Answers

Chapter 1 – Financial Mathematics: Investing Money

Lesson 1.1, page 14

1. a) $38 400 b) $10 500 c) $25 250 d) $29 760
2. a) 6% b) $5900
3. a) 25% b) A: $1365, 25%
4. a) $4460 b) $4920
5. Brad; e.g., Interest rates are almost equal, but Brad's GIC has a term of 1 more year.
6. a) 2.5% b) $16 200
7. a) They will be equal. e.g., The principal, interest rate, and term are equal. Both earn $300 in interest.
   b) No. e.g., With simple interest, there is no advantage to having it paid more often.
   c) e.g., They may need the interest money to pay a monthly bill.
8. a) A: $12 500, B: $11 400, C: $11 330, D: $10 840
   b) No. e.g., The amount of interest not earned in the last 1.5 years is insufficient to change rankings. A: $11 750, B: $11 190, C: $11 045, D: $10 630
9. a) Desiree, $92.50
   b) $7.50
   c) Desiree: 1.25%, Latoya: 1.5%
10. 8 years
11. a) $17 241.38 b) about 14.1 years
12. a) 12.5% b) $14 100
13. a) Increase the interest rate. e.g., The higher the interest rate, the more money earned per time period.
    b) e.g., They both have the same interest rate, but they have different principals.
14. $5570.00
15. $24 294.05

Lesson 1.2, page 19

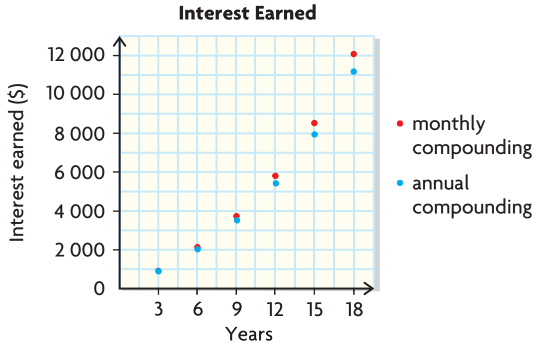
1. Larry's GIC earns $49.96 more interest than Eve's GIC. e.g., Larry's GIC earns more interest because it is compounded.
2. B. e.g., The future value of option A is $7826.00 and the future value of option B is $7840.77.
3. a) e.g., No, it is not possible to tell, as the principals, interest rates, and timelines all differ.
   b) C

Lesson 1.3, page 30

1.

Compound Interest Rate per Annum (%)	Compounding Frequency	Term	Interest Rate per Compounding Period, i (%)	Number of Compounding Periods, n
10.2	semi-annually	4 years	5.1	8
4.1	monthly	6 years	0.3416…	72
13.2	quarterly	7 years	3.3	28
3.5	daily	9 months	0.009 589	e.g., 274

2. a) $744.83, $224.83
   b) $4950.59, $3550.59
3. i) a) 10.59 years, 10.54 years
   b) 7.83 years, 7.56 years
   c) 4.62 years, 4.45 years
   d) 26.67 years, 25.85 years
   ii) a) $69 999.01, $62 999.01
   b) $5314.63, $4464.63
   c) $27 236.58, $14 736.58
   d) $49 572.41, $9572.41
4. a) $14 151.36, $15 067.91
   b)



   c) As compounding frequency increases, interest rate growth increases.
5. a) 15 years, about 14.78 years
   b) 5 years sooner, about 4.81 years sooner
6. $9590.25
7. C, B, A
8. a) 48 years
   b) 24 years
9. 6.5%, $800
10. $54 333.96
11. $2655.41
12. a) e.g., The higher interest rate is payment in exchange for more time before maturity.
    b) i) Option 1: $6884.47, Option 2: $6494.71, e.g., Interest rates are compounded annually, interest rates remain the same in 5 years, can reinvest all $5698.55 in 5 years.
    ii) e.g., Option 1: Higher return as long as rates do not rise, but money is locked in for 10 years. Option 2: If rates rise, the money can be reinvested after 5 years at a higher rate, but if rates do not rise, a lower return may be realized.

13. e.g., In both cases, interest is earned on the principal. For compound interest, interest is also earned on the interest.

14. **a)** $2727.80
b) $5877.96

15. $5168.65

Lesson 1.4, page 40

1. e.g., Option B will require a greater present value because the compounding frequency is less than that of option A.
Option A: $6071.61, Option B: $6084.13.

2. **a)** A: 1.647…, B: 1.644…
b) e.g., Higher ratio because the interest rate is higher and the principal is lower.

3.

Future Value (amount in $)	Present Value (amount in $)	Interest Rate per Annum (%)	Compounding Period	Investment Term (years)
2 500.00	1 370.85	7.8	annually	8
3 500.00	2 000.00	11.5	semi-annually	5
11 000.00	8 254.48	2.4	quarterly	12
100 000.00	609.35	13.6	annually	40
23 500.00	16 150.00	18.9	monthly	2

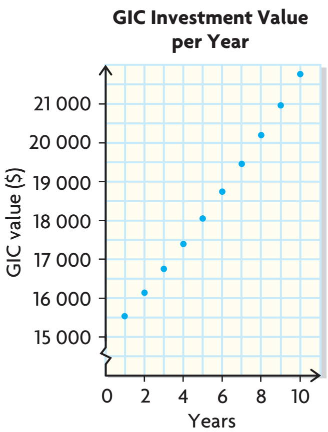
4. **a)** $48 904.10
b) $201 095.90

5. **a)** 33.1%; e.g., No, it would be difficult to find a guaranteed investment with that interest rate.
b) about 5.5 years

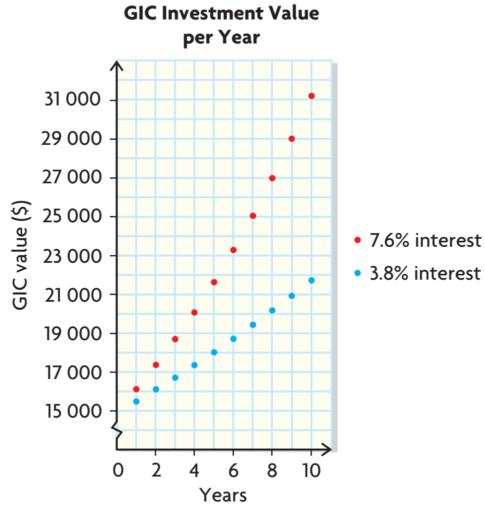
6. $10 073.39

7. **a)** A: 32.49%, B: 32.53%, C: 32.36%; Choose B. e.g., B has the greatest rate of return.
b) $5891.43

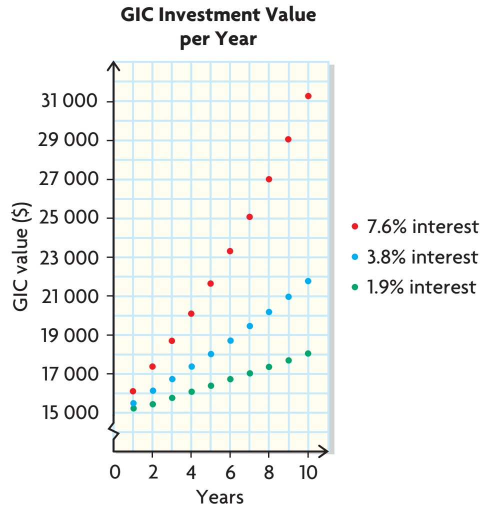
8. **a)** e.g., Investment A



b) e.g., Changed the interest rate to 7.6%, compounded annually.



c) e.g., Changed the interest rate to 1.9%, compounded annually.



d) e.g., Changing the interest rate changes the rate at which the value of the investment increases.

9. C; e.g., Option C has the best interest rate with frequent compounding; option A requires $8678.89, option B requires $9815.74, option C requires $5545.60, and option D requires $9982.77.

10. Franco, $204.20

11. **a)** 8.56%
b) 2.33; e.g, I predict it will decrease because it is compounded less frequently so the future value decreases, 2.27.

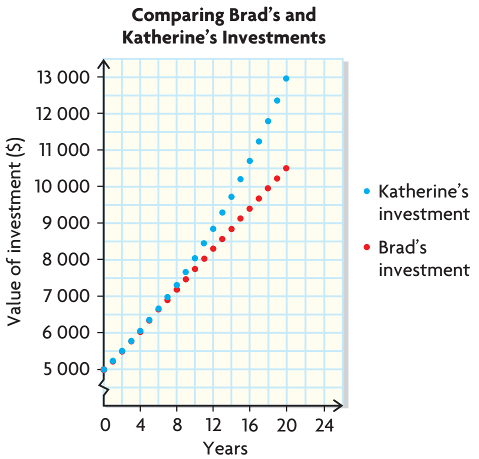
12. $185.30

13. $2906.25

14. e.g., In an investment, you agree to lend a sum of money to another entity (like a company); the amount you lend is called the *present value* of the principal. The *interest rate* dictates the amount of money they pay you for the loan, for a given time period, called the *term*. *Simple interest* pays you a percentage of the loaned amount at the end of the term. With *compound interest,* the interest is paid out more often, defined by the *compounding frequency*. You don't get the *compound interest* immediately, but effectively lend the entity the interest as well, until the end of the term. The *present value* plus the interest you earn is called the *future value*. A higher *interest rate* and a higher *compounding frequency* will earn you more interest.
15. 9.26%
16. **a)** $1050.00
    b) **i)** 4.94%
    ii) 4.91%
    iii) 4.89%
    c) e.g., It could have a return greater than an investment with a higher interest rate if compounded less frequently.

Mid-Chapter Review, page 45

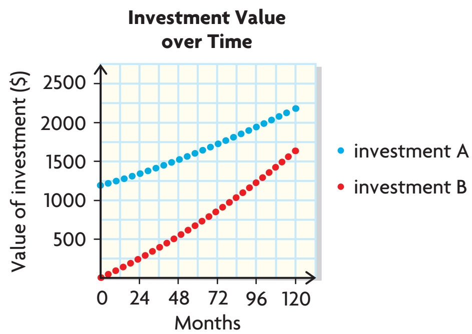
1. 3 years
2. **a)** 3.125 years
   b) 4 years
   c) 3.25 years
3. **a)** Katherine: $12 941.82, Brad: $10 500.00
   b)



   c) e.g., The intersection represents the time when both investments are worth the same amount.
4. **a)** $5173.21
   b) 25.44 years, or about 26 years
   c) **i)** e.g., Future value would increase. $5245.18
   ii) e.g., Future value would decrease. $3961.69
   d) 5.64%
5. $12 947.39
6. A and D: $1340.10, B: $1338.23, C: $1283.35
7. **a)** $8356.45
   b) $8374.84
8. **a)** **i)** $2382.91
   ii) $2320.40
   b) Higher; ratio for a) i) is about 4.62, ratio for a) ii) is about 4.74, ratio for b) is about 4.85.
9. 4.47%
10. e.g., 24 years, about 23.45 years or 23.5 years

Lesson 1.5, page 55

1. **a)** $498 526.60
   b) $126 127.32
   c) $63 820.79
   d) $195 389.47
2. **a)** 2.68%
   b) $250.00
   c) 10.5 years
3. $154 030.54, $78 430.54
4. A; e.g., The total amount is invested at the beginning and earns interest for the full term. A: $6691.13, B: $5637.09
5. $14 150.77, $2150.77
6. $196.60
7. **a)** **i)** $76.22
   ii) $568.60
   b) **i)** 3023.79% **ii)** 318.74%; e.g., She should choose option i).
8. **a)** $1665.90
   b) $1356.16
   c) Aaron; e.g., He will continue to invest $25 each month and his interest compounds monthly; Aaron: $3720.33, Casey: $2046.39.
9. 6.13%
10. 4.5 years
11. **a)** Dee: $66 298.98, Pete: $73 624.90
    b) e.g., Pete's money was invested for a longer period of time.
12. Trey's by $62.75
13. No. e.g., He will be short $78.87.
14. **a)** $40 006.80
    b) $605 469.20
15. No. e.g., He will be short by $35 320.92.
16. **a)** Same: The amount invested, interest rate, and term are all equal. Different: Investment A has a lump sum payment while investment B has regular payments.
    b)



    e.g., Investment A is the better investment. Investment B grows at a faster rate, but does not have time to surpass investment A.
17. No. e.g., The probability that his tips will equal the same amount each month is very unlikely.
18. $150.93
19. $5522.35
20. No. e.g., He will be short by $1933.99.

Lesson 1.6, page 64

1. **a)** e.g., He can invest the $300 in a GIC and deposit $15 into a high interest savings account so he has access to funds if required.
   b) No; $27.21
2. $4078.92

Answers

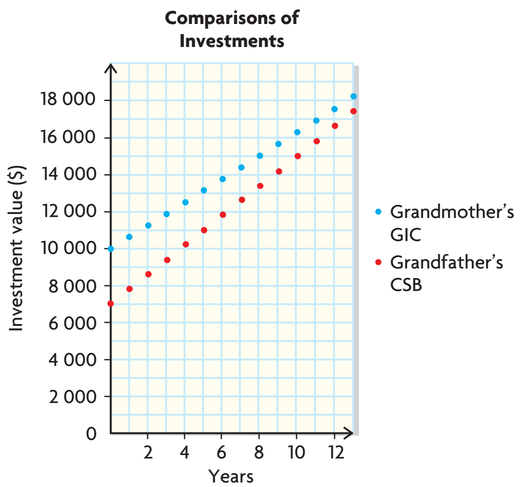
3. $57 125.96
4. a) $4788.51
   b) Yes, she has $2887.83.
   c) She will be short by $1080.55.
5. a) $2160.00
   b) Gayla; e.g., Funds were invested for a longer period of time. Gayla: $3940.98, Corey: $2622.73
6. $65 078.24, 160.3%
7. a) $38 191.53 b) 205.5%
8. a) No
   b) e.g., She needs about $20 per week. Exact amount is $19.01.
9. A; e.g., It is worth $263.17 more.
10. e.g., An investment portfolio may consist of single payment investments or regular payment investments. Some investments may lock in funds for a period of time, limiting access, but often offer a higher interest rate. Investments offering a lower interest rate usually allow access to the funds. A higher investment amount usually tends to offer higher interest rates. The greater the principal, term, interest rate, and compounding frequency, the faster the investment will grow.
11. a) $2121.31 b) $2374.04
12. $143 664

Chapter Self-Test, page 68

1. a) estimate 12 years; 11.9 years
   b) 16.67 years
2. A; e.g., Option A will take about 24 years and option B will take about 28 years.
3. a) $1200.00, $2256.24, 88%
   b) $1388.25
   c) $32.50
4. a) Alex: $12 600.42, Jamie: $12 064.27
   b) Alex, at 15.6%; e.g., Alex used investments with greater principals that compounded more often.

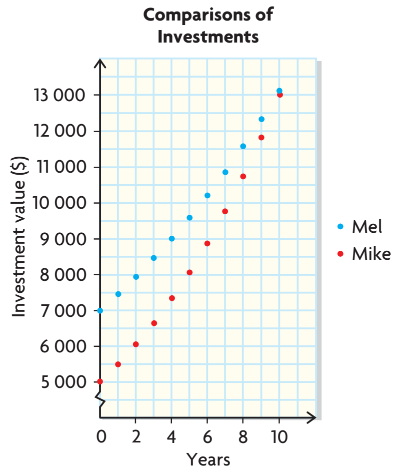
Chapter Review, page 71

1. a) 15%
   b) $2850; e.g., She will not receive any interest for the last 6 months.
2. a) Grandmother: $18 190.00, Grandfather: $17 374.00
   b)



   c) e.g., Given time, a higher interest rate can compensate for a lower initial investment.

3. a) e.g., I predict Sonia because it has a much higher interest rate.
   b) Sonia: 31.0%, Trent: 30.2%
   c) e.g., Sonia's interest rate was applied on the principal only while Trent's interest rate was applied to the principal and any accrued interest.
4. a) e.g., Johnny, because it is compounded more frequently. James: $876.19, Johnny: $894.71
   b) James: 43.8%, Johnny: 44.7%
5. 10.54%
6. a) Phil, $27 027.06
   b) $395 323.07
7. a) Mel
   b)



   c) e.g., Mike's investment will soon be worth more because it is compounded more frequently.
8. $2180.78
9. a) Josh: $212 743.51, Jeff: $69 827.91
   b) $45 000.00 each
   c) Josh: $167 743.51, Jeff: $24 827.91
   d) $9140.05
10. a) e.g., They have different compounding frequencies and payments.
    b) John
11. a) i) $31 529.31
       ii) 37.1%
    b) No, she will need an extra $611.26 for her fourth year.
12. a) e.g., Portfolio 2, the $25 000 portion is compounded more frequently, the $10 000 portion is compounded versus simple interest, and the regular payment portion has the same total investment amount but it is invested sooner each year.
    b) Portfolio 1: $105 273.55, Portfolio 2: $109 852.24; Yes

Chapter 2 – Financial Mathematics: Borrowing Money

Lesson 2.1, page 92

1. a) $2560 b) $60
2. a) $1268.79 b) $68.79
3. a) $11 154.61 b) $845.39
4. a) 31 months b) $1140.33
5. a) $18 845.60
   b) i) More. e.g., He is borrowing the money for a longer period of time, so he will pay more interest. He will pay $6522.73 more.
   ii) Less. e.g., He is borrowing money for a shorter period of time, so he will pay less interest. He will pay $2602.50 less.
6. a) $11 347.95 b) $652.05
7. a) $17 990
   b) $161 910
   c) $284.63
   d) 454 weeks or 8 years 38 weeks
   e) $60 101.74
8. a) 42 months or 3 years 6 months
   b) $695.61
9. a) $1275.15 b) $24.85
10. $16 545.65
11. a) $2082.42
   b)

Payment Period (half year)	Payment ($)	Interest Paid ($)	Principal Paid ($)	Balance ($)
0				15 899.00
1	2082.42	166.94	1915.48	13 983.52
2	2082.42	146.83	1935.59	12 047.93
3	2082.42	126.50	1955.92	10 092.01
4	2082.42	105.97	1976.45	8 115.56
5	2082.42	85.21	1997.21	6 118.35
6	2082.42	64.24	2018.18	4 100.17
7	2082.42	43.05	2039.37	2 060.80
8	2082.42	21.64	2060.78	0.02

   in 2 years 6 months
   c) $760.38
12. a) $41 278.72; $11 278.72
   b) i) $585.58
   ii) $24 740.54; $19 134.42; $13 158.79; $6789.31; $0.00
   iii) $5134.82
13. a) $406.87
   b) i) 45 months
   ii) $466.26
14. a) $594.93
   b) i) $29 033.09
   ii) $19 999.61
   iii) $10 336.54
   iv) $0.00
   c) i) $2263.77
   ii) $3938.98
   iii) $4984.59
   iv) $5356.74
15. a) $2742.85
   b) $2341.85
   c) $401.00
   d) e.g., Mike would make smaller payments to the store each month, and has one additional year to pay off the loan.
16. a) bank: 26 months; investors: 13 months
   b) bank: $156.68; investors: $84.00
   c) bank: $3156.68; investors: $3084.00
   d) e.g., Elise should take the loan from the investors, if she can make the $250 monthly payments, because she pays less interest.
17. a) option A: $453.77; option B: $456.89
   b) option A: $780.81; option B: $447.88
   c) e.g., Connor should take option B, if he can afford the $5000 down payment, because he pays less interest.
18. principal = $5000; interest rate per period = 2.25%; number of payments = 8; payment amount = $689.93; total interest paid = $519.38
19. e.g.,

	Payment ($)	Term (years)	Total Interest Paid ($)
Option A	8829.81/year	20	56 596.20
Option B	727.18/month	20	54 522.33
Option C	17 883.50/year	10	58 835.39
Option D	1455.93/month	10	54 711.74

   Choose Option B, as it has a manageable monthly payment, it has the longest term, and it incurs the least amount of interest over the life of the loan.
20. Option B is the better option, because Gabe would pay $71.77 less interest.
21. a) $28 797.46
   b) 55 months or 4 years 7 months
   c) 6 months sooner

Lesson 2.2, page 100

1. a) credit card: $5933.80; bank loan: $5615.43
   b) credit card: $1053.80; bank loan: $615.43
   c) credit card: 30 months or 2 years 6 months; bank loan: 29 months or 2 years 5 months
   d) e.g., She should use the bank loan, because she will pay it off sooner and pay less overall.
2. Card Blue. e.g., She will pay $34.54 less.
3. Annie's credit card
4. a) card A: $1261.56; card B: $1327.49 b) i)

Mid-Chapter Review, page 103

1. a) $1081.20 b) $1082.97
2. a) $445.05
   b) i) $1249.37 ii) $244.34
   c) e.g., For the first schedule, interest is charged on the full original principal every month. For the second schedule, the remaining principal is reduced every month, and therefore the interest is also reduced.
3. a) 83 weeks or 1 year 31 weeks
   b) Yes. e.g., it takes 41 weeks
4. a) 3.4%
   b) e.g., The interest rate would have been less, because the rate would be applied to the entire balance for a year.
5. a) $112.82
   b) no
   c) $103.88
   d) Yes. e.g., It's equivalent to having two of the original loans.
6. Lauren's credit card. e.g., The 2% cash back is more than the difference in interest she will pay.

7. Store credit card. e.g., He will pay $434.44 less.
8. Store financing. e.g., He will pay $113.58 less.

Lesson 2.3, page 114

1. a) $48.77
   b) i) $38.89 ii) $118.56
2. a) dealership financing: $827.34; bank loan: $694.76
   b) dealership financing: $39 712.37; bank loan: $41 685.80
   c) dealership financing: advantages: lower total interest, lower total payment, no shipping charge, debt paid off sooner; disadvantages: higher monthly payments; bank loan: advantages: lower monthly payments; disadvantages: higher total interest, higher total payment, must pay shipping charge, debt takes longer to pay off
3. a) tire shop b) credit card c) tire shop
4. a) credit card: $524.53; line of credit: $514.24
   b) credit card: $329.82; line of credit: $226.94
5. a) 13 months b) $26.37
6. a) 8 months b) $12.22 c) $20.47 more
7. No. e.g., With the rebate, Joanne will pay less in total with the credit card option.
8. a) 9 months
   b) It will not take her any longer, but the last payment will be more.
9. a) line of credit: 72 months or 6 years; credit card: 70 months or 5 years 10 months
   b) e.g., line of credit, as he would pay less overall
   c) line of credit: $240.93; credit card: $249.17
10. $216.59
11. a) $910 b) 85.3% simple interest
12. a) $349.48; $17.22 b) 16 months; $98.98
13. a) 19.4% b) Clint; $25.48
14. a) $99.95 d) $1240.34
    b) $5602.70 e) $1190.68
    c) 1.82% f) $1614.08
15. e.g., Interest rate: The lower the rate the better the credit option if all other factors are equal. Total number of payments: The higher the number of total payments, the more interest is paid in total. As the interest rate rises, so does the total number of payments.
16. e.g., Line of credit payments of $330 will take 26 months to pay off the loan of $8000, and the total interest paid will be $384.14. Art gallery loan payments of $311.05 will take 26 months to pay off the loan of $7500, and the total interest paid is $402.52.
17. a) bank loan: 6 months; credit card A: 42 months or 3 years 6 months; credit card B: 30 months or 2 years 6 months
    b) 9 months
    c) $594.66

Lesson 2.4, page 129

1. a) e.g., renting: costs (120 days): $9000; benefits: cleaning service, utilities
   leasing: costs: $8500; benefits: $1600 refund if no damage
   b) e.g., I would recommend leasing. Even if the deposit is lost, it is less expensive.
2. a) leasing: costs: $23 889.60; benefits: e.g., could upgrade earlier than 3 years; no need to resell equipment
   purchasing: costs: $28 537.60; benefits: e.g., own equipment, which is cheaper for the fourth year onward
   b) e.g., It would be better to lease, because the total cost of leasing is less than that of purchasing.
3. a) Rent. e.g., It is cheaper.
   b) 78 days
   c) 3 years
   d) e.g., They might need to buy new equipment sooner than 3 years.
4. e.g., She should pay $700 per half day if she is confident she can finish in 4 days or less, weather permitting, or she should pay $6000 for the week so she has a few extra days for unforeseen problems that may arise.
5. a) about 17 years b) about 47 years
6. Jake: $84 000; Archie: $49 170.74
7. a) rent: $340 per month; buy new: $1302.80; buy used: $781.68
   b) buying used
8. a) $18 715.15
   b) $12 520
   c) $8085
   d) e.g., Lease, if she plans to use the equipment for only 2 years, or purchase equipment if she plans to continue to use the equipment beyond the 2 years.
9. a) $9200.08
   b) $6318.18, including trade-in value
   c) $10 920.00; leasing: profit of $1719.92; buying: profit of $4601.82
   d) e.g., If the manager is sure she will stay with the same store after 18 months, she should buy. Otherwise, she should lease to keep her options open.
10. a) renting: $70 000; buying: $43 288.18; leasing: $40 960.00
    b) e.g., Lease, it is the least expensive option.
11. a) e.g., Lease, because it seems cheaper than renting and no need to purchase, because tractor is only required for 9 months.
    b) buying: $19 036.71; renting (275 days): $16 500; leasing: $14 105
    c) e.g., Renting is the most flexible option; the tractor may not be needed for the full 9 months. Renting is better than buying if the depreciated value of the tractor is less than $2536 in 9 months.
12. a) $3788.69
    b) about 2.83%
    c) e.g., The bond is less risky; if he bought the house, he would not need to pay rent.
13. a) $2071.00 b) $707.11
14. e.g., Renting, buying, or leasing a car for 3 years: Renting may be the best option if only occasional use is needed. Either buying or leasing may be the cheapest option, depending on financing, but leasing may give the flexibility to change cars earlier.
15. 9 days a month
16. e.g., Equipping an office with a computer network for 2 years. Option 1: Buying the equipment for $7500 financed at 5.5% interest, compounded monthly, paid back over 2 years, with depreciation of 40% per year on resale; Option 2: Leasing the equipment at $230 per month. Cost of buying is $5237.22; cost of leasing is $5520; buying is slightly cheaper, but leasing avoids the costs and uncertainties of resale.

