6 axis of sym: $x = 2$ $R = (-\infty; 2]$

d) $y = 3(x - 5)^2 - 3$

vertex: $(5; -3)$ $D = \mathbb{R}$ y-int: 72 x-int: $3(x-5)^2 = 3$
 axis of sym: $x = 5$ $R = [-3; +\infty)$ $(x-5)^2 = 1$
 $x-5 = \pm 1$
 $x = 4$ $x = 6$

e) $y = (2x + 1)(x - 3) = 2x^2 - 5x - 3$

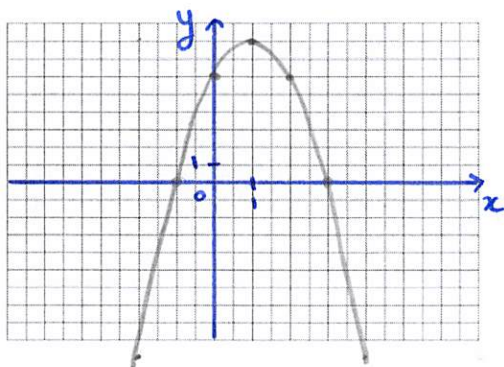
x-int: $-\frac{1}{2}$ and 3 $D = \mathbb{R}$ vertex: $\frac{-b}{2a} = \frac{5}{4}$ $(\frac{5}{4}; -\frac{49}{8})$
 y-int: -3 axis of sym: $x = \frac{5}{4}$ $R = [-\frac{49}{8}; +\infty)$

f) $y = x^2 - 6x - 7$

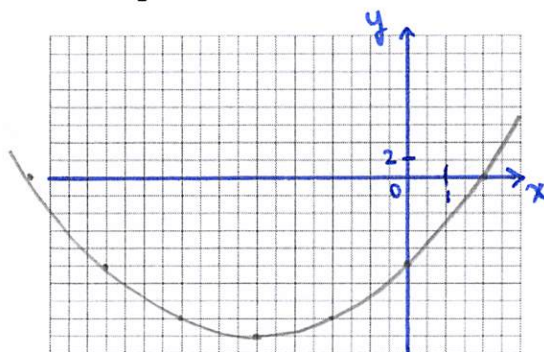
$= (x-7)(x+1)$ $D = \mathbb{R}$ $R = [-16; +\infty)$
 x-int: -1 & 7 vertex: $(3; -16)$ axis of sym: $x = 3$
 y-int: -7

