**Chapter - FACTORING**

**Definitions:** Factoring, means changing a sum into a product.
 Expanding, means changing a product into a sum.

A sum is made of separate terms separated by « + » or « – » outside of any brackets.

Examples:

1. $5x^{2}-3x+1$ is a sum of 3 terms: $5x^{2}$, $-3x$ et $1$.
2. $2(x-3)^{2}+5$ is a sum of 2 terms: $2(x-3)^{2}$ et $5$.
3. $\left(x+2\right)\left(x-3\right)-x(2x+1)$ is a sum of 2 terms: $\left(x+2\right)\left(x-3\right)$ et $-x(2x+1)$

A product has only one term that is made of factors that multiply each.

Examples:

1. $(x+3)(x-2)$ is a product of 2 factors: $x+3$ et $x-2$.
2. $5x(x-1)^{2}$ is a product of 4 factors: $5$, $x$, et $x-1$ that is double.

**Reminder on expanding:**

We use distributivity:

Examples :

1. $\left(x-3\right)\left(2x+5\right)-3\left(2x-1\right)=$
2. $(3x-1)^{2}-\left(x+5\right)\left(3x-2\right)=$
3. $\left(x+1\right)\left(x-2\right)\left(x+3\right)=$
4. $\left(3x-5\right)\left(3x+5\right)-\left(x+1\right)^{2}=$

We need to notice in particular that: $(a+b)^{2}=a^{2}+2ab+b^{2}$

**Factoring by common factor:**

Your first reflex when factoring should be to look and see if all the terms have a common factor.

Examples:

1. $12x^{2}-8x=$
2. $\left(x-3\right)\left(x+1\right)+5x\left(x+1\right)=$
3. $(x-2)^{2}-3\left(x-2\right)=$
4. $5x^{3}-25x^{2}+5x=$

ATTENTION: Factoring by common factor should always be the first technique to use!

**Factoring trinomials of the form** $ax^{2}+bx+c$**:**

1. If $a=1:$ Which means that there is “no number before” $x^{2}$.

You already learned how to do it in grade 10.

Examples:
a) $x^{2}-5x+6=$

b) $x^{2}+7x+6=$

c) $25x^{2}-50x+25=$
2. If $a\ne 1:$ which means that there is a “number in front of” $x^{2}$.
It’s more complicated! We still look for 2 numbers but they don’t give us the factored form right away:

Examples:
a) $2x^{2}+7x-15=$

b) $6x^{2}-13x-5=$

c) $6x^{2}-5x+1=$

 **Factoring Special trinomials:**

When we expand certain products, we can notice some particular results:

1. **Differences of squares:**

**Definition:** Two expressions are said to beconjugates if one of them is a sum of two terms and the other one is the difference of the same terms.

Examples: a) $5x+3$ et $5x-3$
 b) $-1+x$ et $1+x$

IMPORTANT: When we multiply 2 conjugates expressions, we get a difference of squares.

$$\left(a+b\right)\left(a-b\right)=a^{2}-b^{2}$$

Examples: a) $\left(x+3\right)\left(x-3\right)=$

 b) $\left(3x+5\right)\left(3x-5\right)=$

As a consequence, when we notice a difference of squares, we can factor it into a product of conjugates.

Examples:

1. $x^{2}-25=$
2. $9x^{2}-1=$
3. $2x^{2}-32=$

ATTENTION: A *sum* of squares can’t be factored!!

1. **Perfect Squares :**

We call perfect squares an expression that will have 2 identical factors once factored.

Example : $x^{2}-6x+9=(x-3)^{2}$ 🡪 It’s a perfect square!

To recognize a perfect square, you need 2 perfect squares terms and the double product!!

Examples :

1. $x^{2}-10x+25$
2. $4x^{2}+4x+1$
3. $x^{2}+2x+4$
4. $18x^{2}-12x+4$

**Other factoring techniques :** **Changes of variables :**

Example 1 : $2\left(x+4\right)^{2}-11\left(x+4\right)+15$

Example 2 : $\left(x^{2}-6\right)^{2}+7\left(x^{2}-6\right)-30$

Your turn : a) p 222

Example 3 : $(x^{2}+3)^{2}-9(x^{2}-1)^{2}$

Your turn : b) p 222

Hwk : p 229 # 5, 6

**Why factor?**

Factoring is mostly useful to study the sign of an expression (see chapter 9) and solve equations (see 4.2).

Review : worksheet (from textbook 10) + p 258 # 6