Chapter Self-Test, page 134

1. a) $5772.70; $772.70
   b) $5751.28; $751.28
   c) $5383.96; $383.96
2. a) $12 043.83 b) $230.05; $1759.24
3. a) credit card: $110.51; line of credit: $109.35
   b) Line of credit. e.g., It will be cheaper.

4. e.g., She should purchase the Laser, because it is cheaper, and keep it if she plans to return to the cottage in the future. She should sell it when she will no longer go to the cottage

Chapter Review, page 136

1. a) 6.76% b) $33.75
2. a) 14 months b) 25 months c) $224.71
3. a) $189.78
   b) i) No. e.g., He would still save $68.93 using card A.
      ii) Yes. e.g., He would save $50.97 using card B.
4. Bank loan. e.g., She will pay $1.41 less overall.
5. $1670.00
6. e.g., Casey should purchase a snowplow; it is less expensive than leasing or renting over 2 years.
7. Rent. e.g., It is cheaper.

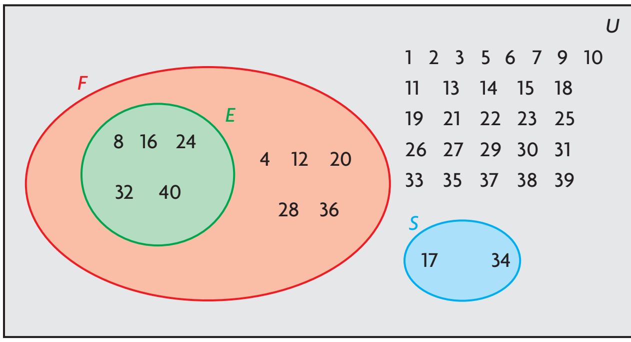
Cumulative Review, Chapters 1–2, page 140

1. a) $24 200, $4200 b) $5896, $396
2. 4 years
3. a) $5773.18, $773.18 b) $43 536.44, $19 536.44
4. B. e.g., It has a more frequent compounding period that earns more interest.
5. estimate: $4000; actual: $3970.31
6. $1330.78
7. a) Emma b) Hans
8. a) $5449.90 b) $449.90
9. $243.29
10. Option B. e.g., Option A is worth $28 613.14 and option B is worth $29 245.11.
11. a) $1536.33 b) $36.33
12. a) $462.67 b) $2760.37
13. a) 26 months b) 14 months sooner c) $662.16
14. a) Misty's parents: $48 000; Danielle's parents: $86 135.48
    b) e.g., Danielle's parents, because if they resell the house they should make a profit.

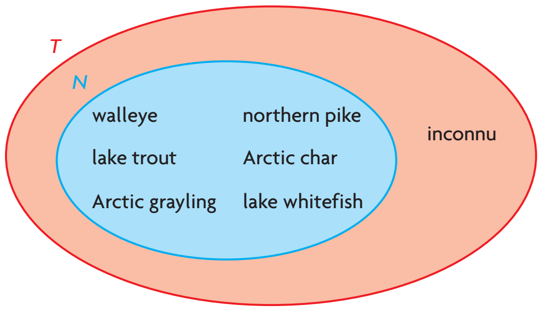
Chapter 3 – Set Theory and Logic

Lesson 3.1, page 154

1. a) Yes
   b) e.g.,
      - C = {produce}
      - O = {orange produce} = {oranges, carrots}
      - Y = {yellow produce} = {bananas, pineapples, corn}
      - G = {green produce} = {apples, peas, beans}
      - B = {brown produce} = {potatoes, pears}
   c) e.g., $S \subset F$ because all fruits you can eat without peeling are also fruits. $S \subset C$ because all fruits you can eat without peeling are also produce.
   d) S and V, F and V
   e) Yes. e.g., $C = \{F \text{ and } V\}$, so $F' = V$.
   f) $n(V) = 5$
   g) oranges, pineapples, bananas, peas, carrots, corn, beans, potatoes
2. a)

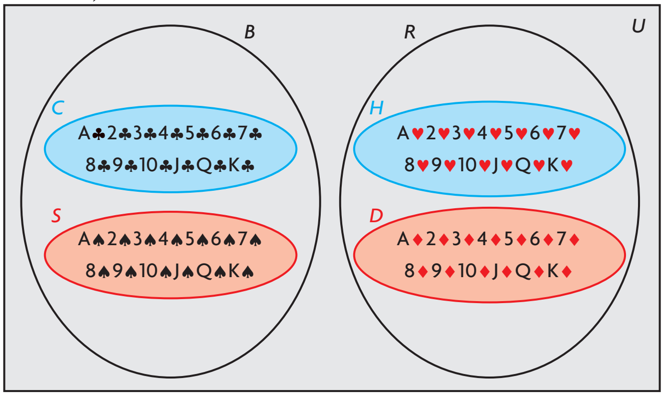


   b) E and S, F and S
   c) i) True. e.g., Multiples of 8 are also multiples of 4.
      ii) False. e.g., Not all multiples of 4 are multiples of 8.
      iii) True. e.g., All multiples of 8 are multiples of 8.
      iv) False. e.g., F' = {all numbers from 1 to 40 that are not multiples of 4}
      v) True. e.g., The universal set includes natural numbers from 1 to 40.
3. a)



   b) e.g., $N \subset T$ means that all the fish found in Nunavut are also found in the Northwest Territories. $T \not\subset N$ means that not all the fish found in the Northwest Territories are found in Nunavut.

4. a)



b) C and S

c) H and D

d) Yes. e.g., A card cannot be both a spade and a club.

e) Yes. e.g., You cannot draw a card that is a heart and a diamond at the same time.

f) Yes. e.g., $n(S \text{ or } D) = 26$ (there are 26 cards that are spades or diamonds) $=$ the number of cards that are spades (13) plus the number of cards that are diamonds (13) $= n(S) + n(D)$.

5. a) e.g., $C = \{$all clothes$\}$, $S = \{$summer clothes$\}$, $W = \{$winter clothes$\}$, $H = \{$summer headgear$\}$

b) C

c) No. e.g., S' includes jacket, but W does not.

d) S and W, H and W

e) e.g., $C = \{$clothes$\}$, $H = \{$headgear$\} = \{$cap, sunglasses, toque$\}$, $B = \{$clothing for body$\} = \{$shirt, shorts, coat, jacket$\}$, $F = \{$footwear$\} = \{$sandals, insulated boots$\}$

6. 99 988

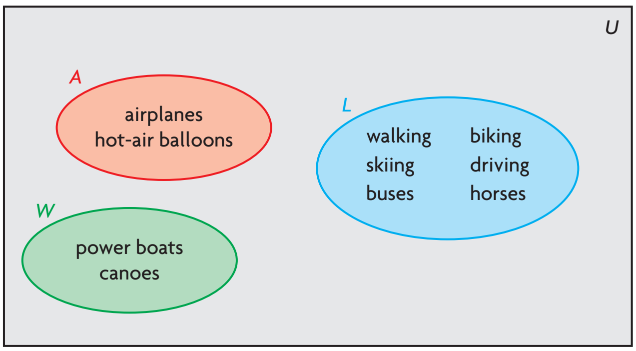
7. not possible; e.g., There may be some elements that are in both X and Y.

8. 76

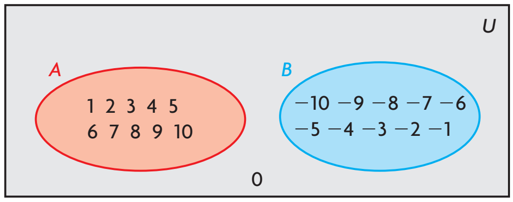
9. a) $S = \{$A, E, F, H, I, K, L, M, N, T, V, W, X, Y, Z$\}$
$C = \{$C, O, S$\}$

b) False. e.g., B is not in S or C.

10. e.g.,



11. a)



b) A and B

c) i) False. e.g., 1 is not in B.

ii) False. e.g., -1 is not in A.

iii) False. e.g., 0 is in A' but not in B.

iv) True. e.g., $n(A) = 10$, $n(B) = 10$

v) True. e.g., No integer from -20 to -15 is in U.

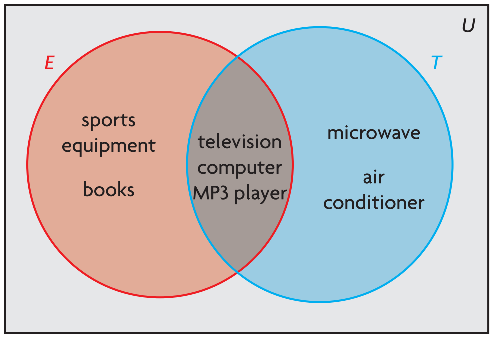
12. a) $S = \{4, 6, 9, 10, 14, 15, 21, 22, 25, 26, 33, 34, 35, 38, 39, 46, 49\}$
$W = \{1, 2, 3, 5, 7, 8, 11, 12, 13, 16, 17, 18, 19, 20, 23, 24, 27, 28, 29, 30, 31, 32, 36, 37, 40, 41, 42, 43, 44, 45, 47, 48, 50\}$

b) e.g., $E = \{$even semiprime numbers$\}$
$E = \{4, 6, 10, 14, 22, 26, 34, 38, 46\}$

c) 33

d) No. e.g., There is an infinite number of prime numbers, so there is an infinite number of semiprime numbers.

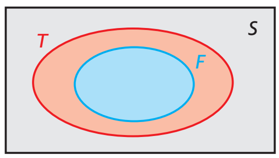
13. e.g.,



14. e.g., Agree. The number of elements in a subset must be equal to or less than the number of elements in the set.

15. a) $S = \{x \mid -1000 \leq x \leq 1000, x \in \mathrm{I}\}$
$T = \{t \mid t = 25x, -40 \leq x \leq 40, x \in \mathrm{I}\}$
$F = \{f \mid f = 50x, -20 \leq x \leq 20, x \in \mathrm{I}\}$
$F \subset T \subset S$

b)

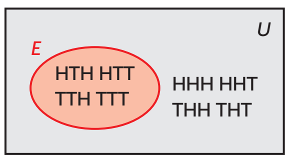


16. a) $U = \{$HHH, HHT, HTH, THH, HTT, THT, TTH, TTT$\}$

b) $E = \{$HTH, HTT, TTH, TTT$\}$

c) $n(U) = 8$; $n(E) = 4$

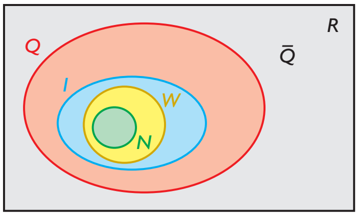
d) Yes. e.g.,



e) e.g., E' is the set of elements of U where the second coin turns up heads; $n(E') = n(U) - n(E) = 8 - 4 = 4$;
$E' = \{\text{HHH, HHT, THH, THT}\}$

f) Yes. e.g., A coin cannot show both heads and tails at the same time.

17. a)



b) e.g., N' is the set of all non-natural numbers. W' is the set of all non-whole numbers. I' is the set of non-integer numbers. Q' is the set of numbers that cannot be described as a ratio of two integers. $\overline{Q}'$ is the set of numbers that can be described as a ratio of two integers.

c) N and $\overline{Q}$, W and $\overline{Q}$, I and $\overline{Q}$, Q and $\overline{Q}$

d) Yes. e.g., Q' is the set of numbers that cannot be described as a ratio of two integers, which is the set of irrational numbers.

e) W, I, Q, R

f) No. e.g., The area of a region in a Venn diagram is not related to the number of elements in the set.

18. a) 8
b) 9
c) 283

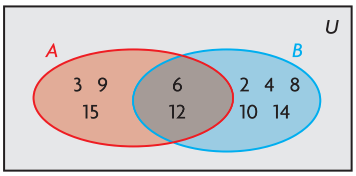
19. a) e.g., $A \subset B$ if all elements of A are also in B. For example, all weekdays are also days of the week, so weekdays is a subset of days of the week.

b) e.g., A' consists of all the elements in the universal set but not in A. For example, all days of the week that are not weekdays are weekend days. So weekend days is the complement of weekdays.

20. e.g., Disagree; since both the subsets are empty, they both contain the same elements and are therefore the same subset.

Lesson 3.2, page 160

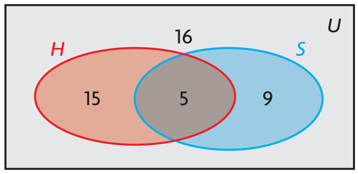
1. a)



b) i) 5
ii) 3
iii) 7
iv) 5
v) 2
vi) 10
vii) 5

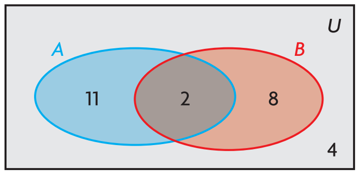
2. a) 8
b) 11; 6
c) 19; 14
d) 25
e) 13

3. a) 5
b) 24
c)



4. a) 11
b) 14
c) 21

5. a) $n(A \text{ and } B) = 2$, $n(A \text{ only}) = 11$, $n(B \text{ only}) = 8$
b)

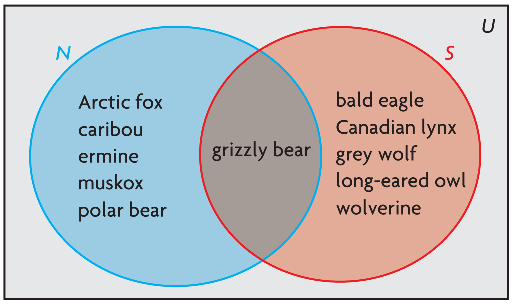


Lesson 3.3, page 172

1. a) $\{-10, -8, -6, -4, -2, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
b) 16
c) $\{0, 2, 4, 6, 8, 10\}$
d) 6

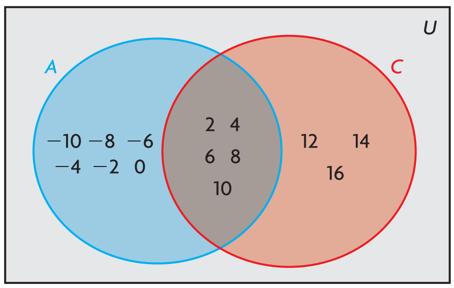
2. a) union = {Arctic fox, caribou, ermine, muskox, polar bear, grizzly bear, bald eagle, Canadian lynx, grey wolf, long-eared owl, wolverine}
intersection = {grizzly bear}

b)



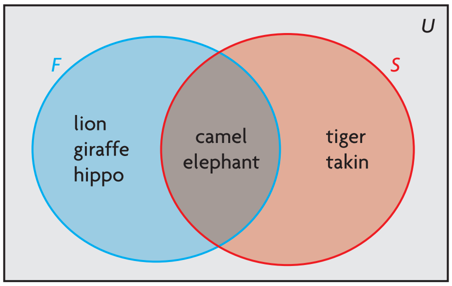
3. a) $A \cup C = \{-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10, 12, 14, 16\}$; $n(A \cup C) = 14$
$A \cap C = \{2, 4, 6, 8, 10\}$; $n(A \cap C) = 5$

b)



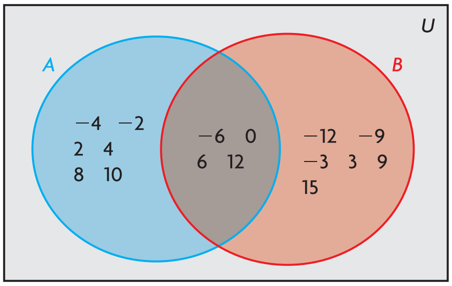
4. a) {half-tons, quarter-tons, vans, SUVs, crossovers, 4-door sedans, 2-door coupes, sports cars, hybrids}
b) 9
c) {crossovers}

5. a)



b) union = {lion, giraffe, hippo, camel, elephant, tiger, takin}
intersection = {camel, elephant}

6. a)



b) $A \cup B = \{-12, -9, -6, -4, -3, -2, 0, 2, 3, 4, 6, 8, 9, 10, 12, 15\}$; $n(A \cup B) = 16$
$A \cap B = \{-6, 0, 6, 12\}$; $n(A \cap B) = 4$

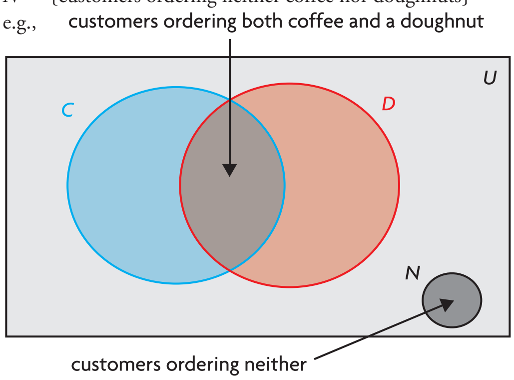
7. 6; 10; 5

8. 40

9. 12

10. e.g., She could draw a Venn diagram showing the set of multiples of 2 and the set of multiples of 3. The intersection of the sets would be the multiples of 6.

11. a) U = {all customers surveyed}, C = {customers ordering coffee}, D = {customers ordering doughnuts}, N = {customers ordering neither coffee nor doughnuts}
b) e.g.,



c) 20

12. 33

13. 10

14. 16

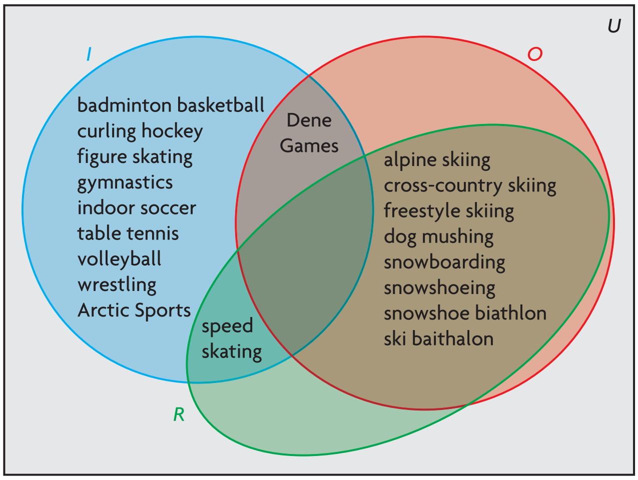
15. 6

16. No. e.g., Some students take a bus but do not drive a car. So these regions should only be partially overlapping. The total number of students in Beyondé's diagram is only 43.

17. a) A and B
b) A and C
c) Yes; B and C; e.g., C intersecting A and A and B being disjoint says nothing about the intersection, if any, of B and C.

18. e.g., The union of two sets is more like the addition of two numbers because all the elements of each set are counted together, instead of just those present in both sets.

19. a) e.g., indoor, outdoor, races
b) e.g., U = {all sports}, I = {indoor sports} = {badminton, basketball, curling, figure skating, gymnastics, hockey, indoor soccer, speed skating, table tennis, volleyball, wrestling, Arctic Sports, Dene Games}, O = {outdoor sports} = {alpine skiing, cross-country skiing, freestyle skiing, snowshoe biathlon, ski biathlon, dog mushing, snowboarding, snowshoeing, Dene Games}, R = {races} = {speed skating, alpine skiing, cross-country skiing, biathlon, dog mushing, snowboarding, snowshoeing}
c) e.g.,

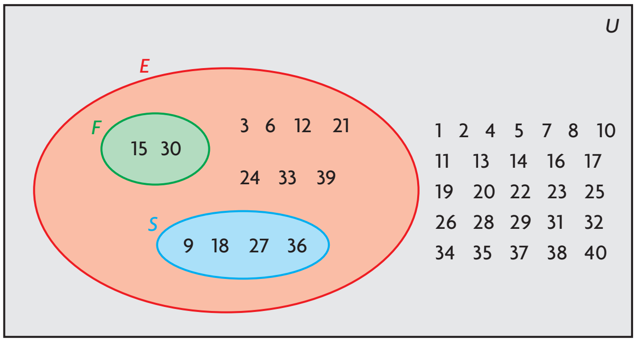


d) Yes

Mid-Chapter Review, page 178

1. a) $V \subset N$, $M \subset N$, $F \subset N$, $F \subset M$
b) e.g., N = {all foods}, V = {fruits and vegetables}, M = {meats}, F = {fish}
c) No. e.g., Pasta is not part of M or V.
d) V and M, V and F

2. a)



b) S and F

c) i) False. e.g., 6 is in E but not in F.

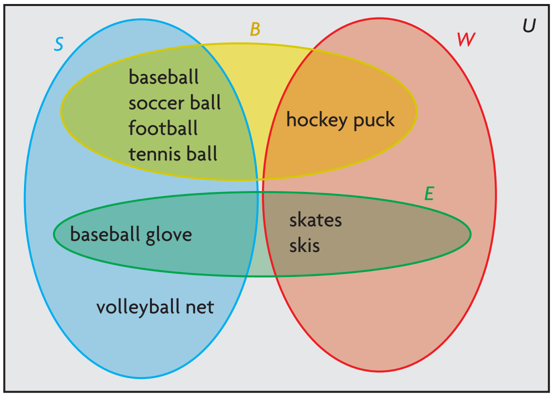
ii) True. e.g., All elements in S are in E.

iii) False. e.g., 9 is not a multiple of 15.

iv) True. e.g., $F = \{15, 30\}$

v) True. e.g., A set is a subset of itself.

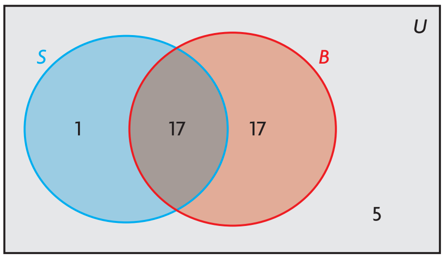
3. e.g.,



4. a) 17

b) 18

c)



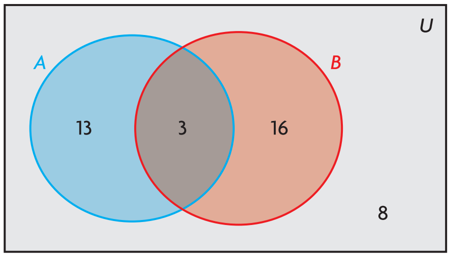
5. a) 4

b) 9

c) 2

6. a) e.g., Tanya did not put any elements in the intersection of A and B.

b)



7. 0

Lesson 3.4, page 191

1. e.g., $p = 14, q = 9, r = 12$

2. a) 32 **b)** 27 **c)** 63 **d)** 7

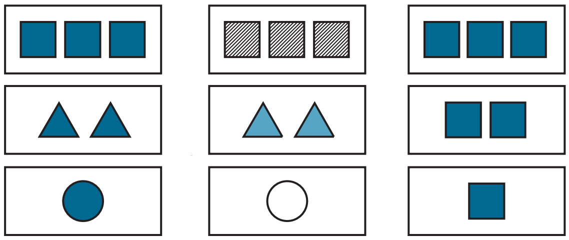
3. e.g., Staff could look at how many David Smiths were on that bus route or they could look at the books in the bag and see how many David Smiths are taking courses that use those books.

4. C = {contact lens wearers}, G = {glasses wearers}; $n(C \cap G) = 51$ or about 10.8%

5. e.g., “Canadian Rockies,” “ski accommodations,” “weather forecast,” “Whistler.” By combining two or more of these terms, Jacques can search for the intersection of web pages related to these terms. For example, “ski accommodations” and “Canadian Rockies” is more likely to give him useful information for his trip than either of those terms on its own.

6. 7

7. e.g.,



8. a) e.g., The dealer might use exterior colour, interior colour, or year.

b) e.g., The dealer might prioritize the search according to options Travis wants or by distance from where Travis lives.

9. e.g., He counted the students who like two of the three restaurants and undercounted those that like all three. 63

10. a) e.g., He can search for *colleges and* (*Calgary or Edmonton*).

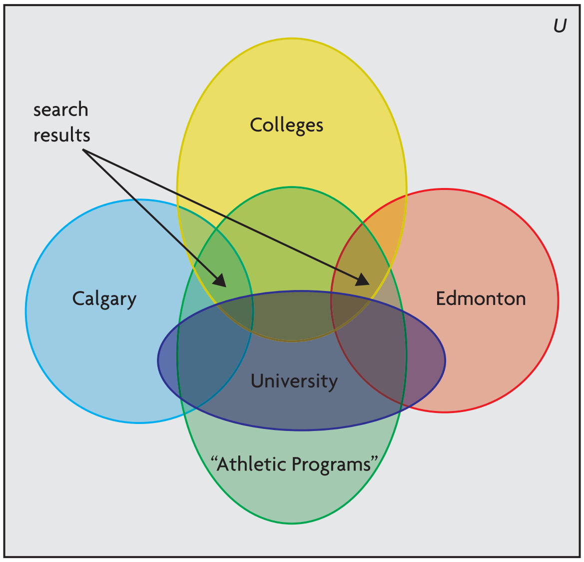
b) “and”

c) “or”

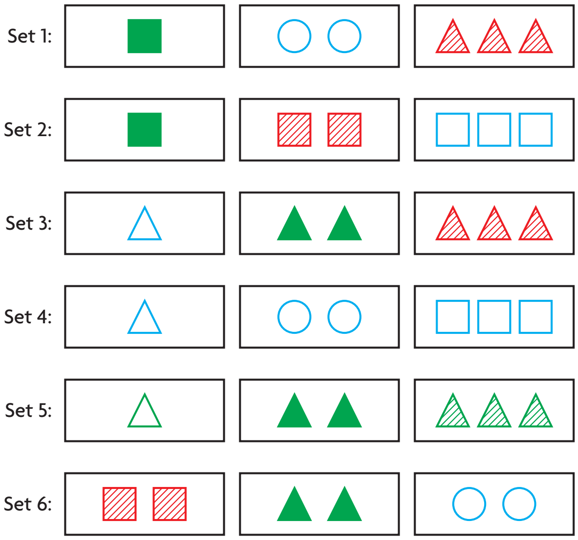
d) e.g., *colleges and* (*Calgary or Edmonton*) *and “athletics programs”-university*

e) e.g., about 1500

f) e.g.,



11.