2. Graph the following quadratic functions.

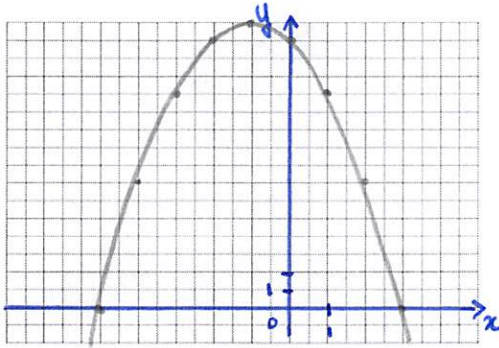
a) $y = -2(x - 1)^2 + 8$



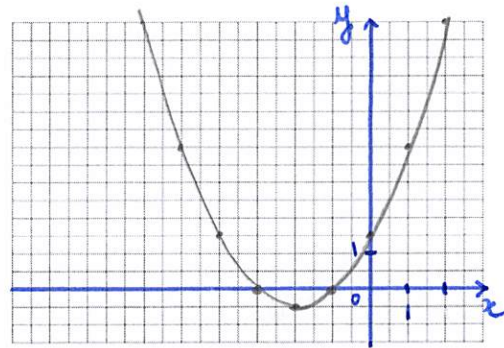
b) $y = \frac{1}{2}(x + 4)^2 - 18$



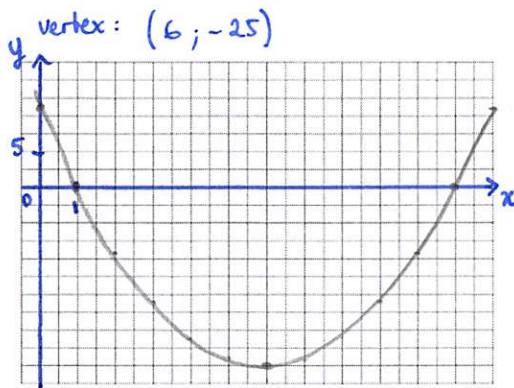
c) $y = -(x - 3)(x + 5)$



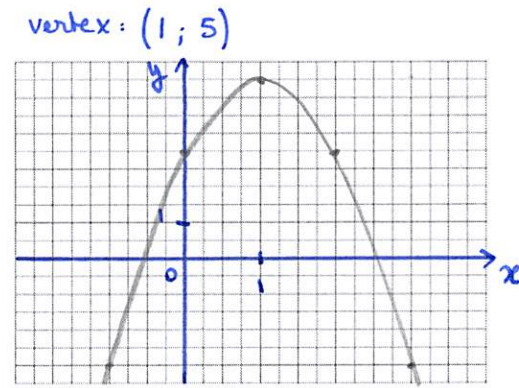
d) $y = \frac{1}{2}(x + 1)(x + 3)$



e) $y = x^2 - 12x + 11$



f) $y = -2x^2 + 4x + 3$



3. Rewrite the following functions in vertex form:

a) $y = x^2 - 6x + 2$

$$\frac{-b}{2a} = \frac{6}{2} = 3 \quad (3; -7)$$

$$y = (x - 3)^2 - 7$$

b) $y = -3x^2 + 12x - 3$

$$\frac{-b}{2a} = \frac{-12}{-6} = 2 \quad (2; 9)$$

$$y = -3(x - 2)^2 + 9$$

c) $y = 4x^2 + 3x - 2$

$$\frac{-b}{2a} = -\frac{3}{8} \quad \left(-\frac{3}{8}; -\frac{41}{16}\right)$$

$$y = 4\left(x + \frac{3}{8}\right)^2 - \frac{41}{16}$$

d) $y = (x - 3)(x + 7)$

zeros: 3 & $-7 \Rightarrow$ vertex at $x = -2$
 $(-2; -25)$

$$y = (x + 2)^2 - 25$$

4. Rewrite the following functions in general form:

a) $y = -2(x + 1)(2x - 3)$

$$y = -2(2x^2 - 3x + 2x - 3)$$

$$y = -4x^2 + 2x + 6$$

b) $y = 3(x + 1)^2 - 4$

$$y = 3(x^2 + 2x + 1) - 4$$

$$y = 3x^2 + 6x - 1$$

c) $y = \frac{1}{2}(3x + 5)(x - 2)$

$$y = \frac{1}{2}(3x^2 - 6x + 5x - 10)$$

$$y = \frac{3}{2}x^2 - \frac{1}{2}x - 5$$

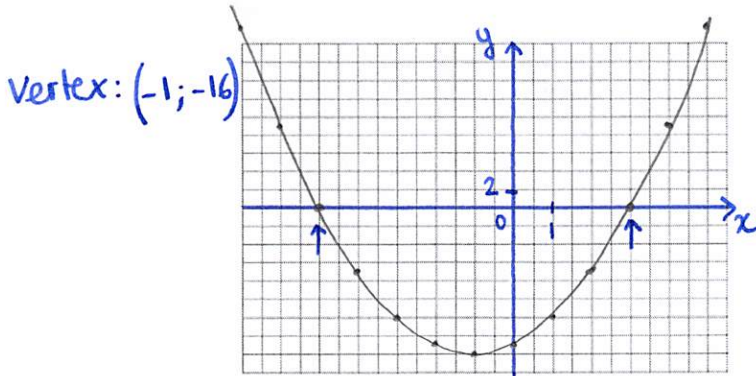
d) $y = \frac{1}{4}(x + 5)^2 + 2$

$$y = \frac{1}{4}(x^2 + 10x + 25) + 2$$

$$y = \frac{1}{4}x^2 + \frac{5}{2}x + \frac{33}{4}$$

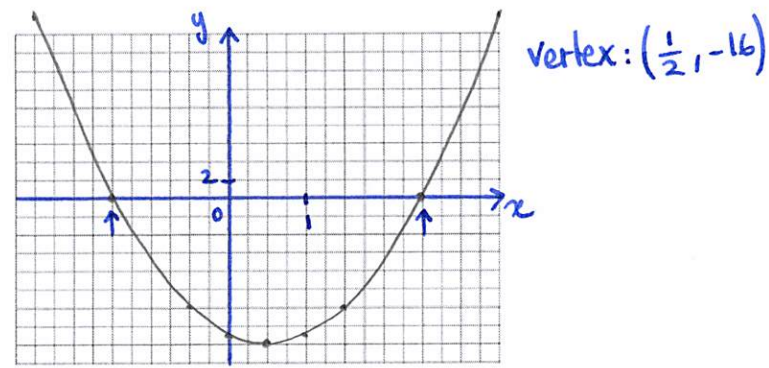
5. Solve the following equations graphically:

a) $x^2 + 2x - 15 = 0$



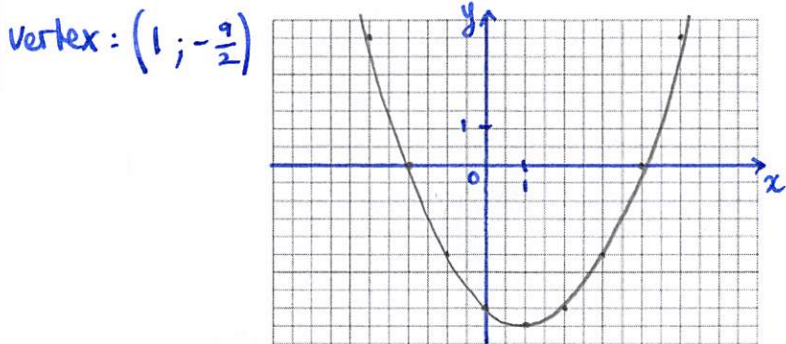
solutions: $\{-5; 3\}$

b) $4\left(x - \frac{1}{2}\right)^2 - 16 = 0$



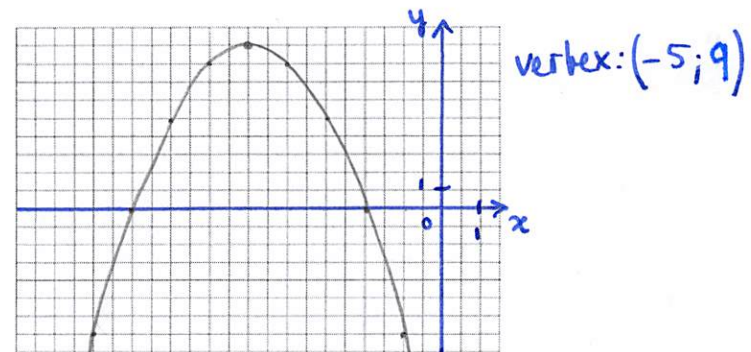
solutions: $\{-\frac{3}{2}; \frac{5}{2}\}$

c) $\frac{1}{2}x^2 - x = 4$ $\frac{1}{2}x^2 - x - 4 = 0$



solutions: $\{-2; 4\}$

d) $-(x + 5)^2 + 9 = 0$



solutions: $\{-8; -2\}$

6. Solve algebraically alternating methods. Circle exact values, and give approximations to the nearest hundredth when relevant:

a) $x^2 + 6x + 8 = 0$

$$(x+4)(x+2) = 0$$

$$\boxed{x = -4} \quad \boxed{x = -2}$$

b) $(x+7)^2 = 121$

$$x+7 = \pm 11$$

$$x = \pm 11 - 7$$

$$\boxed{x = -18} \quad \boxed{x = 4}$$

c) $x^2 - 10x + 10 = 0$

$$\Delta = 100 - 40$$

$$= 60$$

$$\boxed{x = \frac{10 \pm \sqrt{60}}{2}}$$

$$x \approx 1.13 \quad x \approx 8.87$$

d) $4x^2 + 27x = 24$

$$4x^2 + 27x - 24 = 0$$

$$\Delta = 27^2 - 4(4)(-24)$$

$$= 1113$$

$$\boxed{x = \frac{-27 \pm \sqrt{1113}}{8}}$$

$$x \approx -7.55 \quad x \approx 0.80$$

e) $-3(x+1)^2 + 48 = 0$

$$-3(x+1)^2 = -48$$

$$(x+1)^2 = 16$$

$$x+1 = \pm 4$$

$$x = \pm 4 - 1$$

$$\boxed{x = -5} \quad \boxed{x = 3}$$

f) $5x^2 = 8 - 2x$

$$5x^2 + 2x - 8 = 0$$

$$\Delta = 4 - 4(5)(-8)$$

$$= 164$$

$$\boxed{x = \frac{-2 \pm \sqrt{164}}{10}}$$

$$x \approx -1.48 \quad x \approx 1.08$$

7. Which tool gives us the number of solutions of a quadratic equation?

the discriminant (Δ)

