Factoring with Square Roots

1. Factor the following Expressions

a) $(x-3)\sqrt{x-5}-\frac{8}{\sqrt{x-5}}$

$= \frac{(x-3)(x-5)-8}{\sqrt{x-5}}$ $= \frac{(x-7)(x-1)}{\sqrt{x-5}}$

$= \frac{x^2-8x+7}{\sqrt{x-5}}$

b) $(x+1)^{\frac{1}{2}}(3x^2-1)+(x+1)^{\frac{3}{2}}(x-2)$

$= (x+1)^{1/2}\left[3x^2-1+(x+1)(x-2)\right]$ $= (x+1)^{1/2}(4x+3)(x-1)$

$= (x+1)^{1/2}(4x^2-x-3)$

c) $(x+3)^{-\frac{1}{2}}(7x-12)+2(x+3)^{\frac{1}{2}}(x-3)$

$= \frac{7x-12+2(x+3)(x-3)}{\sqrt{x+3}}$ $= \frac{(2x-5)(x+6)}{\sqrt{x+3}}$

$= \frac{2x^2+7x-30}{\sqrt{x+3}}$

d) $(x+3)\sqrt{2x+1}-\frac{4}{\sqrt{2x+1}}(x+1)^2$

$= \frac{(x+3)(2x+1)-4(x+1)^2}{\sqrt{2x+1}}$

$= \frac{-2x^2-x-1}{\sqrt{2x+1}}$ ← can't be factored ($\Delta=-7$)

e) $(x-2)^{\frac{1}{2}}(x^2-4)-5x(x-2)^{\frac{3}{2}}$

$= (x-2)^{1/2}\left(x^2-4-5x(x-2)\right)$ $= -2(x-2)^{1/2}(2x-1)(x-2)$

$= (x-2)^{1/2}(-4x^2+10x-4)$ $= -2(x-2)^{3/2}(2x-1)$

$= -2(x-2)^{1/2}(2x^2-5x+2)$

f) $2(5x-4)^{-\frac{1}{2}}(x-8)-2(5x-4)^{\frac{1}{2}}(x-3)$

$= \frac{2(x-8)-2(5x-4)(x-3)}{\sqrt{5x-4}}$ $= \frac{2}{\sqrt{5x-4}}\left(-5x^2+20x-20\right)$

$= \frac{2}{\sqrt{5x-4}}\left[x-8-5x^2+19x-12\right]$ $= -\frac{10(x^2-4x+4)}{\sqrt{5x-4}}$

$= -\frac{10(x-2)^2}{\sqrt{5x-4}}$

2. Solve the following Equations

a) $2(x-1)\sqrt{x+1} - \frac{4}{\sqrt{x+1}}(x-1)^2 = 0$ Restr : $x > -1$

$$\frac{2(x-1)(x+1) - 4(x-1)^2}{\sqrt{x+1}} = 0$$

$$2(x-1)\left[x+1 - 2(x-1)\right] = 0$$

$x = 1$ $\quad -x + 3 = 0$

$x = 3$

b) $(x+5)^{\frac{1}{2}}(x^2+8) = 3x(x+5)^{\frac{3}{2}}$ Restr : $x \geq -5$

$$(x+5)^{1/2}\left[x^2+8-3x(x+5)\right] = 0$$

$$(x+5)^{1/2}(-2x^2-15x+8) = 0$$

$x = -5$

$\Delta = 289$

$x = \frac{15 \pm 17}{-4}$ → $x = 1/2$

→ $x = -8$ Restr ×

c) $3(x-4)^{-\frac{1}{2}}(2x-3) - 2(x-4)^{\frac{1}{2}}(x+1) = 0$ Restr : $x > 4$

$$\frac{3(x-3) - 2(x-4)(x+1)}{\sqrt{x-4}} = 0$$

$$-2x^2 + 12x - 1 = 0$$

$\Delta = 136$

$x = \frac{-12 \pm \sqrt{136}}{-4}$ → $x \simeq 5.9$ ⇒ $x = \frac{12 + \sqrt{136}}{4}$

i.e $x = \frac{6 + \sqrt{34}}{2}$

→ $x \simeq 0.08$ Restr ×