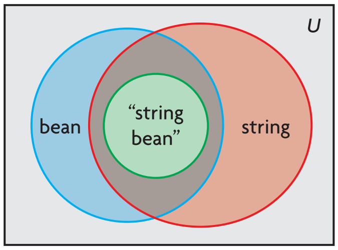
12. **a)** 81 **b)** 27 **c)** 27 **d)** 54 **e)** 45 **f)** 18

13. **a)** 36; e.g., 35 sites deal with boats. 21 sites deal with fishing, 20 of which also deal with boats, so only 1 deals with just fishing.

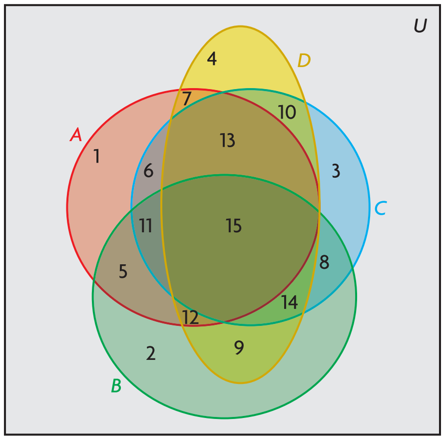
b) e.g., Because *fishing* and *boats* will turn up sites that deal with boats and fishing, but not just fishing boats.

c) 1

14. e.g., No, they did not get the same results. Elinor got all of James' results, plus others dealing with either *string* or *bean*, but not both.



15. **a)** and **b)** e.g.,



c) e.g., $1 = A \backslash (B \cup C \cup D)$; $2 = B \backslash (A \cup C \cup D)$;
$3 = C \backslash (A \cup B \cup D)$; $4 = D \backslash (A \cup B \cup C)$;
$5 = (A \cap B) \backslash (C \cup D)$; $6 = (A \cap C) \backslash (B \cup D)$;
$7 = (A \cap D) \backslash (B \cup C)$; $8 = (B \cap C) \backslash (A \cup D)$;
$9 = (B \cap D) \backslash (A \cup C)$; $10 = (C \cap D) \backslash (A \cup B)$;
$11 = (A \cap B \cap C) \backslash D$; $12 = (A \cap B \cap D) \backslash C$;
$13 = (A \cap C \cap D) \backslash B$; $14 = (B \cap C \cap D) \backslash A$;
$15 = A \cap B \cap C \cap D$

16. e.g., Let $B =$ {blue}, $Y =$ {yellow}, $R =$ {red}, and $G =$ {green}. There is no area representing $(B \cap R) \backslash (G \cup Y)$ or $(G \cap Y) \backslash (B \cup R)$.

Lesson 3.5, page 203

1. **a)** $p =$ I am swimming in the ocean; $q =$ I am swimming in salt water

b) True

c) If I am swimming in salt water, then I am swimming in the ocean. False. e.g., Some swimming pools are salt water.

2. **a)** Yes

b) If a number is divisible by 2, then it is divisible by 4; false

c) e.g., A counterexample of the converse is the number 2.

3. **a)** If it is an equilateral triangle, then it has three equal sides.

b) If it has three equal sides, then it is an equilateral triangle.

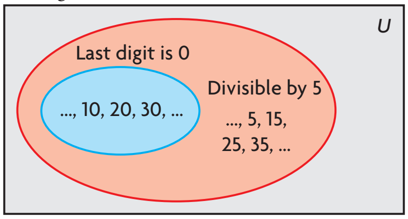
c) True; true

d) Yes. e.g., The statement and its converse are both true.

4. **a)** e.g., If we cannot get what we like, then let us like what we get.

b) Hypothesis: We cannot get what we like.; Conclusion: Let us like what we get.

5. a) No
b) If a number's final digit is 0, then it is divisible by 5.
c) Yes. e.g.,



6. a) not biconditional; e.g., You might live in Mexico.
b) biconditional; e.g., You live in the capital of Canada if and only if you live in Ottawa.

7. biconditional; e.g.,

$\sqrt{x^2} = x$	x is not negative	$\sqrt{x^2} = x \Rightarrow x$ is not negative
true	true	true
true	false	false
false	true	true
false	false	true

8. a) If a glass is half-empty, then it is half-full. A glass is half-empty if and only if the glass is half-full.
b) If a polygon is a rhombus, then it has equal opposite angles. Not biconditional; e.g., a rectangle.
c) If a number is a repeating decimal, it can be expressed as a fraction. Not biconditional; e.g., 0.3 can be expressed as $\frac{3}{10}$, but it is not a repeating decimal.

9. a) If *AB* and *CD* are parallel, then the alternate angles are equal. If the alternate angles are equal, then *AB* and *CD* are parallel.
b) True. e.g., Since *AB* and *CD* are parallel, *t* intersects them at the same angle, and the alternate angles must be equal. Since the alternate angles are equal, *t* must intersect both *AB* and *CD* at the same angle, so *AB* and *CD* must be parallel.
c) True. e.g., Both the conditional statement and its converse are true.

10. a) If your pet is a dog, then it barks. It is not biconditional.
b) If your pet wags its tail, then it is a dog. It is not biconditional.

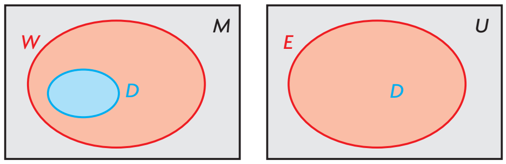
11. a) True
b) True

12. e.g., If a number appears in the same row, column, or large square as the shaded square, then it is not in the shaded square. The numbers in the column should be, from top to bottom: 8, 3, 9, 5, 6, 4, 1, 7, 2.

13. a) i) If a figure is a square, then it has four right angles.
ii) If a figure has four right angles, then it is a square.
iii) True; false. e.g., A rectangle has four right angles.
iv) n/a
b) i) If a figure is a right triangle, then $a^2 + b^2 = c^2$.
ii) If $a^2 + b^2 = c^2$, then the figure is a right triangle.
iii) True; true
iv) A figure is a right triangle if and only if $a^2 + b^2 = c^2$.
c) i) If a shape is a trapezoid, then it has two sides that are parallel.
ii) If a shape has two sides that are parallel, then it is a trapezoid.
iii) True; false. e.g., A regular hexagon has two sides that are parallel.
iv) n/a

14. a) i) \$1674.56
ii) \$836.16
b) 2164; e.g., If you can afford to make your mortgage payments more frequently, then you will save a lot of money on interest.

15. a) e.g., If it is December, then it is winter. If a number is even, then it is divisible by 2.
b) e.g.,



c) e.g., If the sets are the same (i.e., there is one area in the Venn diagram), then the converse is true. If there are two or more areas in the Venn diagram, then the converse is false.

16. a) e.g., If the first letter is a consonant, then the second letter is a vowel.
b) How wonderful it is that nobody need wait a single moment before starting to improve the world. –Anne Frank

17. a) 231 b) \$38 850

Lesson 3.6, page 214

1. a) converse: If you are looking in a dictionary, then you will find *success* before *work*.
inverse: If you do not find *success* before *work*, then you are not looking in a dictionary.
contrapositive: If you are not looking in a dictionary, then you will not find *success* before *work*.
b) converse: If you can drive, then you are over 16.
inverse: If you are not over 16, then you cannot drive.
contrapositive: If you cannot drive, then you are not over 16.
c) converse: If a quadrilateral's diagonals are perpendicular, then it is a square.
inverse: If a quadrilateral is not a square, then its diagonals are not perpendicular.
contrapositive: If a quadrilateral's diagonals are not perpendicular, then it is not a square.
d) converse: If $2n$ is an even number, then n is a natural number.
inverse: If n is not a natural number, then $2n$ is not an even number.
contrapositive: If $2n$ is not an even number, then n is not a natural number.

2. a) converse: If an animal is a giraffe, then it has a long neck.
contrapositive: If an animal is not a giraffe, then it does not have a long neck.
b) No. e.g., Ostriches have long necks. No. e.g., Llamas have long necks.

3. a) converse: If a polygon is a pentagon, then it has five sides.
inverse: If a polygon does not have five sides, then it is not a pentagon.
b) Yes. e.g., The definition of a pentagon is that it has five sides.

4. a) Disagree. e.g., x could be -5.
b) Yes. e.g., If $x = 5$, then $x^2 = 25$.
c) Yes. e.g., If x^2 is not 25, then x is not equal to 5.
d) No. e.g., $(-5)^2 = 25$

5. a) i) True
ii) If you are in the Northwest Territories, then you are in Hay River; false.
iii) If you are not in Hay River, then you are not in the Northwest Territories; false.
iv) If you are not in the Northwest Territories, then you are not in Hay River; true.

b) i) True
ii) If a puppy is not female, then it is male; true
iii) If a puppy is not male, then it is female; true
iv) If a puppy is female, then it is not male; true
c) i) True
ii) If the Edmonton Eskimos were number 1 in the West, then they would have won every game; false
iii) If the Edmonton Eskimos did not win every game this season, then they would not be number 1 in the West; false
iv) If the Edmonton Eskimos were not number 1 in the West, then they would not have won every game; true
d) i) False. e.g., The number could be zero.
ii) If an integer is positive, then it is not negative; true
iii) If an integer is negative, then it is not positive; true
iv) If an integer is not positive, then it is negative; false

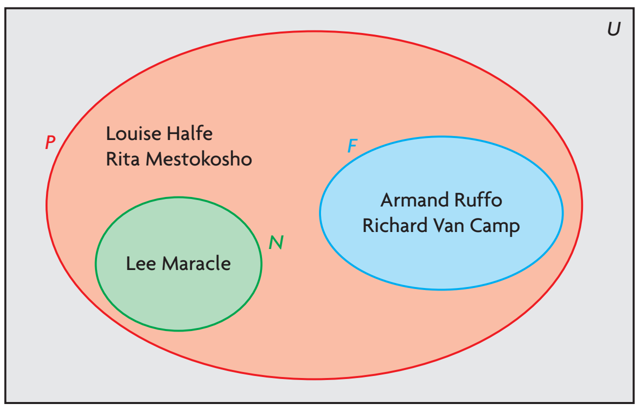
6.

	Conditional Statement	Inverse	Converse	Contrapositive
a)	true	false	false	true
b)	true	true	true	true
c)	true	false	false	true
d)	false	true	true	false

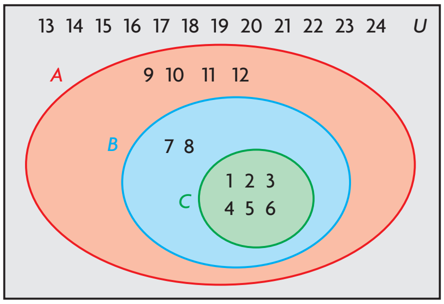
7. a) They are either both true or both false.
b) They are either both true or both false.
8. a) e.g., no conclusion
b) e.g., no conclusion
9. a) converse: If a polygon is a quadrilateral, then it is a square.
inverse: If a polygon is not a square, then it is not a quadrilateral.
contrapositive: If a polygon is not a quadrilateral, then it is not a square.
b) conditional statement: True
converse: False; e.g., trapezoid
inverse: False; e.g., trapezoid
contrapositive: True, e.g., A square is a quadrilateral.
10. a) converse: If the y-intercept is 2, then the equation of the line is $y = 5x + 2$.
inverse: If the equation of the line is not $y = 5x + 2$, then the y-intercept is not 2.
contrapositive: If the y-intercept is not 2, then the equation of the line is not $y = 5x + 2$.
b) converse: False; e.g., $y = x + 2$
inverse: False; e.g., $y = x + 2$
contrapositive: True
11. e.g., It is biconditional.
12. a) i) e.g., False. A pin will not burst a hot air balloon.
ii) False. e.g., A pin can burst a bubble.
iii) False. e.g., The Moon could be a bubble.
iv) e.g., False. The Moon could be a hot air balloon.
b) i) True
ii) True
iii) True
iv) True
c) i) True
ii) False; e.g., 3 is not a perfect square.
iii) False, e.g., 3 is not a perfect square.
iv) True
d) i) True
ii) False. e.g., $\frac{1}{3}$ cannot be written as a decimal.
iii) False. e.g., $\frac{1}{3}$ cannot be written as a decimal.
iv) True
e) i) True
ii) False. e.g., $y = x^2$
iii) False. e.g., $y = x^2$
iv) True
f) i) False. e.g., -1 is not a whole number.
ii) True
iii) True
iv) False. e.g., -1 is an integer.
g) i) True
ii) False. e.g., I might be 20.
iii) False. e.g., I might be 20.
iv) True
h) i) False. e.g., Some Canadians do not enjoy hockey.
ii) False. e.g., Some Americans enjoy hockey.
iii) False. e.g., Some Americans enjoy hockey.
iv) False. e.g., Some Canadians do not enjoy hockey.
13. a) e.g., The contrapositive assumes as its hypothesis that the original conclusion is false, which means that the original hypothesis must also not be true. If the original hypothesis is not true, then the conditional statement must be false.
b) e.g., The inverse of a statement is the contrapositive of the statement's converse.
14. a) e.g., If you are tall, then you like chocolate.
contrapositive: If you do not like chocolate, then you are not tall. This is false: I am tall and do not like chocolate.
b) e.g., If a traffic light is green, it is not red.
contrapositive: If a traffic light is red, it is not green. This is true: A traffic light cannot be two colours.
15. a) e.g., If it is Saturday, then it is the weekend.
inverse: If it is not Saturday, then it is not the weekend.
converse: If it is the weekend, then it is Saturday.
b) e.g., If a polygon has six sides, then it is a hexagon.
Inverse: If a polygon does not have six sides, then it is not a hexagon.
converse: If a polygon is a hexagon, then it has six sides.

Chapter Self-Test, page 217

1. e.g.,



2. **a)** e.g.,



b) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$; $n(A \cup B) = 12$; $A \cap C = \{1, 2, 3, 4, 5, 6\}$; $n(A \cap C) = 6$
c) $\{7, 8\}$
d) $\{13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24\}$

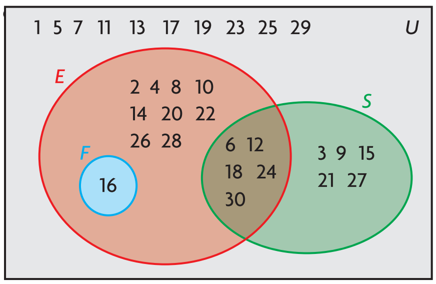
3. 8%

4. **a)** If you win an election, then you got the most votes.
inverse: If you do not win an election, then you did not get the most votes.
The statement is not biconditional; e.g., in some electoral systems, you need a majority to win.
b) If the planet is Earth, then it is the third planet from the Sun.
inverse: If the planet is not Earth, then it is not the third planet from the Sun.
biconditional: The planet is Earth if and only if it is the third planet from the Sun.
c) If a number is between 1 and 2, then it is not a whole number.
inverse: If a number is not between 1 and 2, then it is a whole number.
The statement is not biconditional. e.g., 3.5 is not between 1 and 2 and is not a whole number.

5. **a)** **i)** False. e.g., The age of majority in British Columbia is 19.
ii) True
iii) True
iv) False. e.g., The age of majority in British Columbia is 19.
b) **i)** False. e.g., A 44-year-old may know how to drive.
ii) False. e.g., You might be older.
iii) False. e.g., You might be older.
iv) False. e.g., A 16-year-old may know how to drive.

Chapter Review, page 220

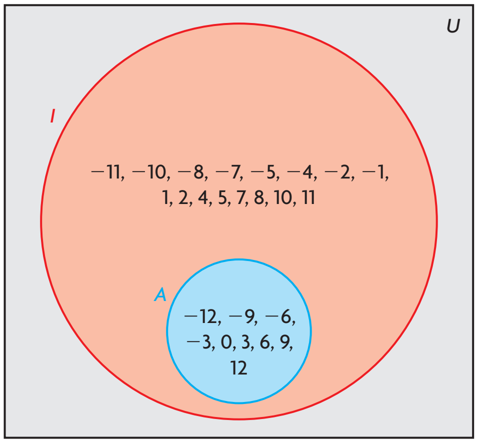
1. **a)**



b) F and S
c) $F \subset E$
d) $S' = \{$natural numbers from 1 to 30 not divisible by 3$\}$; It is different from E', which is the set of natural numbers that are not multiples of 2.
e) e.g., $H = \{$multiples of 50$\}$

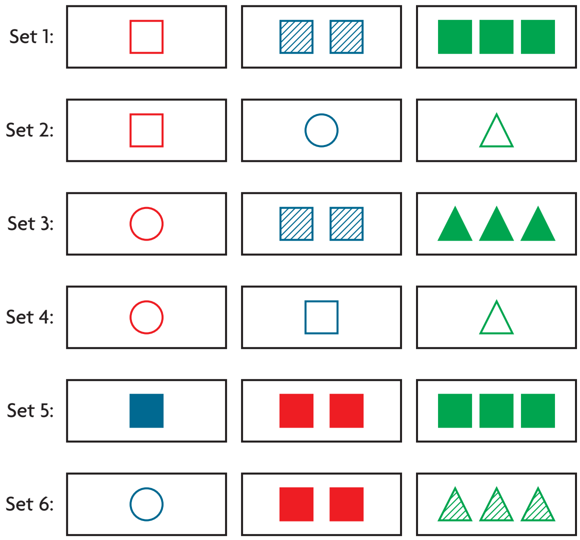
2. **a)** 8; e.g., Of the 19 students with black hair, 8 have blue eyes.
b) 11
c) 0

3. **a)** $A \cup B = \{-12, -11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$; $n(A \cup B) = 25$; $A \cap B = \{-12, -9, -6, -3, 0, 3, 6, 9, 12\}$; $n(A \cap B) = 9$
b)



4. 1

5.



6. **a)** biconditional; x is positive if and only if $10x > x$.
b) not biconditional; e.g., You might live in Port Hardy.
c) biconditional; xy is an odd number if and only if both x and y are odd numbers.
d) not biconditional; e.g., Two odd numbers have an even sum.

7. **a)** \$443.26
b) 11 months

8. **a)** conditional statement: true
converse: false, e.g., The number could be zero.
inverse: false, e.g., The number could be zero.
contrapositive: true
b) conditional statement: true
converse: false, e.g., Friday could be a holiday.
inverse: false, e.g., Friday could be a holiday.
contrapositive: true

Chapter 4 – Counting Methods

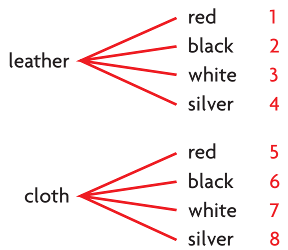
Lesson 4.1, page 235

1. a) There are six outfit variations.

	khaki	black
red	red/khaki	red/black
blue	blue/khaki	blue/black
green	green/khaki	green/black

b) $O =$ Number of different outfits
$O =$ (Number of shirts) $\cdot$ (Number of shorts)
$O = 3 \cdot 2$
$O = 6$

2. a) **Upholstery** **Colour**

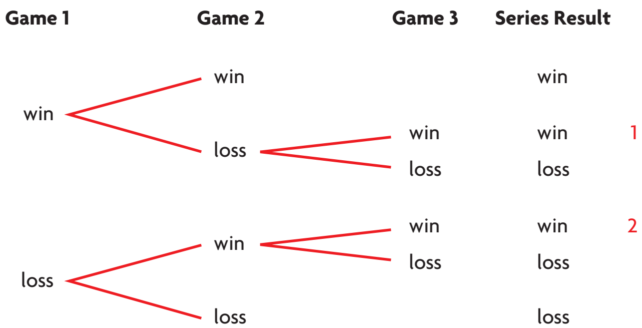


There are eight different choices of upholstery and colour.

b) $C =$ Number of different upholstery-colour choices
$C =$ (Number of upholstery options) $\cdot$ (Number of colours)
$C = 2 \cdot 4$
$C = 8$

3. a) No. e.g., The possibilities are related by the word OR.
b) Yes. e.g., The outfit consists of a shirt, a tie, AND shoes.
c) No. e.g., The possibilities are related by the word OR.
d) Yes. e.g., The possibilities include type of transmission, air conditioning option, type of window, AND GPS option.

4. a)



b) There are two ways to win the series despite having one loss.

5. 20
6. 360
7. 120
8. 18
9. a) 59 049 b) 15 120
10. 256
11. a) 17 576 000 b) 9 261 000
12. 20 000
13. 19 683
14. 65
15. a) 320 b) 4
16. a) 13 824 000 b) 124 416 000
17. e.g., If multiple tasks are related by AND, it means the Fundamental Counting Principle can be used and the total number of solutions is the product of the solutions to each task. For example: A 4-digit PIN involves choosing the 1st digit AND the 2nd digit AND the 3rd digit AND the 4th digit. So the number of solutions is $10 \cdot 10 \cdot 10 \cdot 10 = 10\ 000$. OR means the solution must meet at least one condition so you must add the number of solutions to each condition, and then subtract the number of solutions that meet more than one condition. For example: Calculating the number of 4-digit PINs that start with 3 OR end with 3. The solution is the number of PINs that start with 3, plus the number of PINs that end with 3, minus the number of PINs that both start and end with 3: $1000 + 1000 - 100 = 1900$.
18. a) i) 8 in 52 or 2 in 13
ii) 26 in 52 or 1 in 2
iii) 16 in 52 or 4 in 13
b) No, e.g., because the Fundamental Counting Principle only applies when tasks are related by the word AND.
19. 36
20. 1 in 1024
21. 359

Lesson 4.2, page 243

1. a) 720 b) 362 880 c) 20 d) 8 e) 12 f) 2520
2. a) $3 \cdot 2 \cdot 1 = 6$ b) $3!$
3. a) $5!$ b) $\frac{9!}{6!}$ c) $\frac{15!}{12! \cdot 4!}$ d) $\frac{100!}{98!}$
4. **a)**, **c)**, and **d)**; e.g., Factorial notation is only defined for natural numbers.
5. a) 40 320 c) 28 e) 720
b) 132 d) 42 f) 33
6. a) n
b) $(n + 4)!$
c) $n + 1$
d) $n(n - 1)(n - 2)$
e) $(n + 5)(n + 4)$
f) $\frac{1}{(n - 1)}$
7. $9! = 362\ 880$
8. $5! = 120$
9. $6! = 720$
10. $28!$ or about 3.05×10^{29}
11. a) $n = 9$ b) $n = 1$ c) $n = 9$ d) $n = 6$
12. 40 320
13. $7! = 5040$, e.g., There are 7 numbers and each one can be used only once.
14. 120

15. 5040

16. a) e.g., YKONU, YUKNO, YKNOU
 b) 5!, e.g., Each letter can be used only once.

17. a) $n \geq 4$
 b) $\{1, 2, 3\}$

18. 725 760

Lesson 4.3, page 255

1. a) 20 b) 20 160 c) 30 240 d) 1 e) 5040 f) 360 360

2. a) e.g.,

	President	Vice-President
1	Katrina	Jess
2	Katrina	Nazir
3	Katrina	Mohamad
4	Jess	Katrina
5	Jess	Nazir
6	Jess	Mohamad
7	Nazir	Jess
8	Nazir	Katrina
9	Nazir	Mohamad
10	Mohamad	Nazir
11	Mohamad	Jess
12	Mohamad	Katrina

 b) ${}_4P_2 = 12$

3. a) 360
 b) 6

4. e.g., ${}_{10}P_8$ is greater than ${}_{10}P_2$; there are more ways to arrange 8 of the 10 objects than 2 of the 10 objects.

5. 504

6. 32 760

7. 40 320

8. 124 925 010 000

9. 100 000 000 in each group

10. a) 95 040 b) 7920 c) 1440

11. a) $n \geq 1$ b) $n \geq -2$ c) $n \geq 0$ d) $n \geq -3$

12. a) 360
 b) 1296
 c) e.g., Yes; if you replace the marble, there are more possibilities for the next draw.

13. a) 1 860 480 b) 3 200 000

14. a) 5040 b) 4960

15. a) $n = 5$ b) $n = 8$

16. a) $r = 2$ b) $r = 3$

17. For $n \in \mathrm{N}$, ${}_nP_n = \dfrac{n!}{(n-n)!} = \dfrac{n!}{0!} = \dfrac{n!}{1} = n!$ and

$${}_nP_{n-1} = \frac{n!}{(n-(n-1))!} = \frac{n!}{1!} = \frac{n!}{1} = n!.$$

18. a) e.g., The formulas for both ${}_nP_n$ and ${}_nP_r$ have a numerator of $n!$. However, the formula for ${}_nP_n$ has a denominator of 1 and the formula for ${}_nP_r$ has a denominator of $(n-r)!$.
 b) e.g., A group of friends each order a different flavour of ice cream from a shop with 12 flavours. How many possibilities are there if the group is 12 people? If the group is 7 people?