8. Discuss the number of solutions of the following equations depending on the values of k :

a) $2x^2 - 3x + k = 0$

$$\Delta = 9 - 4(2)k \\ = 9 - 8k$$

if $\Delta > 0$: 2 solutions

$$9 - 8k > 0 \text{ i.e. } \boxed{k < \frac{9}{8}}$$

if $\Delta = 0$: 1 solution \therefore when $k = \frac{9}{8}$

if $\Delta < 0$: no solution \therefore when $k > \frac{9}{8}$

b) $kx^2 + 2x - 5 = 0$

$$\Delta = 4 - 4(k)(-5) \\ = 4 + 20k$$

if $\Delta > 0$: 2 solutions

$$4 + 20k > 0 \text{ i.e. } \boxed{k > -\frac{1}{5}}$$

if $\Delta = 0$: 1 solution \therefore when $k = -\frac{1}{5}$

if $\Delta < 0$: no solution \therefore when $k < -\frac{1}{5}$

9. The profit, p , earned from the sale of a particular product by a business is given by $p(d) = -0.25d^2 + 5d + 80$, where d is the number of days the product has been for sale.

a) Determine on what day the best profit is reached.

$$\text{Vertex: } \frac{-b}{2a} = \frac{-5}{-0.5} = 10 \Rightarrow \text{on } \underline{\text{day 10}}, p \text{ is at its max.}$$

b) Determine the last day on which the product will be profitable.

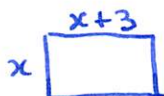
$$\text{zeros: } \Delta = 25 - 4(-0.25)(80) = 105$$

$$d = \frac{-5 \pm \sqrt{105}}{-0.5} \begin{matrix} \nearrow 30.5 \\ \searrow -10.5 \end{matrix}$$

\Rightarrow last day is day 30

(after that, p becomes negative)

10. One side of an envelope is 3 cm longer than the other side. The area of the envelope is 108 cm^2 . Determine the dimensions of the envelope.



$$x(x+3) = 108$$

$$x^2 + 3x - 108 = 0$$

$$\Delta = 9 - 4(-108)$$

$$= 441$$

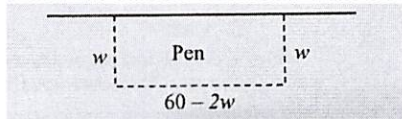
$$x = \frac{-3 \pm \sqrt{441}}{2}$$

$$x = \cancel{-12} \quad x = \underline{9}$$

impossible
 x is a length

\Rightarrow dimensions: $\boxed{9 \text{ by } 12}$

11. A rectangular pen is to be built along the side of a barn to house chickens. Find the maximum area that can be enclosed with 60 m of fencing if the barn is one side of the enclosure.



$$A = w(60 - 2w)$$

$$= -2w^2 + 60w$$

$$A_{\max} = 450 \text{ m}^2$$

vertex: $\frac{-b}{2a} = \frac{-60}{-4} = 15$

$(15; 450)$

12. Mary stands on the top of a building and fires a gun upwards. The bullet travels according to the equation $h(t) = -16t^2 + 384t + 50$, where h is the height of the bullet off the ground in metres at t seconds after it was fired.

a) How high above the ground is Mary holding her gun?

50 m (y-intercept)

b) How long does it take the bullet to reach its maximum height and what was the maximum height of the bullet?

vertex: $\frac{-b}{2a} = \frac{-384}{-32} = 12$

$(12; 2354)$

\Rightarrow it took 12 seconds
max height: 2354 m

c) After how many seconds does the bullet hit the ground?

zeros: $\Delta = 384^2 - 4(-16)(50)$

$$= 150656$$

$$t = \frac{-384 \pm \sqrt{150656}}{-32}$$

\swarrow 24.1s
 \searrow -0.1s

24 s after being shot

13. A cannon is fired from a cliff above the water. The height h (in metres) of the cannon above the water is given by $h(x) = -0.005x^2 + x + 100$, where x is the horizontal distance of the cannon from the base of the cliff.

a) How many metres above the water is the cliff?

100 m (y-intercept)

b) How far from the cliff is the cannon shell reaching its maximum height, and what is its maximum height?

vertex: $\frac{-b}{2a} = \frac{-1}{-0.01} = 100$

$(100, 150)$

max height: 150 m
reached 100 m from the cliff.

c) How far from the base of the cliff will the cannon shell land in the water?