19. a) 311 875 200
 b) 7 893 600 in 311 875 200 or about 2.53%
 c) 154 400 in 311 875 200 or about 0.05%

20. e.g.,

$$\begin{aligned}
{}_{n+1}P_2 - {}_nP_1 &= \frac{(n+1)!}{(n+1-2)!} - \frac{n!}{(n-1)!} \\
&= \frac{(n+1)!}{(n-1)!} - \frac{n!}{(n-1)!} \\
&= \frac{(n+1)! - n!}{(n-1)!} \\
&= \frac{n+1 \cdot n \cdot n-1 \cdot n-2 \ldots - n \cdot n-1 \cdot n-2 \ldots}{n-1 \cdot n-2 \ldots} \\
&= \frac{n \cdot n-1 \cdot n-2 \ldots (n+1-1)}{n-1 \cdot n-2 \ldots} \\
&= n(n) \\
&= n^2
\end{aligned}$$

21. e.g.,

$$\begin{aligned}
{}_nP_{r+1} &= \frac{n!}{(n-(r+1))!} \\
&= \frac{n!}{(n-r-1)!} \\
&= \frac{n!}{(n-r-1)!} \cdot \frac{n-r}{n-r} \\
&= \frac{(n-r)n!}{(n-r)!} \\
&= (n-r)\frac{n!}{(n-r)!} \\
&= (n-r){}_nP_r
\end{aligned}$$

Mid-Chapter Review, page 259

1. 1620

2. 54 756

3. 4

4. 720

5. a) 40 320 b) 4320 c) 504 d) 18

6. 362 880

7. a) $(n+5)!$ c) $n-4$
 b) $(n+4)(n+3)$ d) $(n+2)(n+1)$

8. a) $n = 9$ b) $n = 7$

9. a) 72 c) 120
 b) 19 958 400 d) 239 500 800

10. a) a: $n \geq -4$, b: $n \geq -2$, c: $n \geq 5$, d: $n \geq 0$
 b) a: $n \geq 2$, b: $n \geq 3$

11. 27 907 200

12. 182

13. Agree. e.g., The number of ways to choose a president and a vice-president from a group of five students is $\dfrac{5!}{(5-2)!} = 20$. I could also use the Fundamental Counting Principle because there are five choices for president and four choices remaining for vice-president: $5 \cdot 4 = 20$.

Lesson 4.4, page 266

1. a) 420 b) 5040 c) 12 600 d) 83 160
2. 90
3. 20
4. 2 450 448
5. 1260
6. a) 120 c) 20 160
   b) 2520 d) 39 916 800
7. a) 756 756 b) 6
8. e.g., A shish kabob skewer has 4 pieces of beef, 2 pieces of green pepper, and 1 piece each of mushroom and onion. How many different combinations are possible?
9. a) 126 b) 1716
10. 1287
11. a) 560 b) 180
12. 56
13. a) 5040 b) 1260
14. e.g., ${}_nP_n$ will be too high; it gives the number of arrangements of all n items, but some of the arrangements will be identical because of the a identical items in the group.
15. a) 1260 ways, assuming each group consists of identical coins.
    b) 35 ways, assuming each group consists of identical coins.
16. 168
17. a) 560 b) 10 080
18. e.g., BANDITS has 7 different letters, so the number of permutations is 7! BANANAS also has 7 letters, but there are 3 As and 2 Ns so you must divide 7! by $3! \cdot 2! = 12$.
19. 1680
20. a) about 2.38×10^{15}
    b) about 3.06×10^{11}
21. a) 14
    b) $\frac{1}{14}$

Lesson 4.5, page 272

1. a) 6
   b)

Canned Goods	Dry Goods	Fruits and Vegetables
Brian	Rachelle	Linh
Brian	Linh	Rachelle
Rachelle	Brian	Linh
Rachelle	Linh	Brian
Linh	Rachelle	Brian
Linh	Brian	Rachelle

   c) 1
   d) Parts a) and b) involve permutations because the order matters; part c) involves combinations because the order does not matter.
2. e.g., Order matters for the permutations but not for the combinations. One group of 4 objects is one combination, but the 4 objects can be put in 4! = 24 different orders to make 24 different permutations.
3. 210
4. 220

Lesson 4.6, page 280

1. a)

Flavour 1	Flavour 2
vanilla	strawberry
vanilla	chocolate
vanilla	butterscotch
strawberry	vanilla
strawberry	chocolate
strawberry	butterscotch
chocolate	vanilla
chocolate	strawberry
chocolate	butterscotch
butterscotch	vanilla
butterscotch	strawberry
butterscotch	chocolate

   b)

vanilla	strawberry
vanilla	chocolate
vanilla	butterscotch
strawberry	chocolate
strawberry	butterscotch
chocolate	butterscotch

   c) The number of two-flavour permutations is double the number of two-flavour combinations because each two-flavour combination can be written in two different ways.
2. a) 10
   b) 10
   c) e.g., The numbers are the same because by choosing 3 out of 5 people for a committee, you are also choosing 2 out of 5 people not to be on the committee. Therefore, the number of ways of choosing 3 out of 5 is the same as the number of ways of choosing 2 out of 5.
3. 924
4. a) 10 c) 15 e) 924
   b) 9 d) 1 f) 8
5. 210
6. 3 478 761
7. 752 538 150
8. a) combinations; e.g., The order within the starting lineup is not important.
   b) 3003

9. **a)** Agree. e.g., I calculated the values and both are 15.
b) In each case, they are equal.
c) $\binom{n}{r} = \binom{n}{n-r}$

10. 560

11. **a)** 252 **b)** 120 **c)** 6 **d)** 6 **e)** 66

12. 27 720

13. **a)** **i)** 5 objects, 3 in each combination
ii) 10 objects, 2 in each combination
iii) 5 objects, 3 in each combination
b) e.g., i) How many ways can you choose 3 coins from a bag containing a penny, a nickel, a dime, a quarter, and a loonie?

14. **a)** **i)** 1
ii) 1, 1
iii) 1, 2, 1
iv) 1, 3, 3, 1
v) 1, 4, 6, 4, 1
b)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

c) e.g., The numbers on the left and right sides are all 1s; every other number is the sum of the two numbers above it.
d) 1, 5, 10, 10, 5, 1; 1, 6, 15, 20, 15, 6, 1
e) e.g., The number in each square of Pascal's Triangle is equal to the number of pathways to it from the top square.

15. **a)** $n = 6, n \geq 2$
b) $n = 7, n \geq 4$
c) $n = 7$ or $n = 2, n \geq 2$
d) $r = 2$ or $r = 4, 0 \leq r \leq 6$

16. **a)** 1
b) 90 858 767
c) e.g., No. Even if everyone in the city plays, it is very unlikely that anyone will win since each player only has a 1 in 90 858 767 chance of winning.

17. The number of diagonals is given by ${}_nC_2 - n$.

18. **a)** 2 boys, 3 girls: $({}_7C_2) \cdot ({}_{13}C_3) = 6006$
3 boys, 2 girls: $({}_7C_3) \cdot ({}_{13}C_2) = 2730$
4 boys, 1 girl: $({}_7C_4) \cdot ({}_{13}C_1) = 455$
5 boys: ${}_7C_5 = 21$
$6006 + 2730 + 455 + 21 = 9212$
b) 1 boy, 4 girls: $({}_7C_1) \cdot ({}_{13}C_4) = 5005$
5 girls: ${}_{13}C_5 = 1287$
${}_{20}C_5 - 5005 - 1287 = 9212$
c) e.g., indirect reasoning; fewer calculations are needed.

19. **a)** e.g., Combinations and permutations both involve choosing objects from a group. For permutations, order matters. For combinations, order does not matter. For example, *abc* and *bac* are different permutations, but the same combination.
b) Divide ${}_nP_r$ by $r!$ to get ${}_nC_r$. For example, ${}_6C_4 = 15$ and ${}_6P_4 = 360; \frac{360}{4!} = 15$

20. **a)** 65 780 in 13 019 909 or about 0.51%
b) 544 320 in 13 019 909 or about 4.18%
c) 1 in 13 019 909

21. $\frac{n^3 + 5n}{6}$

22. ${}_{n+1}C_r = \frac{(n+1)!}{r!(n+1-r)!}$

$$\begin{aligned}
{}_nC_r + {}_nC_{r-1} &= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-(r-1))!}\\
&= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}\\
&= \frac{n!(n-r+1)!}{r!(n-r)!(n-r+1)!} + \frac{n!r!}{r!(r-1)!(n-r+1)!}\\
&= \frac{n!(n-r+1)}{r!(n-r+1)!} + \frac{r \cdot n!}{r!(n-r+1)!}\\
&= \frac{n!(n-r+1+r)}{r!(n-r+1)!}\\
&= \frac{n!(n+1)}{r!(n+1-r)!}\\
&= \frac{(n+1)!}{r!(n+1-r)!}
\end{aligned}$$

Lesson 4.7, page 288

1. **a)** combinations; e.g., It doesn't matter what order you choose the toppings.
b) permutations; e.g., Changing the order changes the role of each candidate.
c) permutations; e.g., Each die is different.
d) permutations; e.g., Changing the order changes the position played.

2. Situation A: combinations; e.g., Order does not matter.
Situation B: permutations; e.g., Changing the order changes who holds which position.

3. **a)** 1 **b)** 56

4. 28 561

5. **a)** 304 278 004 800
b) 320 000 000 000

6. 180

7. 113 400

8. 300

9. 240

10. 3 628 800, assuming every player is willing to sit in any seat

11. **a)** 60
b) 6

12. 125 000

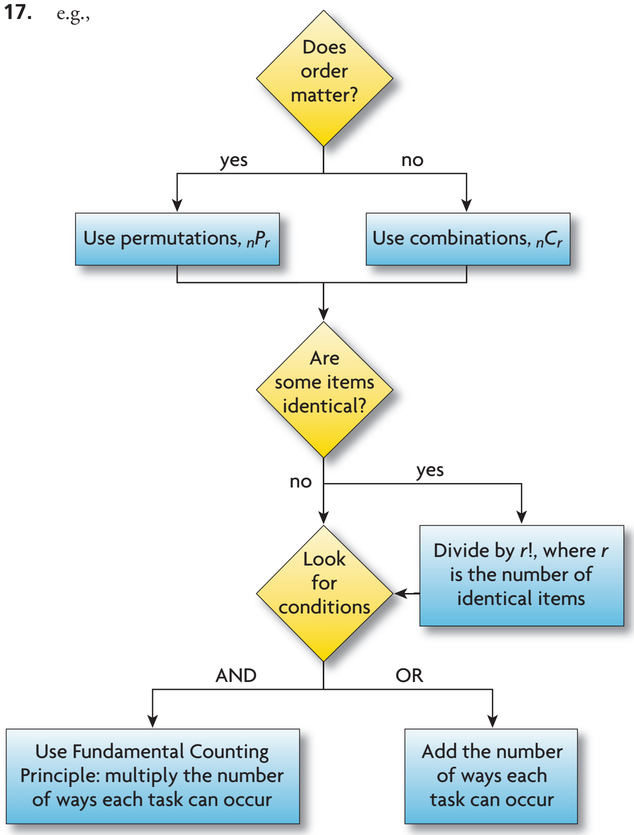
13. 462

14. 1 441 440

15. 24

16. 2 569 788

17. e.g.,



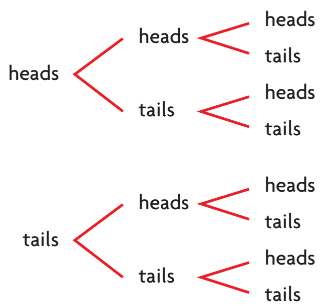
18. 700 in 1716 or about 40.8%
19. 72
20. 33

Chapter Self-Test, page 291

1. **a)** 2 028 000
b) 1 296 000
2. 26
3. **a)** $(n + 10)!$, $n \geq -10$
b) $\frac{1}{n^2 - n}$, $n \geq 2$
4. **a)** 120 **b)** 48
5. **a)** 126
b) 3024
c) Order matters in part b). e.g., Each of the 126 groups of four books can be put in $4! = 24$ different orders.
6. $n = 9$ since $n \geq 2$
7. **a)** 840 **c)** 220
b) 1526 **d)** 1316
8. 30
9. 2880

Chapter Review, page 293

1. e.g., The Fundamental Counting Principle is used when a counting problem has different tasks related by the word AND. For example, you can use it to figure out how many ways you can roll a 3 with a die and draw a red card from a deck of cards.
2. **Quarter** **Toonie** **Loonie**



3. 1 048 576
4. **a)** $n = 3, n \geq 0$
b) $n = 11, n \geq 1$
5. $_6P_6$; e.g., $_6P_6$ is the number of ways to arrange 6 different objects, while $\frac{8!}{6!}$ is the number of ways to arrange only 2 of 8 objects.
6. 479 001 600
7. 13 800
8. **a)** 11 861 676 288 000
b) 19 769 460 480
9. 311 875 200
10. **a)** 4 989 600 **b)** 453 600
11. **a)** 2 522 520 **b)** 27 720
12. $_{11}C_7$
13. 4845
14. No. e.g., Each combination can be arranged in many different ways to make a permutation, so there are more permutations than combinations.
15. **a)** 3876 **b)** 1620 **c)** 210
16. 756 756
17. 66
18. **a)** 195 955 200
b) 39 916 800
19. 844 272

Chapter 5 – Probability

Lesson 5.1, page 303

1. e.g., Reverse the rules for players 1 and 2 on each turn.
2. a) fair
   b) not fair; Gina; e.g., She has a 6 in 8 chance of winning.
   c) fair
3. No. e.g., A certain chance is 100%.
4. No; Player 2; e.g., Player 2 has more opportunities to win.

Lesson 5.2, page 310

1. a) 3 : 5 b) 0.625
2. a) 0.3 b) 7 : 3
3. a) 0.5 b) 1 : 1 c) 3 : 1 d) $\frac{3}{13}$ or about 0.231
4. a) 2 : 3 b) 3 : 2
5. 0.7
6. a) 0.5
   b) 1 : 1
   c) e.g., The odds against and the odds in favour are both 1 : 1.
7. 2 : 3
8. 2 : 23
9. No. e.g., The odds are 1 : 4, but the probability is $\frac{1}{5}$, or 20%.
10. a) 1 : 1
    b) 1 : 4
    c) e.g., Jason's data reflects his record against all goalies, not just Gilles. Gilles' data suggests that he is better than average at blocking penalty shots.
11. 0.62
12. Yes. e.g., The probability of a win is 3 in 5 (60%), the probability of a loss is 1 in 5 (20%), and the probability of a tie is 1 in 5 (20%). The probabilities add up to 100%.
13. a) 13 : 7 b) 2 : 3
14. a) 7 : 3
    b) 29 : 21
    c) 49 : 21, 29 : 21
    d) e.g., Yes, because your chances of not getting sick are much better if you are vaccinated.
15. a) touchdown: 5 : 7; field goal: 5 : 1
    b) field goal
16. a) Eduard: 9 : 11; Julie: 7 : 13; Bill: 1 : 4
    b) 11 : 9
17. a) e.g., Yes. If he pays the $65, he can reduce the 45% chance of having to pay the additional $235.
    b) No. e.g., With odds of 17 : 4, Grant has about a 19% chance of failing even without the practice exams, so he should probably not buy them.
       Yes. e.g., With odds of 3 : 7, his chance of failing is 70% without the practice exams, so he should buy them.
18. a) e.g., If the odds for an event are $m:n$, then $P(A) = \frac{m}{m+n}$ and $P(A') = \frac{n}{m+n}$, so $P(A'):P(A) = \frac{n}{m+n}:\frac{m}{m+n}$. This ratio is equal to $n:m$.
    b) e.g., The probability of the event happening is $\frac{a}{a+b}$. If the odds in favour of rain tomorrow are 2 : 3, then the probability is $\frac{2}{2+3} = \frac{2}{5}$, or 40%.
    c) e.g., The odds against the event happening are $c - a:a$. If the probability of winning the lottery is $\frac{1}{1\,000\,000}$, the odds against are 999 999 : 1.
19. e.g., I prefer using probability because if I express the probability as a percent, it tells me how many times out of a hundred I could expect the event to occur. e.g., I prefer using odds because it compares the chances for and against the event occurring.
20. a) 0.149 b) about 105 : 895

Lesson 5.3, page 321

1. $\frac{120}{10\,000}$ or 0.012
2. $\frac{1287}{752\,538\,150}$ or about 0.000 001 71
3. $\frac{1}{66}$ or about 0.0152
4. a) $\frac{1}{66}$ or about 0.0152
   b) $\frac{5}{11}$ or about 0.455
   c) $\frac{23}{66}$ or about 0.348
5. a) $\frac{1}{325}$ or about 0.003 08
   b) $\frac{1}{338}$ or about 0.002 96
6. a) $\frac{1}{126}$ or about 0.0079
   b) e.g., new probability: $\frac{1}{70}$ or about 0.0143
7. 1 : 5
8. a) $\frac{1}{2730}$ or about 0.000 366 b) $\frac{1}{455}$ or about 0.0022
9. a) $\frac{4994}{9990}$ or about 0.4999
   b) $\frac{99}{9990}$ or about 0.009 91
   c) $\frac{2500}{9990}$ or about 0.250
10. 1680 : 3127
11. $\frac{15}{16}$ or 0.9375
12. a) $\frac{48}{120}$ or 0.4
    b) $\frac{72}{120}$ or 0.6
13. a) $\frac{5}{70}$ or about 0.0714
    b) $\frac{10}{56}$ or about 0.179
14. a) $\frac{4}{752\,538\,150}$ or about 0.000 000 005 32
    b) $\frac{3\,124\,550}{752\,538\,150}$ or about 0.004 15
    c) $\frac{45\,238\,050}{752\,538\,150}$ or about 0.0601

15. $\frac{12}{35}$ or about 0.343

16. $\frac{10\,080}{40\,320}$ or 0.25

17. $\frac{6}{35}$ or about 0.171

18. e.g., I would use permutations in a problem where the order of the items was important, and use combinations in a problem where order was not important. Permutations: Determine the probability that two items are next to each other in a lineup of seven different items that has been placed in a random order. Combinations: Determine the probability that, given eight books, four of which are about math, if I choose five of the eight books, I choose three math books.

19. $\frac{60}{126}$ or about 0.476

20. 35

Mid-Chapter Review, page 327

1. **a)** not fair; Ethan; e.g., There are more products that are greater than the sums.
b) fair

2. **a)**

Sums on Spinners					
	1	**2**	**3**	**4**	**5**
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9

b) **i)** 0.2
ii) 0.15
iii) 0.15

3. **a)** 0.5
b) 1 : 1
c) 3 : 1
d) $\frac{40}{52}$ or about 0.769

4. $\frac{5}{28}$ or about 0.179

5. **a)** slightly less than 1 : 1
b) slightly more than 1 : 1
c) Yes

6. $\frac{1}{364}$ or about 0.002 75

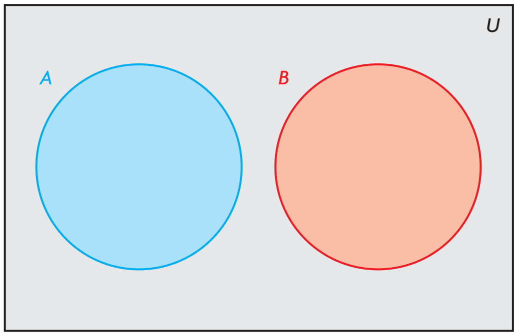
7. **a)** $\frac{80}{42\,504}$ or about 0.001 882
b) $\frac{1584}{42\,504}$ or about 0.037 267
c) $\frac{120}{42\,504}$ or about 0.002 823

8. about 0.000 042 1

9. $\frac{90}{220}$ or about 0.409

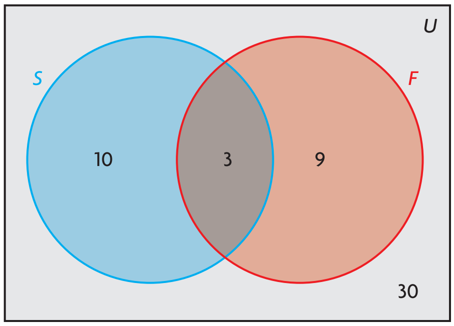
Lesson 5.4, page 338

1. **a)** $A = \{1, 1\}$, $B = \{4, 4\}$



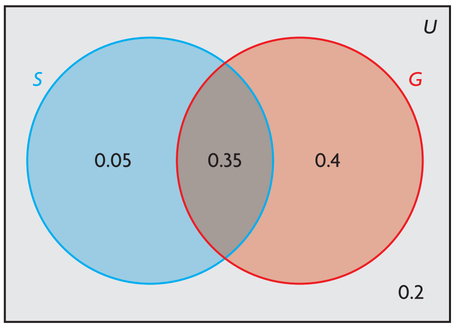
b) mutually exclusive
c) 0.125
d) 0.375

2. **a)**



b) No
c) $\frac{22}{52}$ or about 0.423

3. **a)**



b) No
c) 0.8

4. **a)** No. e.g., 2 is both an even number and a prime number.
b) Yes. e.g., You cannot roll a sum of 10 and a sum of 7 at the same time.
c) Yes. e.g., You cannot walk and ride to school at the same time.

5. **a)** $\frac{144\,945}{389\,045}$ or about 0.373
b) $\frac{119\,920}{389\,045}$ or about 0.308
c) Yes
d) 264 865 : 124 180

6. 12 : 28 or 3 : 7

7. **a)** 28 : 8 or 7 : 2 **b)** 25 : 11

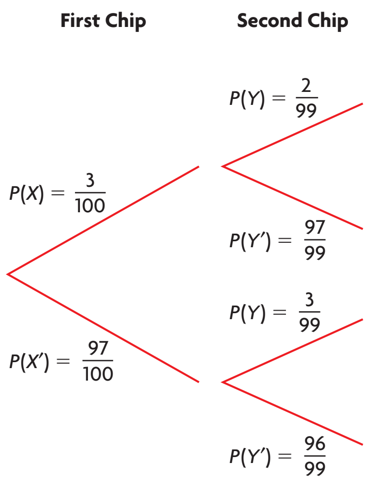
8. a) 0.2 b) No
9. a) No. e.g., One athlete won two or more medals at the Summer and Winter Olympics.
   b) 21 : 286
   c) 68 : 239
10. e.g., Tricia has a probability of 0.3 of cycling to school on any given day, and a probability of 0.2 of getting a ride from her older brother, Steve. Otherwise, she walks to school. What is the probability that she does not walk to school on any given day?
11. e.g., There are 67 Grade 10 students that take art and 37 that take photography. If there are 84 students, how many take both?
12. a) 14.8% b) 17.2%
13. a) $\frac{8}{52}$ or about 0.154
   b) $\frac{32}{52}$ or about 0.615
14. a) 53% b) 16% c) 22%
15. 80%
16. No
17. a) i) 45%
      ii) 16%
      iii) 51%
   b) 44%
   c) 16%
18. e.g., To determine the probability of two events that are not mutually exclusive, you must subtract the probability of both events occurring after adding the probabilities of each event. Example: Female students at a high school may play hockey or soccer. If the probability of a female student playing soccer is 62%, the probability of her playing in goal is 4%, and the probability of her either playing soccer or in goal is 64%, then the probability of her playing in goal at soccer is 62% + 4% − 64% = 2%.
19. 13%
20. a) $P(A \cup B \cup C) = P(A) + P(B) + P(C)$
   b) $P(A \cup B \cup C) =$
   $P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Lesson 5.5, page 350

1. a) dependent
   b) $\frac{1}{12}$ or about 0.0833
2. a) dependent
   b) $\frac{12}{204}$ or about 0.0588
3. a) independent
   b) $\frac{1}{16}$ or 0.0625
4. a) i) $\frac{30}{182}$ or about 0.165
      ii) $\frac{56}{182}$ or about 0.308
      iii) $\frac{86}{182}$ or about 0.473
   b) No
5. a) $\frac{80}{190}$ or about 0.421
   b) $\frac{5}{8}$ or 0.625
6. $\frac{56}{74}$ or about 0.757
7. $\frac{12}{132}$ or about 0.091
8. $\frac{12.4}{39}$ or about 0.317
9. 71.5%
10. $\frac{30}{70}$ or about 0.429
11. e.g., A student selected at random goes to a fast-food outlet that particular day. What is the probability that the student had more than 1 h for lunch?
12. e.g.,
   a) Survey questions for a group of classmates over one month (weekdays only):

How often do you cycle to school?	140
How often do you get to school other than by cycling?	60
How often do you cycle when the weather is fine?	80
How often do you cycle when it is raining or snowing?	60

   b) A randomly selected student cycled to school on a particular day. What is the probability that the weather was fine that day?
13. $\frac{5}{8}$ or 0.625
14. $\frac{6}{7}$ or about 0.857
15. $\frac{2}{9}$ or about 0.222
16. a) $\frac{2}{7}$ or about 0.286
   b) $\frac{2}{7}$ or about 0.286
17. a) Let $X =$ {1st chip is defective} and $Y =$ {2nd chip is defective}.



   b) $\frac{582}{9900}$ or about 0.0588; e.g., Multiply the probability of the first chip being defective by that of the second chip being non-defective, then multiply the probability of the first chip being non-defective by that of the second chip being defective, and then add the products.

18. a) $\frac{6}{22\,350}$ or about 0.000 268
b) $\frac{21\,462}{22\,350}$ or about 0.960
c) $\frac{882}{22\,350}$ or about 0.0395

19. a) 0.57 b) 0.43

20. e.g., Problem 1: On weekdays I have cereal for breakfast 70% of the time. On the weekends I have cereal for breakfast 40% of the time. On a random day, what is the probability that I do not have cereal?
Answer: $\frac{2.7}{7}$ or about 0.386.
Problem 2: I draw without looking two cards from a well shuffled standard deck, drawing the second card without replacing the first one. If my second card is a red card, what is the probability that my first card is black?
Answer: $\frac{26}{51}$ or about 0.510

21. e.g., The probability of event A and event B both occurring is the probability that A occurs, multiplied by the probability that B occurs given that A occurs. Example: If a 6-sided die is rolled twice, P(rolling a 4 the first time and getting a total greater than 7) = P(rolling a 4 the first time) · P(getting a total greater than 7 | rolling a 4 the first time), or $\frac{1}{12} = \frac{1}{6} \cdot \frac{1}{2}$.

22. a) $\frac{24}{970\,200}$ or about 0.000 024 7
b) $\frac{857\,280}{970\,200}$ or about 0.884
c) $\frac{3480}{970\,200}$ or about 0.0036

Lesson 5.6, page 360

1. a) independent
b) independent
c) dependent
d) independent

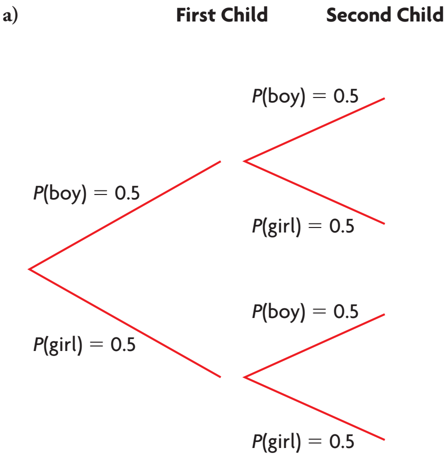
2. a) independent
b) $\frac{1}{6}$ or about 0.167

3. a) dependent
b) $\frac{1}{12}$ or about 0.0833

4. a) $\frac{1}{24}$ or about 0.0417
b) $\frac{1}{36}$ or about 0.0278
c) $\frac{16}{2652}$ or about 0.006 03
d) $\frac{1}{15}$ or about 0.0667

5. a) No b) Yes

6. a)



b) 0.25
c) 0.5

7. $\frac{100}{1560}$ or about 0.0641; dependent

8. a) $\frac{1}{36}$ b) $\frac{1}{4}$ c) $\frac{25}{36}$

9. a) 0.6 b) 0.05 c) 0.35

10. a) e.g., Spinner has 6 equal areas, numbered 1 to 6.
b) e.g., Spinner has 10 equal areas, numbered 1 to 10.

11. a) No. e.g., Anne has a $\frac{15}{90}$ or about 0.167 probability of drawing two blue marbles, while Abby has a $\frac{56}{342}$ or about 0.164 probability of drawing two blue marbles.
b) $\frac{8272}{30\,780}$ or about 0.269
c) No. e.g., Anne has a $\frac{25}{100}$ or 0.25 probability of drawing two red marbles, while Abby has a $\frac{90}{380}$ or about 0.237 probability of drawing two red marbles.

12. a) 0.25
b) $\frac{158}{400}$ or 0.395
c) $\frac{242}{400}$ or 0.605

13. a) 0.0025
b) 0.9025

14. a) e.g., Problem: What is the probability of drawing a card from a shuffled standard deck and getting a red card, then replacing it, shuffling the deck again, drawing a second card, and getting a heart? Answer: $\frac{1}{8}$ or 0.125
b) e.g., Problem: What is the probability of drawing a card from a shuffled standard deck and getting a red card, then drawing a second card without replacing the first one, and getting a spade? Answer: $\frac{13}{102}$ or about 0.127

15. 0.09

16. **a)** The formula is $P(A \cap B) = P(A) \cdot P(B)$ only when A and B are independent events.
b) e.g., Drawing two red marbles from a bag containing 5 red and 15 blue marbles, with replacement: A = {red on 1st draw} and B = {red on 2nd draw} are independent events, so
$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$.
c) e.g., Drawing two red marbles from a bag containing 5 red and 15 blue marbles, without replacement: A = {red on 1st draw} and B = {red on 1st draw} are dependent events, so
$P(A \cap B) = P(A) \cdot P(B \mid A) = \frac{1}{4} \cdot \frac{4}{19} = \frac{1}{19}$.

17. **a)** about 0.366
b) about 0.606
c) about 99.9%

18. **a)** about 0.179
b) about 0.863; assumed that the chance of winning any of the four games was equal

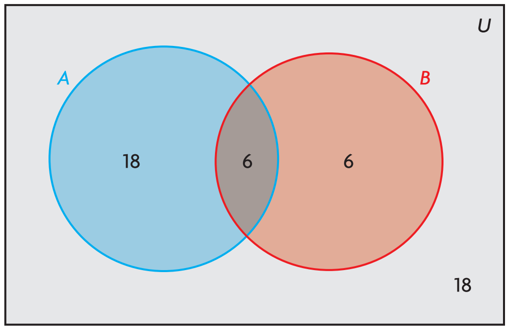
19. $\frac{16}{32}$ or 0.5; $\frac{106}{1024}$ or about 0.1035

Chapter Self-Test, page 364

1. Yes. e.g., The probabilities of winning are equal.
2. 0.7
3. 0.06
4. about 0.000 679
5. 0.2
6. $\frac{4}{104}$ or about 0.0385
7. **a)** 0.49 **b)** 0.09 **c)** 0.42 **d)** 0.91
8. $\frac{30}{552}$ or about 0.0543
9. $\frac{0.072}{0.268}$ or about 0.269

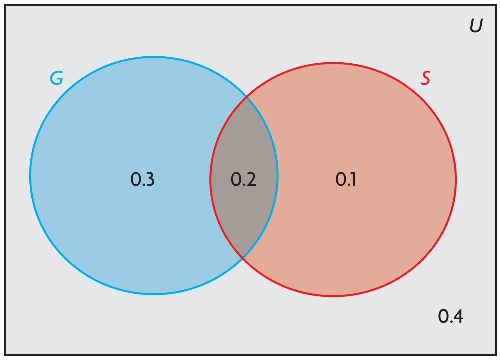
Chapter Review, page 367

1. **a)** not fair; e.g., Camila has a better chance of winning.
b) fair; e.g., Cooper and Alyssa have an equal chance of winning.
2. 0; e.g., The maximum tension is 100%.
3. **a)** 3 : 2 **b)** 2 : 3
4. 3 : 7
5. **a)** 2 : 5 **b)** 5 : 2
6. No. e.g., To calculate the odds, it should be odds against : odds for, which is 4 : 1.
7. second course
8. $\frac{1}{36}$ or about 0.0278
9. $\frac{36}{330}$ or about 0.109
10. **a)** $\frac{1}{2600}$ or about 0.000 385
b) $\frac{6}{17\,576}$ or about 0.000 341
11. **a)** not mutually exclusive; e.g., There are prime numbers that are also odd numbers.
b) mutually exclusive; e.g., You cannot roll a sum of 6 and a sum of 8 at the same time.
c) mutually exclusive; e.g., You cannot eat a peach and an apple at the same time.
12. **a)** A = {face cards}, B = {spades}



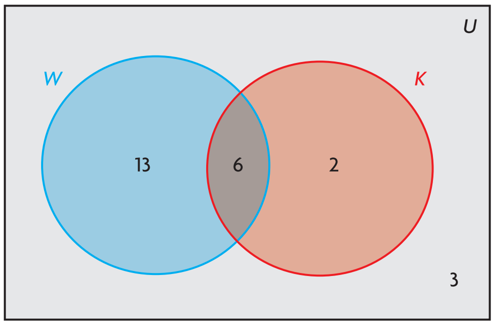
b) not mutually exclusive
c) $\frac{30}{48}$ or 0.625

13. **a)** G = {probability Mya will exercise on Sunday}
S = {probability Mya will shop on Sunday}



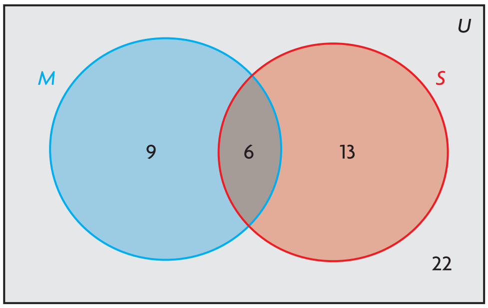
b) not mutually exclusive
c) 0.6

14. e.g., Suppose 6 students can both ski and swim. What is the probability that a randomly selected student cannot ski or swim? Let W = {students who swim} and K = {students who ski}.



Answer: $\frac{3}{24}$ or 0.125

15. e.g., Out of 50 retail outlets, 19 are holding sales this month. 15 outlets sell only sustainably manufactured items, and 6 of these are holding sales this month. What is the probability that a retail outlet sells some non-sustainably manufactured items and is having a sale?
M = {outlets selling sustainably manufactured items}
S = {outlets having sales}



Answer: 0.26

16. $\frac{160}{306}$ or about 0.523

17. 0.8

18. $\frac{400}{1560}$ or about 0.256

19. $\frac{1}{6}$ or about 0.167

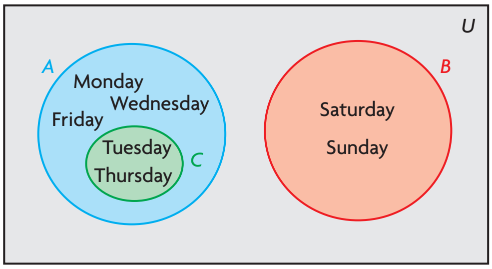
20. Yes. e.g., $P(A \cap B) = P(A) \cdot P(B) = 0.5 \cdot 0.6 = 0.3$

21. **a)** 0.42 **b)** 0.28 **c)** 0.12

Cumulative Review, Chapters 3–5, page 373

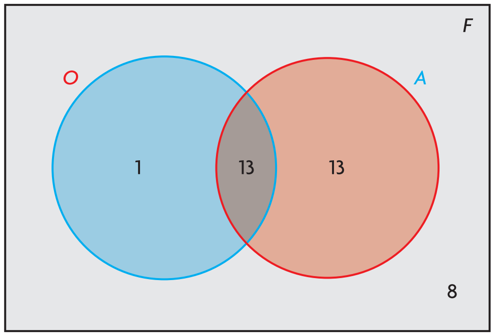
1. **a)** e.g., Manitoba, Québec, PEI, New Brunswick, Nova Scotia
b) e.g., 10, 20, 30, 40, 50

2. **a)**



b) A and B
c) **i)** False; e.g., A and B are disjoint sets.
ii) True; e.g., C is a subset of A.
iii) True; e.g., A is the inverse of B.
iv) False; e.g., A includes C.

3. **a)**



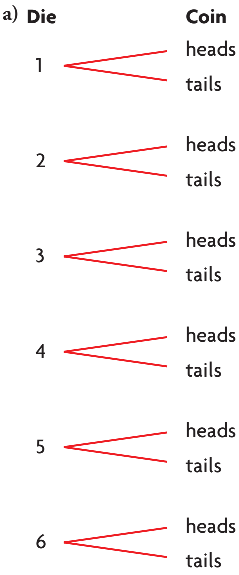
b) 27
c) 14

4. **a)** e.g., odd whole numbers less than 100 and even whole numbers less than 100
b) e.g., odd whole numbers less than 100 and prime numbers less than 100

5. 12

6. **a)** Yes
b) If a number is less than zero, then it is negative. Yes. e.g., All numbers less than zero must be negative.
c) If a number is not negative, then it is not less than zero. Yes. e.g., All numbers that are not negative are either zero or positive, and all of these numbers are not less than zero.
d) If a number is not less than zero, then it is not negative. Yes. e.g., All numbers not less than zero are either zero or positive, so they are all not negative.
e) Yes. e.g., A number is negative if, and only if, it is less than zero.

7. **a)**



b) 6 outcomes from rolling a die and 2 outcomes from tossing a coin; $6 \cdot 2 = 12$ combined outcomes

8. **a)** 1 757 600 **b)** 1 579 500

9. **a)** 479 001 600 **b)** 40 320 **c)** 840 **d)** 15 120 **e)** 126 **f)** 495

10. **a)** 1 307 674 368 000 **b)** 2730 **c)** 91

11. 8568

12. **a)** 120 **b)** 400

13. $n = 27$

14. **a)** 2940 **b)** 8001 **c)** 1456

15. 800

16. Yes. e.g., Both players have an equal chance of winning.

17. **a)** 25 : 1

b) $\frac{25}{26}$ or about 0.962

18. 0.5

19. **a)** $\frac{1}{635\ 013\ 559\ 600}$ or about 0.000 000 000 001 57

b) $\frac{1\ 677\ 106\ 640}{635\ 013\ 559\ 600}$ or about 0.002 64

20. **a)** e.g., choosing an apple or a pear from a bowl of fruit

b) e.g., choosing 2 blue marbles from a bag containing 7 blue and 3 red marbles, without replacement

c) e.g., rolling a standard die and getting 4, tossing a coin and getting heads

21. **a)** $\frac{3}{36}$ or about 0.0833

b) $\frac{27}{36}$ or 0.75

c) $\frac{3}{6}$ or 0.5

d) $\frac{2}{36}$ or about 0.0556

22. **a)** $\frac{10}{16}$ or 0.625 **b)** $\frac{3}{8}$ or 0.375

23. **a)** $\frac{1}{36}$ or about 0.0278

b) $\frac{12}{552}$ or about 0.0217

Chapter 6 – Polynomial Functions

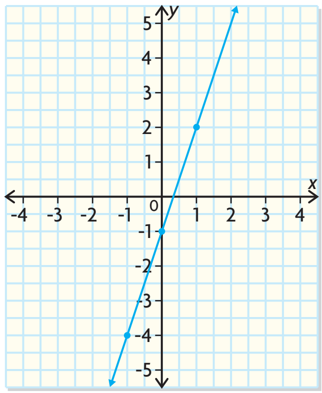
Lesson 6.1, page 383

1. b), c), d)

2.

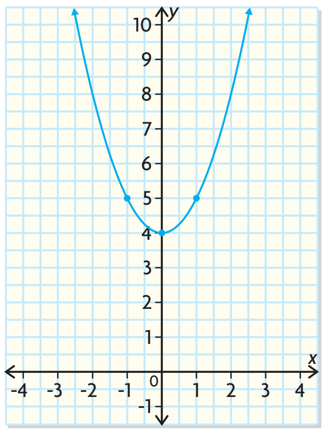
	x-Intercepts	y-Intercept	End Behaviour	Domain	Range	Number of Turning Points
b)	−5, −1	2	QII to QI	$\{x \in R\}$	$\{y \mid y \geq -2.5, y \in R\}$	1
c)	−2, −1, 1	−5	QIII to QI	$\{x \in R\}$	$\{y \in R\}$	2
d)	0.5	2	QII to QIV	$\{x \in R\}$	$\{y \in R\}$	0

3. **a)**



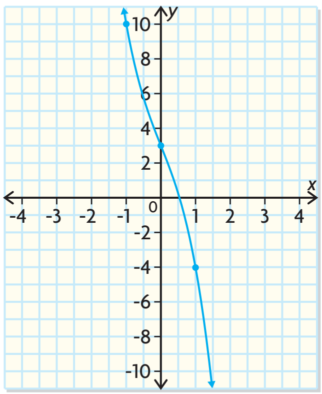
Number of x-Intercepts	y-Intercept	End Behaviour	Domain	Range	Number of Turning Points
1	−1	QIII to QI	$\{x \in R\}$	$\{y \in R\}$	0

b)



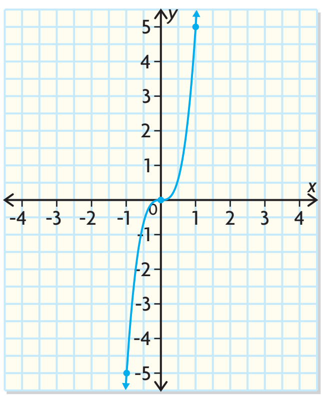
Number of x-Intercepts	y-Intercept	End Behaviour	Domain	Range	Number of Turning Points
0	4	QII to QI	$\{x \in R\}$	$\{y \mid y \geq 4, y \in R\}$	1

c)



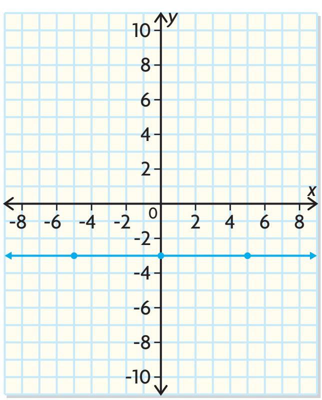
Number of *x*-Intercepts	*y*-Intercept	End Behaviour	Domain	Range	Number of Turning Points
1	3	QII to QIV	$\{x \in \mathrm{R}\}$	$\{y \in \mathrm{R}\}$	0

d)



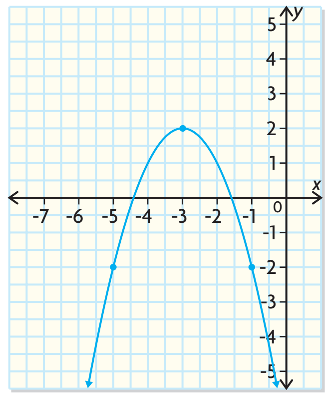
Number of *x*-Intercepts	*y*-Intercept	End Behaviour	Domain	Range	Number of Turning Points
1	0	QIII to QI	$\{x \in \mathrm{R}\}$	$\{y \in \mathrm{R}\}$	0

e)



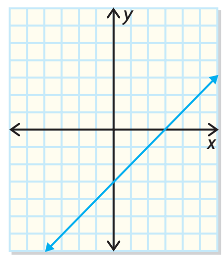
Number of *x*-Intercepts	*y*-Intercept	End Behaviour	Domain	Range	Number of Turning Points
0	−3	QIII to QIV	$\{x \in \mathrm{R}\}$	$\{y \mid y = -3, y \in \mathrm{R}\}$	0

f)

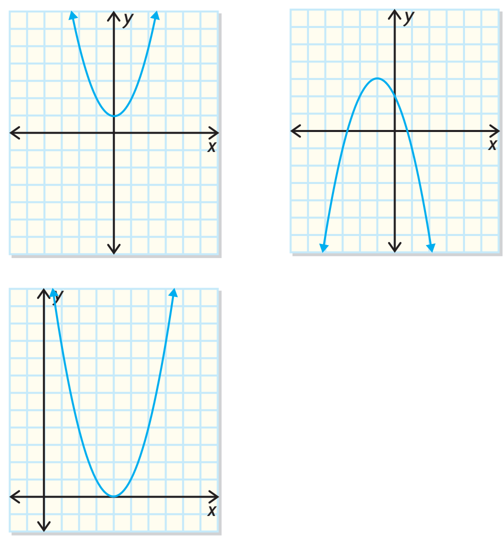


Number of *x*-Intercepts	*y*-Intercept	End Behaviour	Domain	Range	Number of Turning Points
2	−7	QIII to QIV	$\{x \in \mathrm{R}\}$	$\{y \mid y \leq 2, y \in \mathrm{R}\}$	1

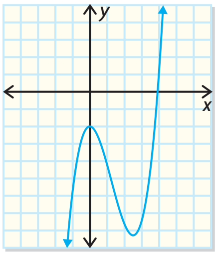
4. **a)** 1 x-intercept

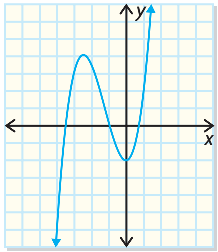


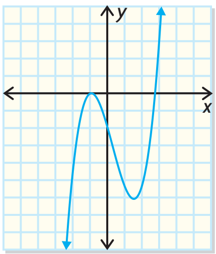
b) 0, 1, or 2 x-intercepts



c) 1, 2, or 3 x-intercepts







Lesson 6.2, page 393

1.

	Degree	Leading Coefficient	Constant
a)	2	6	-2
b)	1	$-\frac{2}{3}$	10
c)	3	-1	6
d)	3	4	-10

2. **a) i)** minimum x-intercepts: 0, maximum x-intercepts: 2
ii) end behaviour: QII to QI, domain: $\{x \in \mathrm{R}\}$, range: $\{y \in \mathrm{R} \mid y \geq \text{minimum}\}$
iii) minimum turning points: 1, maximum turning points: 1
b) i) minimum x-intercepts: 1, maximum x-intercepts: 1
ii) end behaviour: QII to QIV, domain: $\{x \in \mathrm{R}\}$, range: $\{y \in \mathrm{R}\}$
iii) minimum turning points: 0, maximum turning points: 0
c) i) minimum x-intercepts: 1, maximum x-intercepts: 3
ii) end behaviour: QII to QIV, domain: $\{x \in \mathrm{R}\}$, range: $\{y \in \mathrm{R}\}$
iii) minimum turning points: 0, maximum turning points: 2
d) i) minimum x-intercepts: 1, maximum x-intercepts: 3
ii) end behaviour: QIII to QI, domain: $\{x \in \mathrm{R}\}$, range: $\{y \in \mathrm{R}\}$
iii) minimum turning points: 0, maximum turning points: 2

3.

	Degree	Sign of Leading Coefficient	Constant Term
a)	2	$-$	2
b)	1	$+$	2
c)	3	$+$	-1
d)	3	$-$	6

4. **a)** e.g., $y = 5$
b) e.g., $y = 3x + 5$
c) e.g., $y = 4x^2 + 3x + 5$
d) e.g., $y = -5x^3 + x^2 - x + 5$

5. **a)** QIII to QI **c)** QII to QI **e)** QII to QIV
b) QIII to QIV **d)** QII to QIV **f)** QIII to QI

6. **a)** v) **b)** i) **c)** ii) **d)** vi) **e)** iii) **f)** iv)

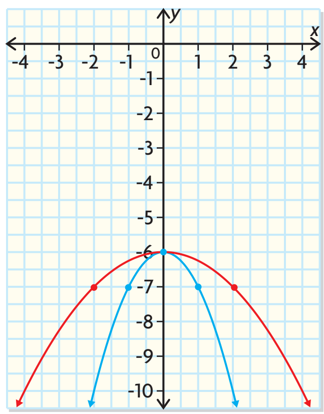
7.

	Number of x-Intercepts	y-Intercept	End Behaviour	Domain	Range	Number of Turning Points
a)	1	5	QII to QIV	$\{x \in \mathrm{R}\}$	$\{y \in \mathrm{R}\}$	0
b)	0, 1, or 2	-6	QII to QI	$\{x \in \mathrm{R}\}$	$\{y \mid y \geq$ minimum, $y \in \mathrm{R}\}$	1
c)	1, 2, or 3	-1	QIII to QI	$\{x \in \mathrm{R}\}$	$\{y \in \mathrm{R}\}$	0 or 2
d)	1, 2, or 3	0	QII to QIV	$\{x \in \mathrm{R}\}$	$\{y \in \mathrm{R}\}$	0 or 2

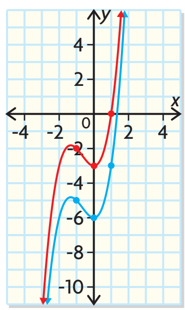
8. **a)** e.g., $y = -x^2 + 2$
b) e.g., $y = x^3 + 2x^2 - x - 2$
c) e.g., $y = x - 3$
d) e.g., $y = x^3 + 2x^2 - x + 5$
e) e.g., $y = -x^2 + 2$

9. e.g., Lukas made an error when he stated, "There is no constant term, so there is no y-intercept." Since there is no constant term, there is a y-intercept at $y = 0$.

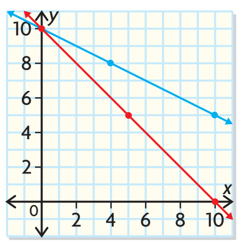
10. **a)** e.g.,



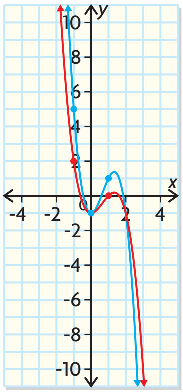
b) e.g.,



c) e.g.,

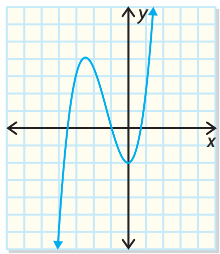


d) e.g.,



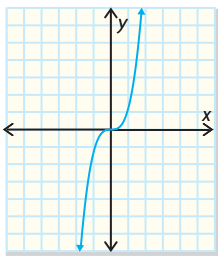
11. e.g., A cubic function may have zero or two turning points. If there are no turning points, the function has only one x-intercept. If there are two turning points, the function may have one, two or three x-intercepts, depending on the values of a, b, c, and d in the function $y = ax^3 + bx^2 + cx + d$ (see figures shown).

2 turning points, 1 x-intercept



2 turning points, 3 x-intercepts

2 turning points, 2 x-intercepts



0 turning points, 1 x-intercept

12. e.g., Cubic functions may have turning points, but they do not have any maximums or minimums.
Quadratic functions have one turning point. The point at which they turn (the vertex) defines the function's maximum or minimum.

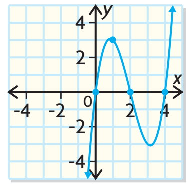
13. **a)** i) **b)** iii) **c)** iv) **d)** ii)

14. **a)** The degree is 3, so it is a cubic function. The leading coefficient is positive, so the function is increasing from left to right. The graph extends from quadrant III to quadrant I.
It has a y-intercept of 25.720 and may have 1, 2, or 3 x-intercepts.
b) the price of gas in 1979

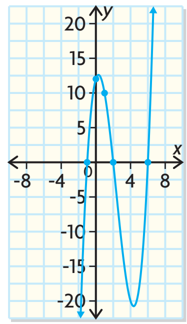
15. **a)** 4.243 m; It is very close.
b) extends from QIII to QI
c) No. e.g., the tides repeat every day, as shown in the table for Jan. 10, but a cubic function will continue to increase.