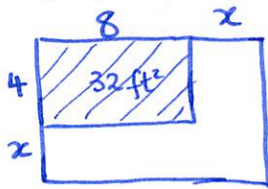
zeros: $\Delta = 1 - 4(-0.005)(100) = 3$

$$x = \frac{-1 \pm \sqrt{3}}{-0.01}$$

\swarrow 273.2
 \searrow -73.2

\Rightarrow 273 m from the base of the cliff.

14. The length and width of a rectangular sheet of plywood is 4ft. by 8ft. How much must be added equally to the length and width to double the area?



$$(8+x)(4+x) = 64$$

$$x^2 + 12x + 32 = 64$$

$$x^2 + 12x - 32 = 0$$

$$\Delta = 12^2 - 4(1)(-32)$$

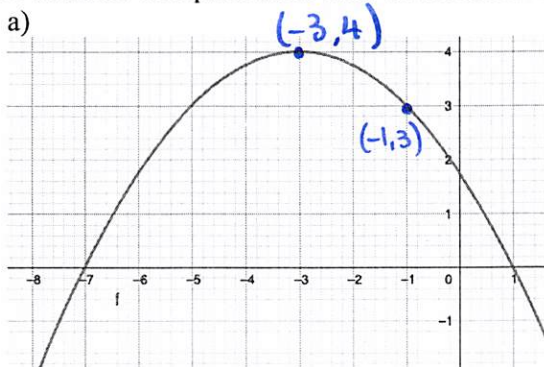
$$= 272$$

$$x = \frac{-12 \pm \sqrt{272}}{2}$$

\swarrow -14.25 length.
 \searrow 2.25

Approx 2.25 ft must be added on each dimension.

15. Determine an equation for each of the following parabolas:



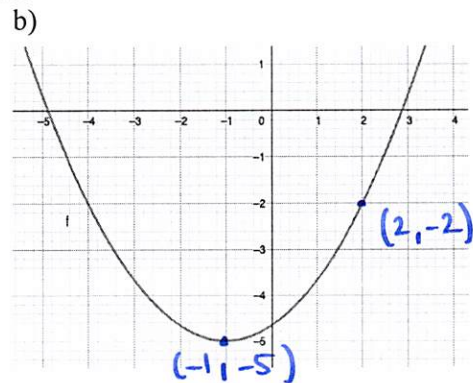
$$y = a(x+3)^2 + 4$$

Point: $3 = a(-1+3)^2 + 4$

$$-1 = 4a$$

$$a = -\frac{1}{4}$$

$$y = -\frac{1}{4}(x+3)^2 + 4$$



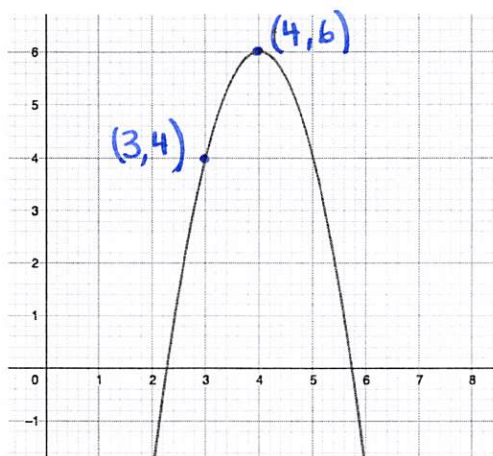
$$y = a(x+1)^2 - 5$$

Point: $-2 = a(2+1)^2 - 5$

$$3 = 9a$$

$$a = \frac{1}{3}$$

$$y = \frac{1}{3}(x+1)^2 - 5$$



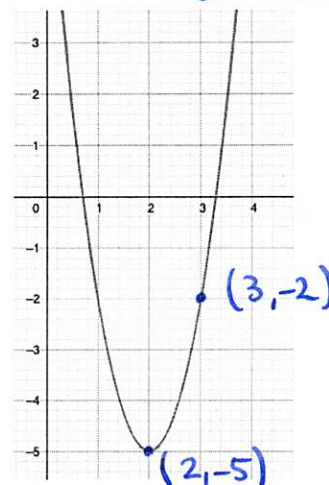
$$y = a(x-4)^2 + 6$$

Point: $4 = a(3-4)^2 + 6$

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$$-2 = a$$

$$y = -2(x-4)^2 + 6$$



$$y = a(x-2)^2 - 5$$

Point: $-2 = a(3-2)^2 - 5$

$$3 = a$$

$$y = 3(x-2)^2 - 5$$