16. e.g., Ask for the degree of the function to determine the end behaviour. Ask for the value of the leading coefficient to determine which quadrants the end behaviour extends to and whether the function is increasing or decreasing from left to right. Ask for the number of x-intercepts to determine how many times the function crosses the x-axis. These questions will enable you to draw a rough sketch of the graph and determine a plausible equation.

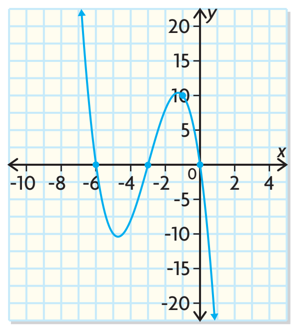
17. **a)** **i)** $(0.85, 3.08)$, $(3.15, -3.08)$
ii) $(0.31, 12.60)$, $(4.36, -20.75)$
iii) $(-1.27, 10.39)$, $(-4.73, -10.39)$
b) **i)**



ii)



iii)



c) e.g., The x-coordinate occurs approximately at the midpoint between the x-intercepts. Once this midpoint is found, substitute this x-value into the equation to determine the y-coordinate. e.g., iii) the x-intercepts are -6, -3, and 0. The midpoint between -6 and -3 is -4.5; find the y-value when x is -4.5. The midpoint between -3 and 0 is -1.5; find the y-value when x is -1.5.

18. **a)** e.g., $P(0) = 0$, $P(0.5) = 0.375$, $P(1) = 0$; The function seems reasonable.
b) D: $\{x \mid 0 \le x \le 1, x \in \mathrm{R}\}$; R: $\left\{y \mid y \le \frac{4}{9}, y \in \mathrm{R}\right\}$
c) $\left(\frac{2}{3}, \frac{4}{9}\right)$; e.g., At this point, the probability of sinking two out of three free throws begins to decrease.
d) $x = 0, 1$; e.g., The probability of sinking two out of three free throws is zero if the probability of sinking one throw is 0. The probability of sinking exactly two out of three free throws is 0 if the probability of sinking one throw is 1. (That is, if the probability of sinking a free throw is 1, you are certain to make all shots, so you would sink 3 out of 3.)

Mid-Chapter Review, page 400

1. a), c), d)

2.

	Degree	Domain	Range	Constant
a)	2	$\{x \in R\}$	$\{y \mid y \leq 6, y \in R\}$	5
c)	0	$\{x \in R\}$	$\{y \mid y = 4, y \in R\}$	4
d)	3	$\{x \in R\}$	$\{y \in R\}$	–2

b) i) d) **ii)** c) **iii)** a)

3. **a)** QII to QIV **b)** QIII to QI **c)** QIII to QI **d)** QIII to QIV

4. **a)**

	Degree	x-Intercepts	y-Intercept	End Behaviour	Domain	Range	Number of Turning Points
i)	3	5	4	QII to QIV	$\{x \in R\}$	$\{y \in R\}$	2
ii)	2	-1.5, 2	-6	QII to QI	$\{x \in R\}$	$\{y \mid y \geq -6.25, y \in R\}$	1
iii)	1	6.5	2	QII to QIV	$\{x \in R\}$	$\{y \in R\}$	0
iv)	3	-1, 3	9	QIII to QI	$\{x \in R\}$	$\{y \in R\}$	2

b) i) leading coefficient: $-$, constant: 4
ii) leading coefficient: $+$, constant: -6
iii) leading coefficient: $-$, constant: 2
iv) leading coefficient: $+$, constant: 9

5.

	Number of x-Intercepts	y-Intercept	End Behaviour	Domain	Range	Number of Turning Points
a)	0, 1, or 2	6	QIII to QIV	$\{x \in R\}$	$\{y \mid y \leq \text{maximum}, y \in R\}$	0
b)	1, 2, or 3	6	QIII to QI	$\{x \in R\}$	$\{y \in R\}$	1
c)	0, 1, or 2	-1	QII to QI	$\{x \in R\}$	$\{y \mid y \geq \text{minimum}, y \in R\}$	0 or 2
d)	1, 2, or 3	0	QIII to QI	$\{x \in R\}$	$\{y \in R\}$	0 or 2

6. **a)** e.g., $y = (x + 1.5874)^3 - x - 4$
b) e.g., $y = (x - 3)(x + 4)$
c) e.g., $y = -x - 3$
d) e.g., $y = 1.5(x - 2)^2$
e) e.g., $y = 1.5(x - 4)^2 - 6$

Lesson 6.3, page 407

1. **a)** e.g., slope: -1, y-intercept: 7.5
b) e.g., slope: 0.1, y-intercept: 3

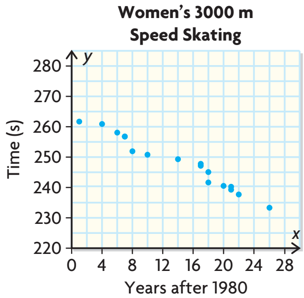
2. **a)** e.g., $y = -x + 7.5$
b) e.g., $y = 0.1x + 2$

3.

	Independent	Dependent
a)	speed	distance
b)	size of family	number of cellphones
c)	time of day	number of people
d)	time of year	hours of daylight

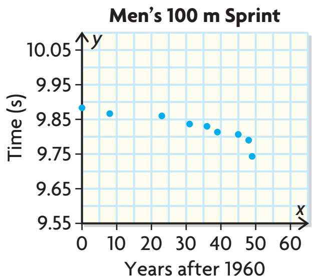
4. **a)** e.g., The line of best fit has a negative slope and has about the same number of points above and below it. The points are very close to the line of best fit, so it should give a strong representation of the data.
b) e.g., about 75; interpolation, because the point is between known values
c) e.g., about 52; interpolation, because the point is between known values
d) e.g., about 105; extrapolation, because the point is outside known values

5. **a)**



b) As the years increase, the world-record times decrease.
c) $y = -1.147...x + 264.178...$ (where y is time in seconds and x is the year); The slope represents the rate at which the world-record time decreases each year, in seconds; the y-intercept represents the world-record time in 1980 (year 0).
d) e.g., 235.489... s or 3:55.49 min
e) 3:55.75 min; My estimate was very close.

6. **a)**

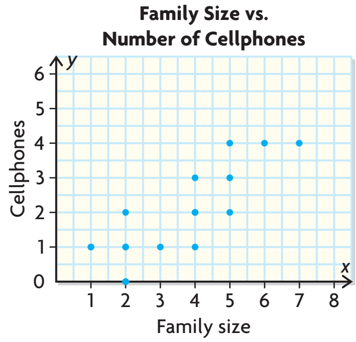


b) As the years increase, the world-record times decrease.
c) $y = -0.006...x + 10.032...$; The slope represents the rate at which the world-record time in seconds decreases each year; the y-intercept represents the world-record time in 1960 (year 0).
d) e.g., 9.75 s
e) 9.74 s; My estimate was very close.

7. **a)** e.g., As latitude increases (i.e., you go further north), the mean temperatures would decrease. My plot verifies that my prediction is correct.
b) $y = -0.637...x + 49.730...$
c) 16.6 °C
d) 49.8° N

8. **a)** As family size increases, the number of cellphones increases.

b)



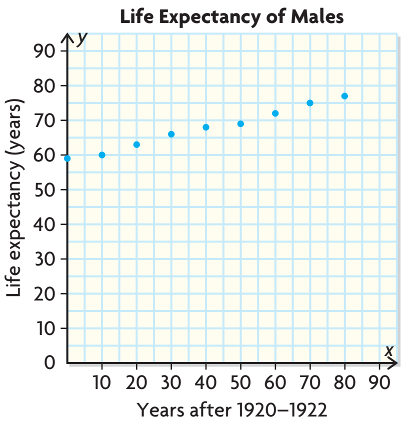
$y = 0.614...x - 0.260...$

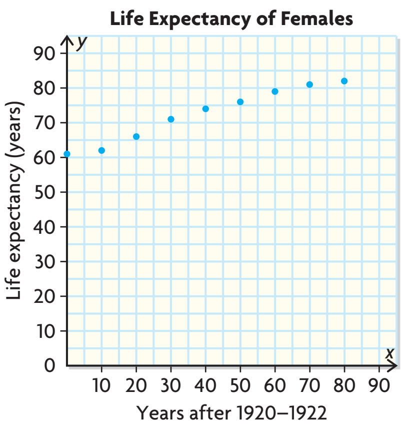
c) e.g., 1.58 cellphones; I would expect a family of 3 to have 2 cellphones.

9. 2 events, to ensure he doesn't go over budget

10. 109 bikes

11. **a)**

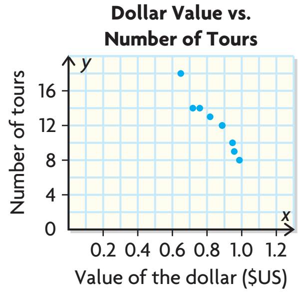




b) males: $y = 0.23x + 58.466...$; females: $y = 0.286...x + 60.977...$

c) males: 79 years; females: 87 years

12.



$y = -24.879...x + 33.314...$

e.g., Using extrapolation, the expected number of tours is 8. There is a strong linear relationship between the value of the dollar and the number of tours.

13. **a)** e.g., Create a scatter plot of the data and perform a linear regression. Insert the known independent value for x and solve for y. If the regression yields $y = 0.5x + 2$, and you want to estimate the value of the dependent variable when the independent variable is 4, input 4 for x and solve for y; this gives us a y-value of 4.

b) e.g., From the linear regression, substitute the known value of the dependent variable for y, and solve for x (the independent variable).

14. **a)** $y = 2.445...x - 23.118...$

b) -10.9 board feet; e.g., This answer does not make sense, because area cannot be negative. Extrapolating does not always make sense, because the trend does not always continue outside the given data range.

c) e.g., Based on the scatter plot, the data does not seem to be linear.

d) e.g., Quadratic regression may be better because the data fit a quadratic function.

Lesson 6.4, page 419

1. **a)** e.g., The number of births increases from 1947 to about 1960; after that it decreases at about the same rate until 1971.

b) about 1960

c) about 425 000

d) between 1953 and 1965

2. **a)** 149 bpm **b)** about 244 W

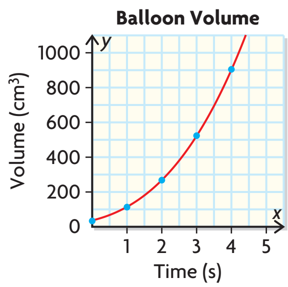
3. **a)** The ball rises to a maximum height at about 3 s and then its height decreases.

b) $y = -10.071...x^2 + 51.408...x + 11$

c) **i)** 11 m **ii)** 76.6 m **iii)** 38.4 m

d) about 5.3 s

4. **a), b)**

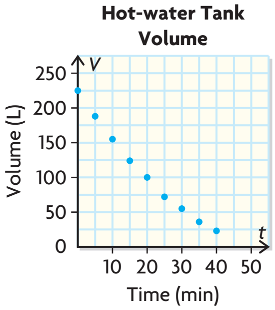


$y = 4.189...x^3 + 25.130...x^2 + 50.267x + 33.510...$

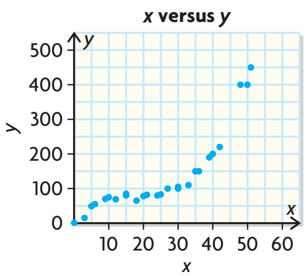
e.g., As time increases, volume increases at a faster rate.

c) 8181.47 cm^3

5. a) 4.0 °C b) 1006.55 cm^3
6. 1.41 m
7. a) e.g., 2 years after 1977, fertility rate is about 3.42 per 1000 females; 7 years after 1977, fertility rate is about 3.43 per 1000 females
   b) e.g., The value when $x = 2$ is close; the value when $x = 7$ is not as close.
8. a)



   e.g., As time increases, the volume decreases.
   b) $V = 0.064...t^2 - 7.650...t + 224.885...$
   c) 17.2 min
   d) 56.2 min
9. a) $y = 0.004...x^3 - 0.294...x^2 + 4.548...x + 74.659...$; 65.342...
   b) Never. The minimum of 65.3 per 100 000 is reached between 2009 and 2010.
10. a)



   b) $y = 0.010...x^3 - 0.557...x^2 + 11.078...x + 1.409...$
   c) 87.7; the result from the regression equation is off by 4.7.
11. a) e.g., Linear: $y = 115.83x + 6999.96$ for $0 \le x \le 9$, Quadratic: $y = -9.783...x^2 + 311.404...x + 6023.16$ for $10 \le x \le 22$, Linear: $y = -115.917x + 10\,689.9$ for $23 \le x \le 30$
   b) e.g., about 14 s

Chapter Self-Test, page 424

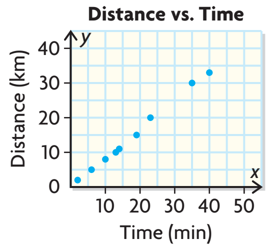
1. a) i) e.g., $y = -x + 3$
   ii) e.g., $y = 14x - 6$
   b) i) e.g., $y = -x^3 + 3$
   ii) e.g., $y = x^3 - 6$
2.

	Possible Number of x-Intercepts	y-Intercept	End Behaviour	Domain	Range	Possible Number of Turning Points
a)	0, 1, or 2	−1	QIII to QIV	$\{x \in R\}$	$\{y \mid y \le 3, y \in R\}$	1
b)	1	1.5	QIII to QI	$\{x \in R\}$	$\{y \in R\}$	0
c)	1, 2, or 3	−42	QIII to QI	$\{x \in R\}$	$\{y \in R\}$	0 or 2
d)	1, 2, or 3	2	QII to QIV	$\{x \in R\}$	$\{y \in R\}$	0 or 2

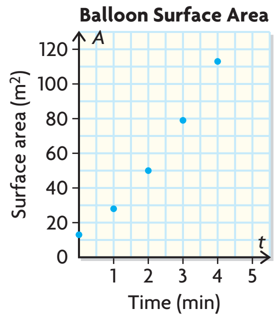
3. a) e.g., $y = -x^3 + 2x - 6$
   b) e.g., $y = -x^2 + 2x + 8$
4.

	Number of x-Intercepts	y-Intercept	End Behaviour	Domain	Range	Number of Turning Points
a)	0	12	QII to QI	$\{x \in R\}$	$\{y \mid y \ge 4, y \in R\}$	1
b)	1	−1	QII to QIV	$\{x \in R\}$	$\{y \in R\}$	0

5. e.g., The line of best fit would have a negative slope of about −1 and about the same number of points above and below the line. The y-intercept would be about 125.
6. a)



   b) $y = 0.844...x - 0.308...$
   c) i) about 30 min
   ii) about 38 km
   iii) about 50.6 km/h
7. a)



   b) $A = 3.214...t^2 + 12.242...t + 12.828...$
   c) e.g., interpolate: at 1.5 min, surface area would be about 38.4 m^2; extrapolate: at 5 min, surface area would be about 154.4 m^2

Chapter Review, page 427

1. a), d), and e)
2.

	Number of x-Intercepts	y-Intercept	Domain	Range	Number of Turning Points
b)	1, 2, or 3	−4	$\{x \in R\}$	$\{y \in R\}$	0 or 2
c)	0, 1, or 2	−32	$\{x \in R\}$	$\{y \mid y \le 0, y \in R\}$	1

3.

	y-Intercept	End Behaviour	Domain	Range
a)	−1	QIII to QIV	$\{x \in R\}$	$\{y \mid y \le$ maximum, $y \in R\}$
b)	5	QIII to QI	$\{x \in R\}$	$\{y \in R\}$
c)	0	QII to QI	$\{x \in R\}$	$\{y \mid y \ge -9, y \in R\}$
d)	5	QIII to QI	$\{x \in R\}$	$\{y \in R\}$

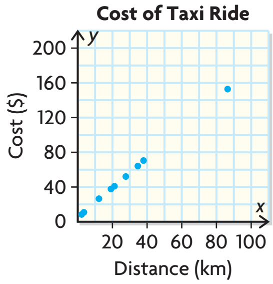
4.

	Number of x-Intercepts	y-Intercept	End Behaviour	Domain	Range	Number of Turning Points
a)	0	−4.5	QIII to QIV	$\{x \in R\}$	$\{y \mid y \le -4, y \in R\}$	1
b)	3	0	QIII to QI	$\{x \in R\}$	$\{y \in R\}$	2

5. **a)** e.g., slope: $\frac{1}{5}$, y-intercept: 0.5

b) e.g., slope: -2, y-intercept: 6

6. **a)**



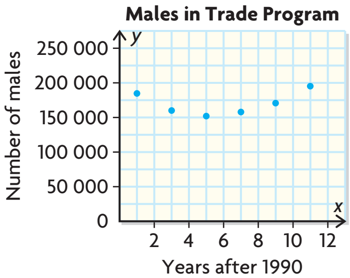
As the distance increases, the cost increases in a linear relationship.

b) $y = 1.711...x + 4.956...$

c) **i)** \$90.51 **ii)** \$4.96

7. **a)** 22 m **b)** 1.06 s

8. **a)**



b) $y = 1471.495...x^2 - 16\,363.017...x + 198\,121.986...$

c)

Years After 1990	Number of Males
2	171 280
4	156 210
6	152 912
8	161 386
10	181 631
12	213 649

9. **a)** $y = 7.062...x^3 - 77.357...x^2 + 1069.99x + 7208.23$

b) 2001

10. **a)** $y = -5x^2 + 20$

b) about 1.4 s

Chapter 7 – Exponential and Logarithmic Functions

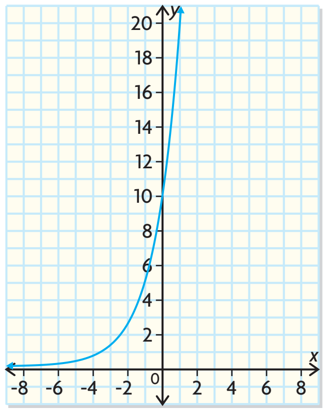
Lesson 7.1, page 439

1. **a)** No. linear
b) Yes
c) No. quadratic
d) No. cubic
e) Yes
f) No. quadratic

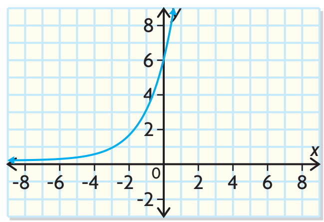
2. **b)** no x-intercept, y-intercept: $y = 1$, end behaviour: extends from QII to QI, domain: $\{x \mid x \in R\}$, range: $\{y \mid y > 0, y \in R\}$

f) no x-intercept, y-intercept: $y = 1$, end behaviour: extends from QII to QI, domain: $\{x \mid x \in R\}$, range: $\{y \mid y > 0, y \in R\}$

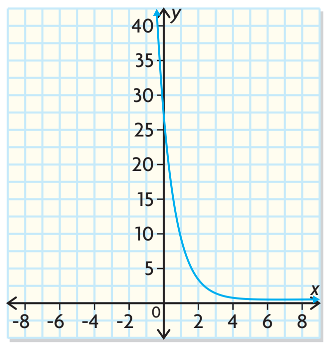
3. **a)**



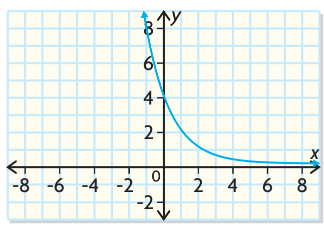
b)



c)



d)



Exponential Function	$y = 10(2)^x$	$y = 6(2)^x$	$y = 27\left(\frac{1}{3}\right)^x$	$y = 4\left(\frac{1}{2}\right)^x$
Number of x-Intercepts	0	0	0	0
y-Intercept	$y = 10$	$y = 6$	$y = 27$	$y = 4$
End Behaviour	extends from QII to QI	extends from QII to QI	extends from QII to QI	extends from QII to QI
Domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
Range	$\{y \mid y > 0, y \in R\}$	$\{y \mid y > 0, y \in R\}$	$\{y \mid y > 0, y \in R\}$	$\{y \mid y > 0, y \in R\}$

Lesson 7.2, page 448

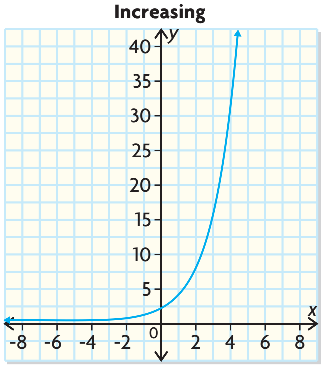
1. a) Yes. For each unit increase in x, the value of y doubles.
b) Yes. For each unit increase in x, the value of y is divided by 4.

2.

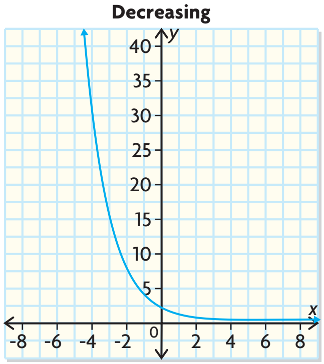
	Number of x-Intercepts	y-Intercept	Domain	Range	End Behaviour
a)	0	4	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$	QII to QI
b)	0	2	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$	QII to QI
c)	0	7	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$	QII to QI
d)	0	3	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$	QII to QI

3. e.g., Increasing exponential functions increase as x increases, whereas decreasing exponential functions decrease as x increases.

Increasing



Decreasing



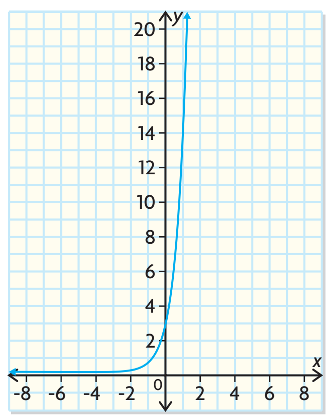
4. a) 5, increasing
b) 2, decreasing
c) 10, increasing
d) 1, decreasing

5. a) i) Yes. $y = 2^x$; y doubles as x increases by 1
ii) 1, increasing
b) i) No. y increases by 2 as x increases by 1
ii) 3, increasing
c) i) Yes. $y = 64\left(\frac{1}{4}\right)^x$; y decreases by $\frac{1}{4}$ as x increases by 1
ii) 64, decreasing
d) i) No. e.g., y decreases, then increases, then decreases again
ii) 1, both

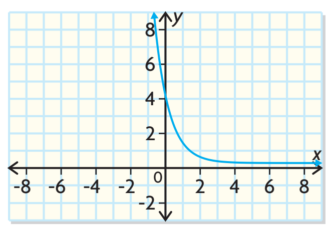
6.

	Number of x-Intercepts	y-Intercept	End Behaviour	Domain	Range
a)	0	3	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$
b)	0	4	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$
c)	0	2	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$
d)	0	3.5	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$
e)	0	25	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$
f)	0	12	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$

a)

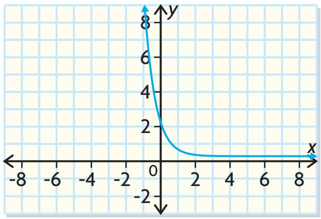


b)

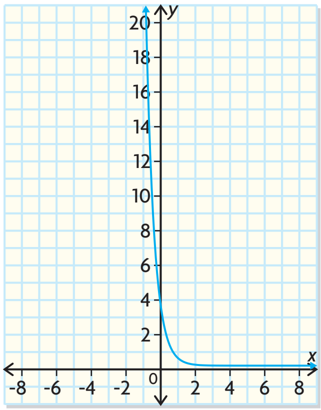


Answers

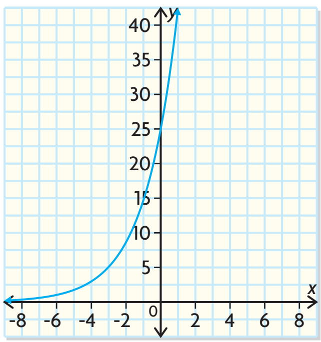
c)



d)



e)



f)

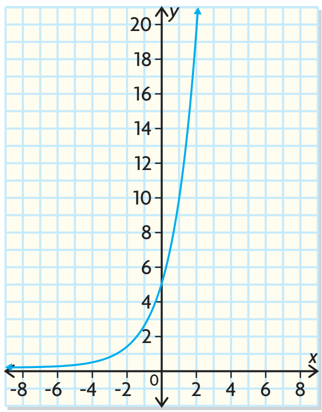
7. **a)** e.g., Since the base is greater than 1, the function is increasing.
b) e.g., Since the base is between 0 and 1, the function is decreasing.
c) e.g., Since the base is greater than 1, the function is increasing.

8.

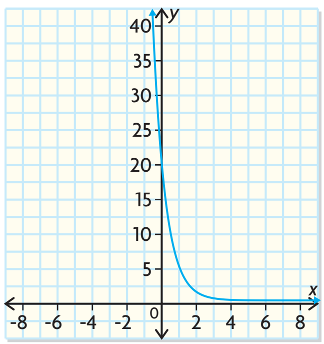
	Number of x-Intercepts	y-Intercept	End Behaviour	Domain	Range
a)	0	4	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$
b)	0	8	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$
c)	0	3	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$
d)	0	10	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$
e)	0	30	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$
f)	0	1	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$
g)	0	3	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$
h)	0	45	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$

9. **a)** Yes. $a > 0$ and $0 < b < 1$
b) No. $a > 0$ and $b > 1$
c) Yes. $a > 0$ and $0 < b < 1$
d) No. $a > 0$ and $b > 1$

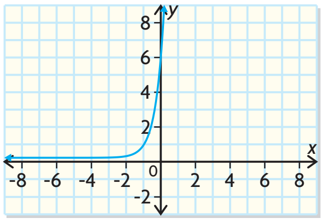
10. **a)** increasing



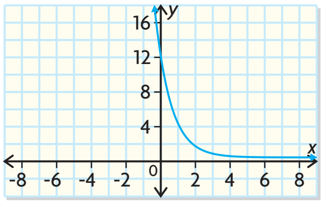
b) decreasing



c) increasing



d) decreasing

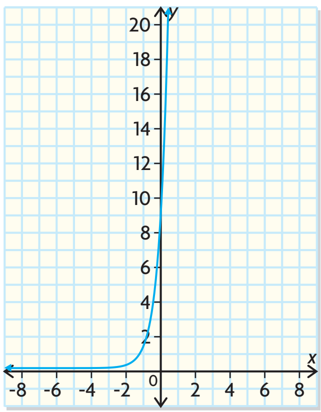


11. **a)** increasing; iii) **b)** decreasing; i) **c)** decreasing; ii) **d)** increasing; iv)

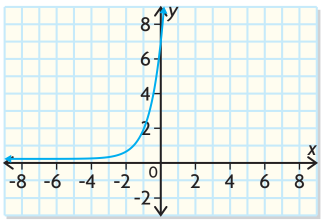
12.

	Function	y-Intercept	Base	Domain	Range	Increasing or Decreasing
a)	$y = 9(7)^x$	9	7	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$	increasing
b)	$y = 7(4)^x$	7	4	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$	increasing
c)	$y = 6\left(\frac{1}{7}\right)^x$	6	$\frac{1}{7}$	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$	decreasing
d)	$y = 2(0.35)^x$	2	0.35	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$	decreasing
e)	$y = 2(e)^x$	2	e	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$	increasing

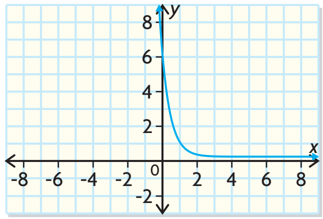
a)



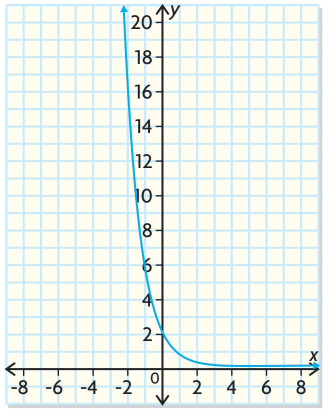
b)



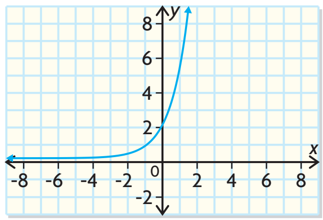
c)



d)



e)



13. **a) i)** range: $\{y \mid y > 0, y \in R\}$, $a = 2$, $b = 0.5$; decreases
ii) range: $\{y \mid y > 0, y \in R\}$, $a = 1$, $b = 3$; increases
iii) range: $\{y \mid y > 0, y \in R\}$, $a = 3$, $b = 0.5$; decreases
iv) range: $\{y \mid y > 0, y \in R\}$, $a = 2$, $b = 4$; increases
b) i) B. e.g., Their *y*-intercepts match.
ii) D. e.g., Their *y*-intercepts match.
iii) A. e.g., Their *y*-intercepts match.
iv) C. e.g., Their *y*-intercepts match.

14. **a)** e.g., increasing: $y = 5(2)^x$, decreasing: $y = 5\left(\frac{1}{4}\right)^x$
b) e.g., Same: number of *x*-intercepts, *y*-intercepts, end behaviour, domain, and range
Different: rate of change (increasing vs. decreasing function)

15. Yes. e.g., All exponential functions are of the form $f(x) = a(b)^x$.

16.

	$f(x) = ax + b$	$f(x) = a(b)^x$
Number of *x*-Intercepts	1	0
y-Intercept	b	a
End Behaviour	QIII to QI OR QII to QIV	QII to QI
Domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
Range	$\{y \mid y \in R\}$	$\{y \mid y > 0, y \in R\}$
Increasing or Decreasing	either increasing ($a > 0$) or decreasing ($a < 0$)	either increasing ($b > 1$) or decreasing ($0 < b < 1$)

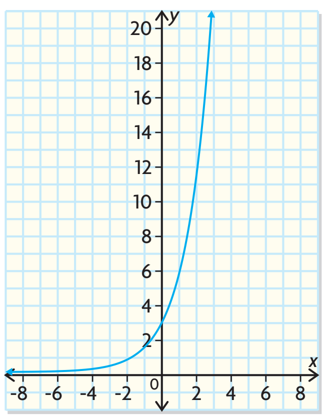
17. The number of x-intercepts, the end behaviour, the domain, and the range are common to all exponential functions. The y-intercept and whether the function increases or decreases is unique to the function.

18. **a)** student A: y-intercept: 80, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$, range: $\{y \mid 0 < y < 80, y \in \mathrm{R}\}$
student B: y-intercept: 100, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$, range: $\{y \mid 0 < y < 100, y \in \mathrm{R}\}$
b) e.g., Concentration of caffeine in blood naturally decreases over time as the kidneys filter it from the blood into the urine.
c) Student B consumed more caffeine. Student B processed the caffeine more quickly.
d) student A: 20 mg, student B: 20 mg
e) It describes the initial amount of caffeine consumed. They are different because the energy drinks could contain different amounts of caffeine.

19. **a)**

Hour	Students Who Are Told
0	3
1	6
2	12
3	24
4	48
5	96

b) Yes. e.g., After each hour, the number of students who are told in that hour about the presentation doubles.
c) $y = 3(2)^x$; domain: $\{x \mid x \geq 0, x \in \mathrm{W}\}$, range: $\{y \mid y \geq 3, y \in \mathrm{W}\}$
d)



; 768 students

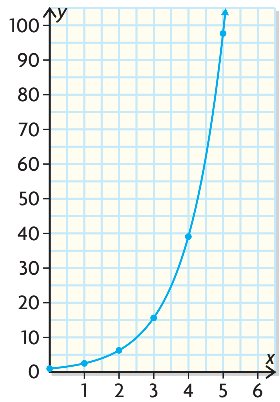
20. **a)** Yes. e.g., Any data with a constant doubling time can be expressed with an exponential function.
b) $y = (2)^{\frac{x}{3}}$; a represents the initial number of requests, b represents the rate of growth of the number of requests, x represents the amount of time in hours since the news broke, y represents the total number of interview requests.
c) domain: $\{x \mid x \geq 0, x \in \mathrm{W}\}$, range: $\{y \mid y \geq 4, y \in \mathrm{W}\}$
d)

Time	x	y
9:00 a.m.	0	4.0
10:00 a.m.	1	5.0
11:00 a.m.	2	6.4
12:00 p.m.	3	8.0
1:00 p.m.	4	10.1
2:00 p.m.	5	12.7
3:00 p.m.	6	16.0
4:00 p.m.	7	20.2

At 4:00 p.m., Philippe had received about 20 requests.

Lesson 7.3, page 461

1. **a)** linear, not exponential; e.g., differences are constant
b) exponential growth; $y = (2.5)^x$, e.g., constant ratios

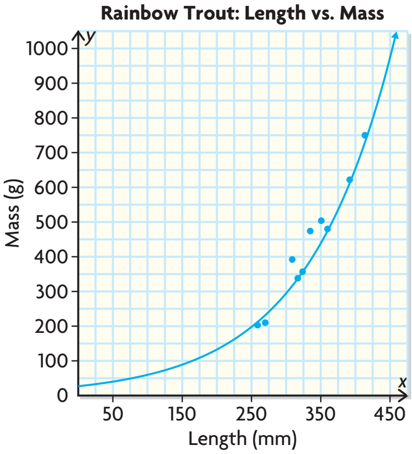


c) quadratic, not exponential; e.g., differences are not constant or a constant ratio
d) cubic, not exponential; e.g., differences are not constant or a constant ratio

2. **a)** $y = 10.097\ldots(0.200\ldots)^x$, decay function, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept: 10.1, end behaviour: extends from QII to QI, decreasing
b) $y = 2.780\ldots(1.054\ldots)^x$, growth function, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept: 2.78, end behaviour: extends from QII to QI, increasing

3. **a)** e.g., Yes, the ratios of consecutive pairs of y-values are close.
b) e.g., We can create the formula by setting a to the first rent value and b to the average ratio of consecutive pairs of y-values.
c) $14 000

4. a)

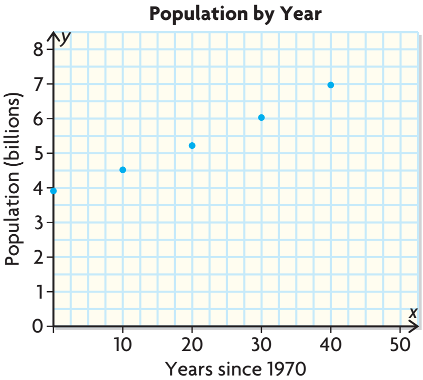


b) e.g., $y = 26.934... (1.008...)^x$

c) e.g., 690 g; I identified the point on the curve that had an x-value of 400.

d) 446 mm; e.g., I identified the point on the curve that had a y-value of 1000.

5. a)



b) e.g., $y = 3.911... (1.014...)^x$, where x is the number of years since 1970 and y is the population in billions

c) e.g., 8.05 billion

d) e.g., 61.5 years after 1970, that is, during 2031

6. a) $y = 14.429... (1.065...)^x$

b) 27.1 cm

c) 95.8 cm; e.g., no, since it was 98.0 cm on day 28

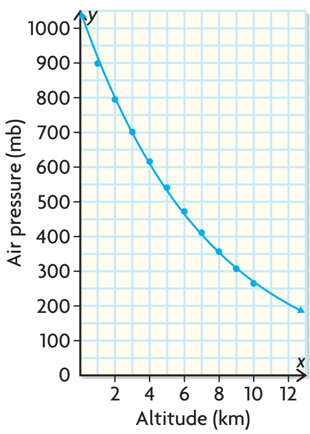
d) day 20

7. a) $y = (2)^x$

b) domain: $\{x \mid x > 0, x \in \mathrm{N}\}$, range: $\{y \mid y > 0, y \in \mathrm{N}\}$

c) 7

8. a)



b) $y = 1050.311... (0.873...)^x$

c) 137.3 mb

d) 5 km, 22 km

9. a) 20%

b) $y = 575.228... (0.799...)^x$

c) e.g., about 3 years; 3.1 years

10. a) e.g., No, the rate of change of atmospheric CO_2 is increasing.

b) e.g., $y = 315.61(1.01)^x$, where x is the number of years since 1960 and y is the atmospheric concentration of carbon dioxide in ppm

c) $y = 315.609... (1.004...)^x$

d) 387 ppm, 403 ppm

11. a) $y = 83.933... (0.9516...)^x$

b) 14 min

c) 29 min

12. a) $y = 40.798... (0.747...)^x$

b) 9 h

c) 2 h

13. a) $y = 4120.075... (1.499...)^x$

b) 237 553

c) $y = 4120.075(1.2)^x$

14. a) 10%

b) 12.2 weeks

15. a) i) 137 barrels

ii) 47 barrels

b) 132nd week

c) 256th week

16. a) $y = 3000(0.9)^x$

b) 365 barrels

c) 61st week

17. a) e.g., Calculate the rate of change in the decrease (b) and use 100 for a. Use software or a calculator to perform exponential regression to get those values.

b) 80.80%

c) 31

18. a) 40

b) e.g., No, the second differences are constant, so the data can be modelled by a quadratic function.

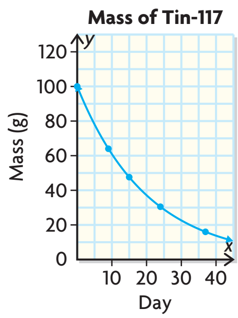
c) 112

19. a) 14

b) $A(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{14}}$

c) $A(t) = 100.001... (0.95...)^t$

d)



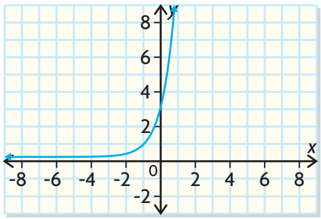
Mid-Chapter Review, page 472

1. a) No. e.g., extends from QII to QIV

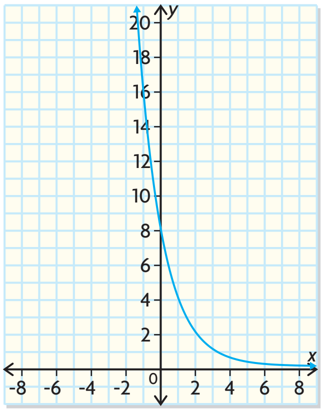
b) Yes. e.g., decreases from QII to QI

c) Yes. e.g., increases from QII to QI

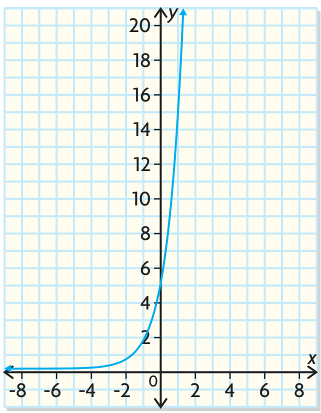
2. **a)** **i)**



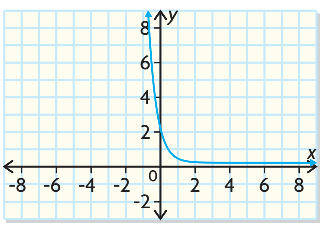
ii)



iii)



iv)



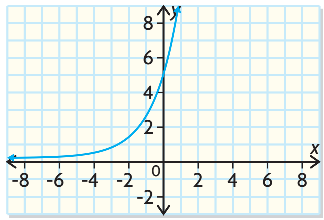
b) **i)** number of x-intercepts: 0, y-intercept: 3, end behaviour: QII to QI, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$

ii) number of x-intercepts: 0, y-intercept: 8, end behaviour: QII to QI, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$

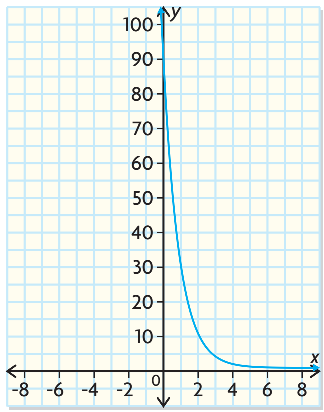
iii) number of x-intercepts: 0, y-intercept: 5, end behaviour: QII to QI, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$

iv) number of x-intercepts: 0, y-intercept: 2, end behaviour: QII to QI, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$

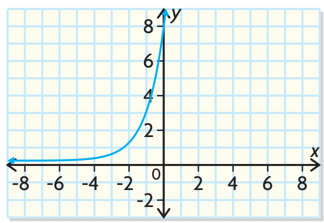
3. **a)** number of x-intercepts: 0, y-intercept: 5, end behaviour: QII to QI, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$; increasing



b) number of x-intercepts: 0, y-intercept: 90, end behaviour: QII to QI, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$; decreasing



c) number of x-intercepts: 0, y-intercept: 8, end behaviour: QII to QI, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$; increasing

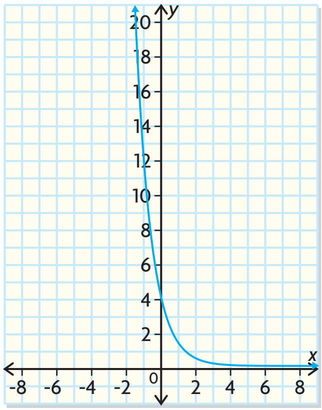


4. **a)** $a = 5$
$a = 3$

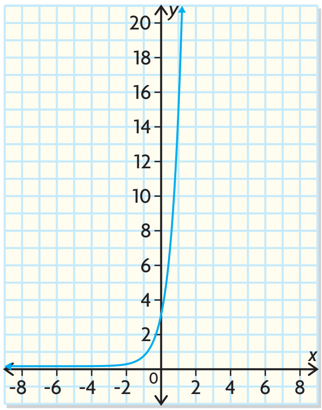
b) $0 < b < 1$
$b > 1$

c) domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$
domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$

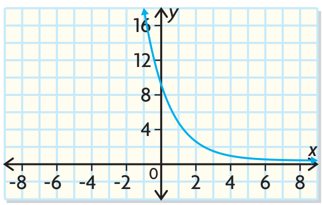
5. **a)** decreasing



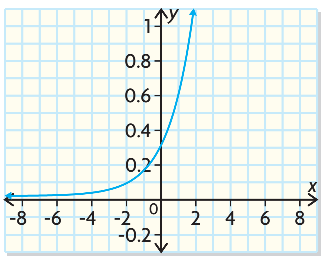
b) increasing



c) decreasing



d) increasing



6. **a)**

Round	Number of Students Who Receive Handouts
0	7
1	49
2	343
3	2401
4	16 807

b) Yes. $y = 7(7)^x$; e.g., The rate of increase of the number of students polled in each round increases at a constant rate.

7. $y = 36.871... (0.663...)^x$

8. **a)** $y = 7.628... (1.742...)^x$

b) **i)** \$10.07
ii) \$17.55
iii) \$30.58
iv) \$53.28

9. **a)** e.g., First divide amounts in consecutive rows to determine the base, then use the initial amount and the base to write an exponential formula.

b) 4% **c)** \$2220.37

10. **a)** 140 cm **c)** fourth bounce
b) 37 cm

11. **a)** exponential function, no x-intercept, y-intercept: 90, end behaviour: QII to Q1, decreasing

b) domain: $\{t \mid t \geq 0, t \in \mathrm{R}\}$, range: $\{C(t) \mid 21 \leq C(t) \leq 90, C(t) \in \mathrm{R}\}$; Domain represents the time period in which the coffee cools. Range represents the temperature of the coffee.

c) about 71 °C
d) 17 min, 47 min
e) about 63 min

Lesson 7.4, page 482

1. x-intercept: 1, number of y-intercepts: 0, end behaviour: QIV to QI, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$

2. **a)** No. e.g., no x-intercept and one y-intercept
b) No. e.g., two x-intercepts and one y-intercept
c) Yes. e.g., one x-intercept and no y-intercept
d) No. e.g., one x-intercept and one y-intercept
e) Yes. e.g., one x-intercept and no y-intercept
f) No. e.g., no x-intercept and one y-intercept

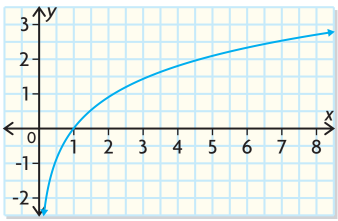
3. c: x-intercept: 1, number of y-intercepts: 0, end behaviour: QIV to QI, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$, $a > 0$, e.g., graph is increasing
e: x-intercept: 1, number of y-intercepts: 0, end behaviour: QI to QIV, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$, $a < 0$, e.g., graph is decreasing

4. $y = \log x$: x-intercept: 1, number of y-intercepts: 0, end behaviour: QIV to QI, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$, increasing
$y = -\log x$: x-intercept: 1, number of y-intercepts: 0, end behaviour: QI to QIV, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$, decreasing

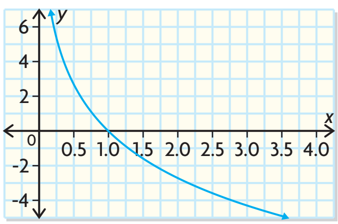
5.

	x-Intercept	Number of y-Intercepts	End Behaviour	Domain	Range
a)	1	0	QIV to QI	$\{x \mid x > 0, x \in R\}$	$\{y \mid y \in R\}$
b)	1	0	QI to QIV	$\{x \mid x > 0, x \in R\}$	$\{y \mid y \in R\}$
c)	1	0	QIV to QI	$\{x \mid x > 0, x \in R\}$	$\{y \mid y \in R\}$
d)	1	0	QIV to QI	$\{x \mid x > 0, x \in R\}$	$\{y \mid y \in R\}$
e)	1	0	QI to QIV	$\{x \mid x > 0, x \in R\}$	$\{y \mid y \in R\}$
f)	1	0	QIV to QI	$\{x \mid x > 0, x \in R\}$	$\{y \mid y \in R\}$

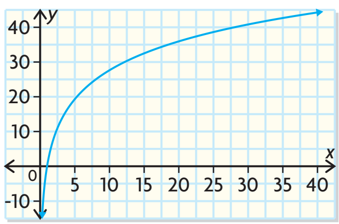
a)



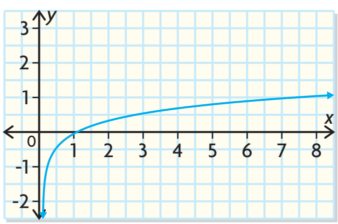
b)



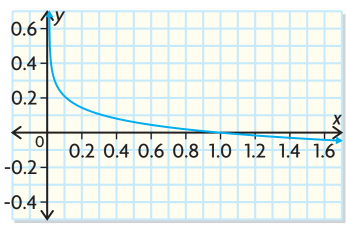
c)



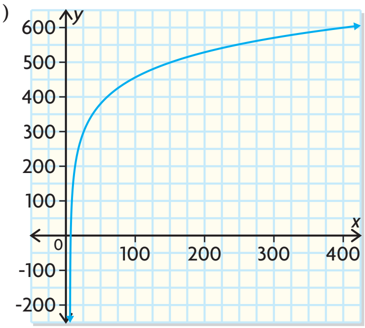
d)



e)

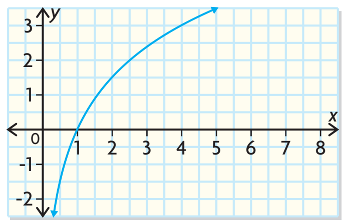


f)

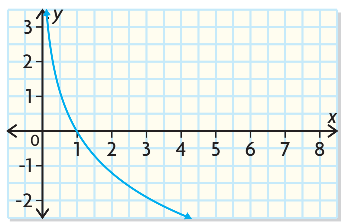


6. e.g., one x-intercept, no y-intercepts, domain: $\{x \mid x > 0, x \in R\}$

7. **a)** e.g., $y = 5 \log x$



b) e.g., $y = -4 \log x$



8. **i)** b, e.g., x-intercept is 1, no y-intercept, graph extends from QIV to QI

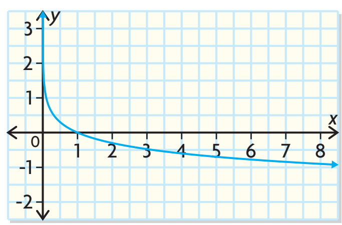
ii) c, e.g., x-intercept is 1, no y-intercept, graph extends from QI to QIV

iii) d, e.g., no x-intercept, y-intercept is 1, graph extends from QII to QI

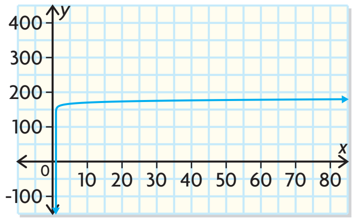
iv) a, e.g., no x-intercept, y-intercept is 2, graph extends from QII to QI

9. Yes. e.g., An exponential function has no x-intercepts, and a logarithmic function has one x-intercept.

10. As hydrogen ion concentration increases, pH decreases.



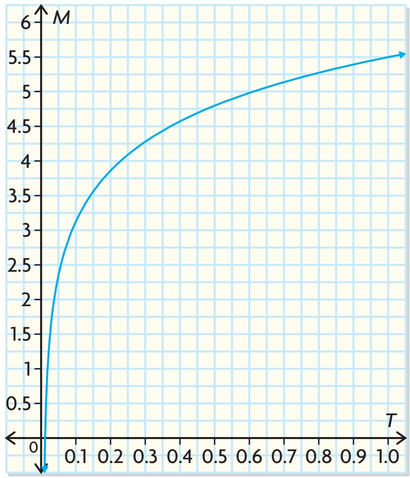
11. As the energy of the sound increases, the number of decibels increases.



12. **a)** P-intercept: 1, t-intercept: none, domain: $\{P \mid 0 < P \leq 1, P \in \mathrm{R}\}$, range: $\{t \mid t \geq 0, t \in \mathrm{R}\}$, function: decreasing, $P = 1$: $t = 0$
b) about 131 years
c) about 30 years

13. **a)** $a < 0$
b) The domain is restricted, $x > 0$, as the functions are logarithmic.
c) The range is unrestricted, as all values of y are possible.

14. **a)**



T-intercept: 0.000 003 16..., M-intercept: none, domain: $\{T \mid T > 0, T \in \mathrm{R}\}$, range: $\{M \mid M \geq 0, M \in \mathrm{R}\}$, function: increasing, $T = 1$: $M = 5.5$
b) about 7.2
c) about 3000 times as intense

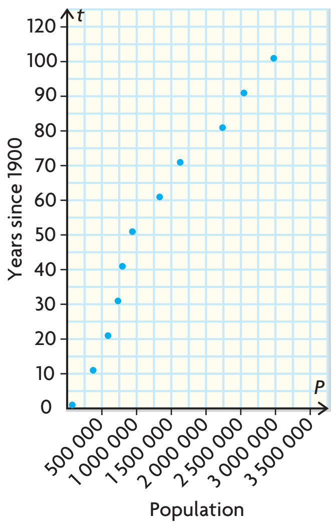
15. **a)** A-intercept: 3000, t-intercept: none, domain: $\{A \mid A \geq 3000, A \in \mathrm{R}\}$, range: $\{t \mid t > 0, t \in \mathrm{R}\}$, function: increasing
b) about 31 years
c) about 18 years

Lesson 7.5, page 494

1. **a)** e.g., Graph is logarithmic because graph is increasing, x-intercept is 1, there is no y-intercept, end behaviour is QIV to QI, domain is $\{x \mid x > 0, x \in \mathrm{R}\}$, and range is $\{y \mid y \in \mathrm{R}\}$.
b) factor of 10
c) increases by a factor of 3

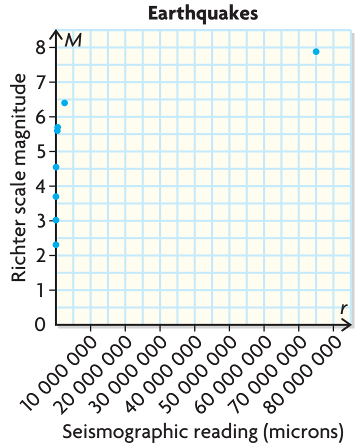
2. $y = -6.653... + 108.49... \ln x$; x-intercept: 1.063, y-intercept: none, end behaviour: QIV to QI, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$, function: increasing

3. **a)**



b) $t = -346.090... + 28.957... \ln P$; P-intercept: 155 110, t-intercept: none, end behaviour is QIV to QI, domain: $\{P \mid P \in \mathrm{N}\}$, range: $\{t \mid t \geq 0, t \in \mathrm{W}\}$, function: increasing
c) about 1974

4. **a)** independent: seismographic reading (r), dependent: Richter scale magnitude (M)
b)

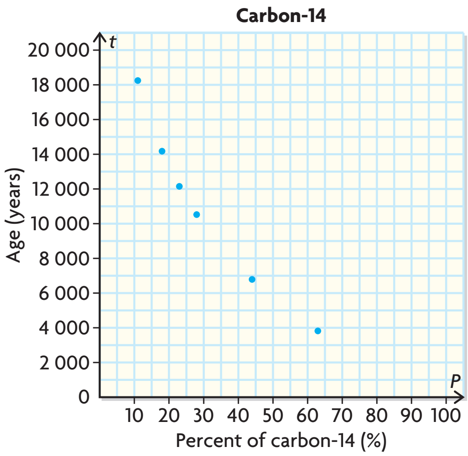


c) $M = -0.006... + 0.434... \ln r$
d) about 15.8 times as intense

5. **a)** independent: pressure (P); dependent: altitude (h)
b) $h = 30\ 665.960... - 6640.436... \ln P$
c) P-intercept: 101.3, h-intercept: none, end behaviour: QI to QIV, domain: $\{P \mid P > 0, P \in \mathrm{R}\}$, range: $\{h \mid h \in \mathrm{R}\}$, function: decreasing
d) 99.2 kPa
e) 26.7 kPa

Answers

6. **a)**



$t = 38\,070.332... - 8266.961... \ln P$

b) 268 years

c) 5730 years

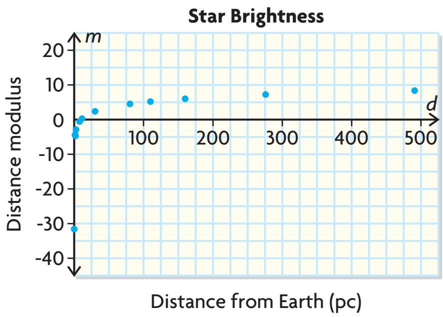
7. **a)** $A = 14\,999.826...\,(1.045...)^t$

b) $t = -218.44... + 22.717... \ln A$

c) about 12 years; e.g., I prefer the logarithmic equation, because it makes the calculations simpler.

8. **a)** independent: distance from Earth (d), dependent: distance modulus (m)

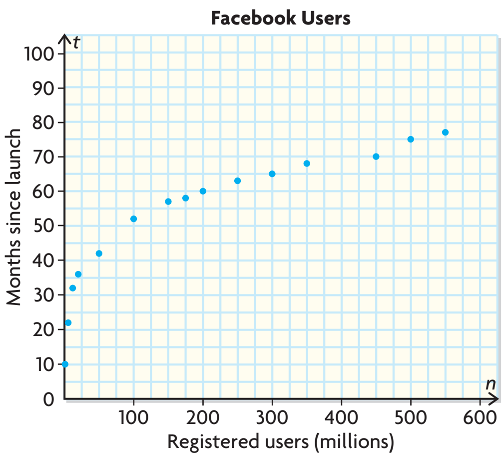
b)



c) $m = -5.086... + 2.174... \ln d$

d) -3.19 **e)** 4.8 pc

9. **a)**



b) $t = 5.888... + 10.833... \ln n$

c) August 2009

10. e.g., Enter the data in my calculator, perform a logarithmic regression, graph the data points in a scatter plot and the logarithmic function on the same axes, and identify the point with the known x-value and the unknown y-value.

11. **a)** **i)** $n = \dfrac{\ln A - \ln 5000}{\ln 1.05}$

ii) $n = \dfrac{\ln A - \ln 5000}{2 \ln 1.025}$

iii) $n = \dfrac{\ln A - \ln 5000}{4 \ln 1.0125}$

iv) $n = \dfrac{\ln A - \ln 5000}{365 \ln 1.000\,136...}$

b) e.g., The denominator approaches $\ln 1 = 0$ as the number of compounding periods increases.

Chapter Self-Test, page 501

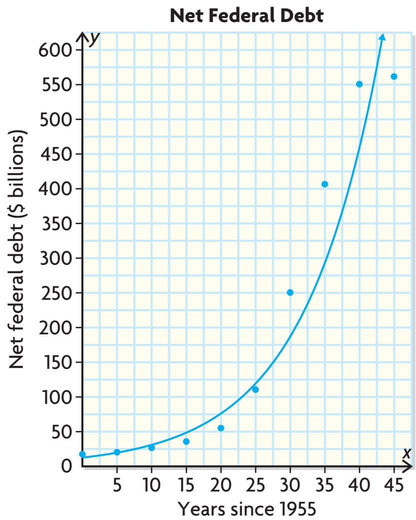
1. **a)** **ii)** e.g., decreasing exponential function with $y > 0$

b) **iv)** e.g., increasing logarithmic function with $x > 0$

c) **iii)** e.g., increasing exponential function with $y > 0$

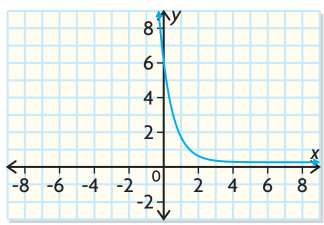
d) **i)** e.g., decreasing logarithmic function with $x > 0$

2. **a)** $y = 12.620...(1.094...)^x$, where y is the debt in billions of dollars and x is the number of years after 1955.



b) \$247.28 billion **c)** 1998

3. number of x-intercepts: 0, y-intercept: 6, end behaviour: QII to QI, domain: $\{x \mid x \in \mathbb{R}\}$, range: $\{y \mid y > 0, y \in \mathbb{R}\}$



4. x-intercept: 1, y-intercept: none, end behaviour: QI to QIV, domain: $\{x \mid x > 0, x \in \mathbb{R}\}$, range: $\{y \mid y \in \mathbb{R}\}$, function: decreasing

5. **a)** $-1.199... + 0.289... \ln x$

b) 9.5 magnitude

Chapter Review, page 504

1. **a)** e.g., Exponential functions extend from QII to QI. Polynomials can extend from either QIII or QII to QI or QIV.
b) e.g., The domain is always $\{x \mid x \in \mathrm{R}\}$, the range is always $\{y \mid y > 0, y \in \mathrm{R}\}$, and they all extend from QII to QI. a is the y-intercept.

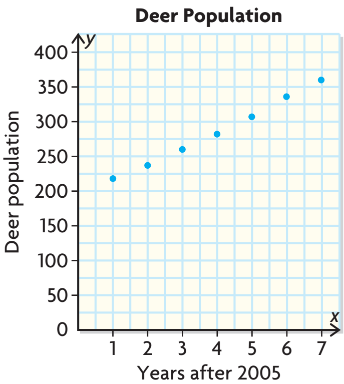
2. **a)** domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept: 9, end behaviour: QII to QI
b) e.g., Change b to a value greater than 1.

3. **a)** **i)** x-intercept: none
ii) y-intercept: 125
iii) QII to QI
iv) domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$
v) decreasing
b) **i)** x-intercept: none
ii) y-intercept: 0.12
iii) QII to QI
iv) domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$
v) decreasing
c) **i)** x-intercept: none
ii) y-intercept: 1
iii) QII to QI
iv) domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$
v) increasing
d) **i)** x-intercept: none
ii) y-intercept: 0.85
iii) QII to QI
iv) domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y > 0, y \in \mathrm{R}\}$
v) increasing

4. **a)** ii, e.g., Graph is increasing.
b) i, e.g., Graph is decreasing.

5. **a)** e.g., $y = 1000(3)^x$, where y is the population and x is the number of quarters after April 1, 1896
b) e.g., domain: $\{x \mid 0 \leq x \leq 3, x \in \mathrm{W}\}$ (quarters), range: $\{y \mid 1000 \leq y \leq 27\,000, y \in \mathrm{W}\}$ (population)
c) e.g., 1732, 5196; e.g., these two points are midpoints of the first two quarters, and so correspond to $x = 0.5$ and $x = 1.5$.

6. **a)**



b) $y = 200.798...(1.088...)^x$
c) 468
d) 2014

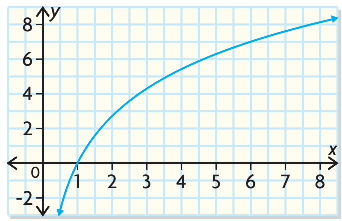
7. **a)** 38%
b) 7400 years

8. x-intercept: 1, number of y-intercepts: none, end behaviour: QIV to QI, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$

9.

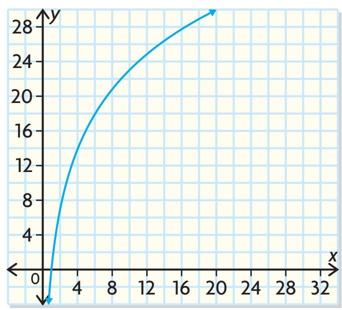
	x-Intercept	Number of y-Intercepts	End Behaviour	Domain	Range
a)	1	0	QIV to QI	$\{x \mid x > 0, x \in \mathrm{R}\}$	$\{y \mid y \in \mathrm{R}\}$
b)	1	0	QI to QIV	$\{x \mid x > 0, x \in \mathrm{R}\}$	$\{y \mid y \in \mathrm{R}\}$
c)	1	0	QIV to QI	$\{x \mid x > 0, x \in \mathrm{R}\}$	$\{y \mid y \in \mathrm{R}\}$
d)	1	0	QI to QIV	$\{x \mid x > 0, x \in \mathrm{R}\}$	$\{y \mid y \in \mathrm{R}\}$

a)

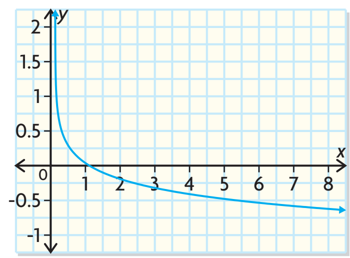


b)

c)



d)



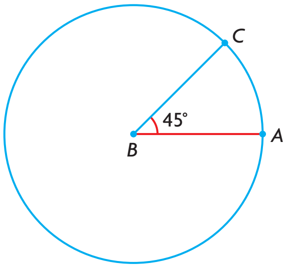
10. **i)** d, e.g., x-intercept is 1, no y-intercept, extends from QI to QIV
ii) b, e.g., no x-intercept, y-intercept is 1, increasing function
iii) a, e.g., x-intercept is 1, no y-intercept, extends from QIV to QI
iv) c, e.g., no x-intercept, y-intercept is 6, decreasing function

11. 0.23%

Chapter 8 – Sinusoidal Functions

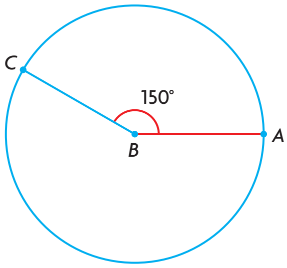
Lesson 8.1, page 519

1. a)



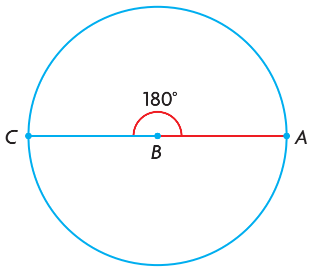
0.8

b)



2.5

c)

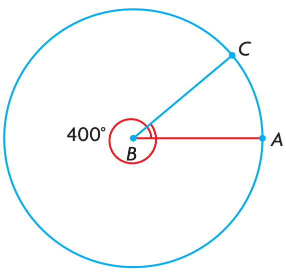


3.0

2. a) 90°
b) 30°
c) 135°
d) 270°

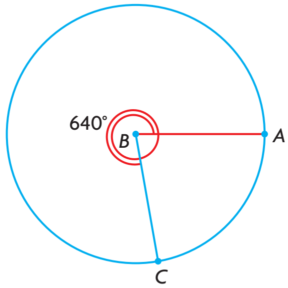
3. a) 60°
b) 235°
c) 390°

4. a)



7.0

b)



11.0

5. a) 450°
b) 600°

6. a) i) 9:20 a.m.
ii) 9:55 a.m.
iii) 10:55 a.m.
b) i) 2.1
ii) 5.8
iii) 12

7. a) about 6.28 m
b) 2, 120°

8. a) 2
b) 45°
c) 5
d) 400°

9. Disagree, e.g., For the circle with radius 5 m, the arc of an angle will be half as long as the arc of the equivalent angle in the circle of radius 10 m, but the ratio of arc length to radius, which gives the radian angle measure, will be the same.

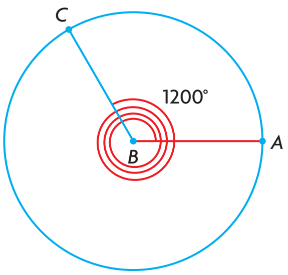
10. e.g., Use benchmarks to estimate the radian measure equivalent of angles greater than 360°, where 1 radian is about 60°. For example, determine the radian measure of
i) 480° ii) 525° iii) 650°

i) $480° = 8 \cdot 60°$, and $8 \cdot 1 = 8$
OR $480° = 360° + 90° + 30°$, and $2\pi + \frac{\pi}{2} + \frac{\pi}{6} = \frac{13\pi}{6}$

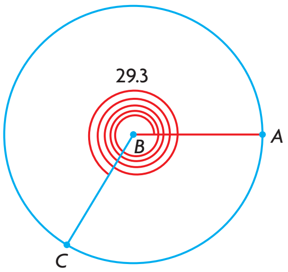
ii) $525° < 540°$
$540° = 9 \cdot 60°$, and $9 \cdot 1 = 9$
OR $540° = 360° + 180°$, and $2\pi + \pi = 3\pi$

iii) $650° < 660°$
$660° = 11 \cdot 60°$, and $11 \cdot 1 = 11$
OR $660° = 360° + 180° + 120°$, and $2\pi + \pi + \frac{\pi}{3} = \frac{10\pi}{3}$

11. **a)**



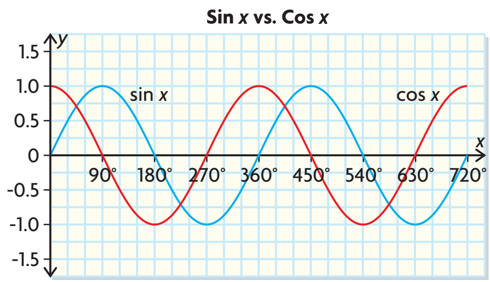
b)



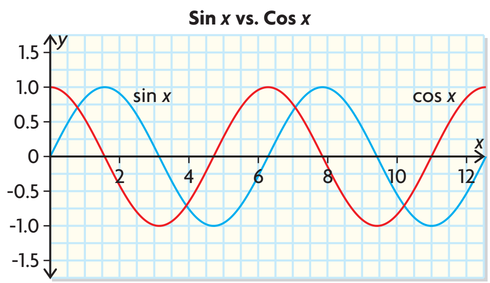
12. **a)** 4.0
b) 3.0
c) 5.9

Lesson 8.2, page 524

1. **a)**



b) 0; 180° and 540°
c) 0; 270° and 630°
d)



2. e.g., The graph of $y = \sin x$ repeats itself after it passes through 360° or 2π.

3. e.g., The graph is symmetrical along the x-axis, with the axis of symmetry being at 90° and 270°, respectively. The graph is rotationally symmetrical around the point (180°, 0).

4. e.g., The graph is symmetrical along the x-axis, with the axis of symmetry being at 180° and 360°, respectively. The graph is rotationally symmetrical around the point (270°, 0).

5. **a)** 0
b) 1

6. **a)** 0°, 180°, 360°, 540°, 720°
b) 90°, 270°, 450°, 630°

7. e.g., The up-and-down pattern in the graph of $y = \sin x$ looks like the repeating pattern of waves in a lake.

8. e.g., Yes, because each point on the sine graph corresponds to a point on the cosine graph with the same y-coordinate but with an x-coordinate of 90° less.

9. e.g., It would look like the graph of $\cos x$, as it would start at 1, decrease to 0 after spinning 90°, then decrease to -1 after spinning 180°, and so on.

Lesson 8.3, page 536

1. **a)** $\{y \mid -3.5 \le y \le 3.5, y \in \mathrm{R}\}$, 3.5
b) $\{y \mid -3 \le y \le 1, y \in \mathrm{R}\}$, 2

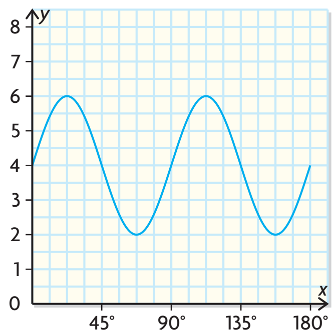
2. **a)** $y = 2$, 1.5
b) $y = -1$; 4

3. **a)** 120°
b) 3

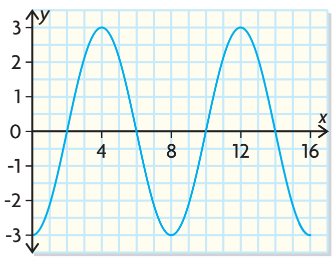
4. **a)** $\{y \mid -7 \le y \le 3, y \in \mathrm{R}\}$, 5, $y = -2$, 180°
b) $\{y \mid -0.5 \le y \le 6.5, y \in \mathrm{R}\}$, 3.5, $y = 3$, 5

5. $\{y \mid -2 \le y \le 3, y \in \mathrm{R}\}$, 2.5, $y = 0.5$, 4.25

6. **a)**



b)



7. e.g., The period would be the same, but the amplitude (and the minima and maxima) would be smaller.

8. **a)** 1.6 m
b) 0.8 m
c) 2.5 s
d) about 1 m, 1.6 m

9. **a)** $y = 0$; e.g., velocity of the air between breaths.
b) 0.8 L/s
c) 5 s; e.g., time to breathe in and out completely

10. **a)** e.g., Both: the increased amplitude means that the breaths are deeper and the decreased period means more breaths per minute.
b) e.g., amplitude and period have changed
c) 1.2 L/s

11. e.g., The gymnast completes a jump every 3 seconds, going from 3 feet above the ground to 30 feet, with a total jump height of 27 feet.

12. range: $\{y \mid -7 \le y \le 3, y \in \mathrm{R}\}$; amplitude: 5; midline: $y = -2$; period: 0.5

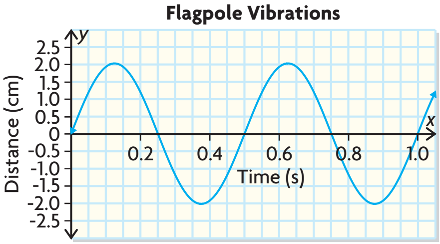
13. **a)** A: period $= 1$ s, minimum $= 10$ cm, maximum $= 26$ cm, amplitude $= 8$ cm;
B: period $= 0.5$ s, minimum $= 10$ cm, maximum $= 18$ cm, amplitude $= 4$ cm
b) e.g., A, because its amplitude is greater.

14. **a)** e.g., hoop's radius
b) hoop 3
c) hoop 1; hoop 3 or hoop 2
d) e.g., hoop 3, because the midline of the graph is lower

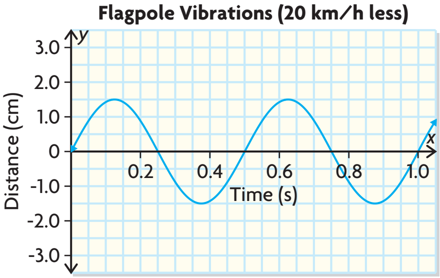
15. e.g., The range is the difference of the maximum and minimum y-values. The amplitude is half the difference of the maximum and minimum values. The equation of the midline is $y =$ average of the minimum and maximum values.

16. e.g., The period and midline stay the same, while the amplitude decreases.

Flagpole Vibrations



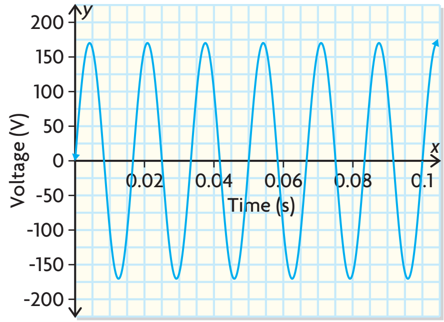
Flagpole Vibrations (20 km/h less)



17. **a)** graph 2, graph 1, graph 3
b) e.g., Each A pitch has double the frequency of the previous one.
c) graph 2: 110 Hz, graph 3: 440 Hz
d) e.g., Sound waves can be approximated by a sinusoidal function. The volume of the sound is dictated by the amplitude. The period, or frequency, varies with the size of the instrument (e.g., a tuba creates low-frequency/long-period waves while a piccolo creates high-frequency waves.)

18. **a)** about 17 ms
b)

Alternating Current



Mid-Chapter Review, page 545

1. **a)** 2.4
b) 3.6
c) 7.0
d) 9.0

2. **a)** 60°
b) 180°
c) 360°
d) 540°

3. 0.1, 20°, 0.5, 1.5, π, 190°, 200°, 310°, 400°

4. **a)** $\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$, 1, 360° or 2π, $y = 0$
b) e.g., Their range, amplitude, period and equation of midline are the same; their y-intercept is different.

5. **a)** 0°, 180°, 360°, 540°, 720°
b) maximum: 90°, 450°, minimum: 270°, 630°

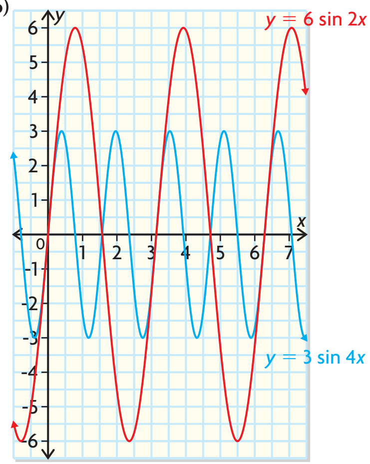
6. **a)** $\{y \mid -5 \le y \le 1, y \in \mathrm{R}\}$, 3, 180°, $y = -2$
b) $\{y \mid -3 \le y \le 1, y \in \mathrm{R}\}$, 2, 360°, $y = -1$

7. **a)** range: $\{y \mid -4 \le y \le 6, y \in \mathrm{R}\}$, amplitude: 5, equation of midline: $y = 1$, period: 7
b) range: $\{y \mid 0.5 \le y \le 3.5, y \in \mathrm{R}\}$, amplitude: 1.5, equation of midline: 2, period: 0.5

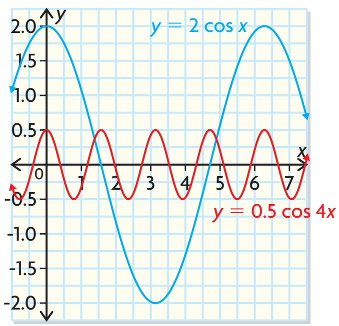
8. **a)** $\frac{1}{3}$ s
b) $y = 0$ amperes
c) 4.5 amperes

Lesson 8.4, page 558

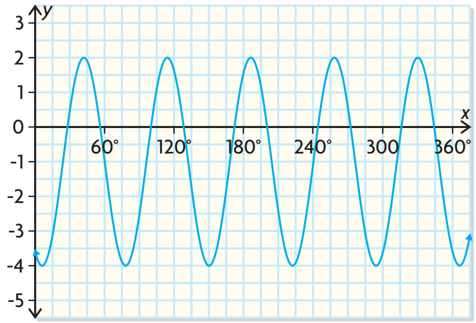
1. c), a), b); e.g., I ordered the functions based on the values of *a*.
2. c), a), b); e.g., The range is defined by the function's amplitude, so I ordered the functions based on the values of *a*.
3. a), b), c); e.g., The magnitude of the period is inversely proportional to *b*.
4. **a)** 45° left
   b) 180° right
   c) 45° right
5. **a)** 7, $\{y \mid -7 \le y \le 7, y \in \mathbf{R}\}$
   b) 13, $\{y \mid -13 \le y \le 13, y \in \mathbf{R}\}$
6. **a)** $y = 5$, 8, $\{y \mid -3 \le y \le 13, y \in \mathbf{R}\}$
   b) $y = -7$, 6, $\{y \mid -13 \le y \le -1, y \in \mathbf{R}\}$
7. **a)** $y = 2$; 7; -3
   b) $y = -3$; 0; -6
8. **a)** 30° right
   b) 100° left
   c) 4.5 right
   d) 3 left
9. **a)** 90°; 45° right
   b) 4π; 1 right
10. e.g.,
   a) $y = 6 \sin 2x$
   b)



11. e.g.,
   a) $y = 0.5 \cos 4x$
   b)



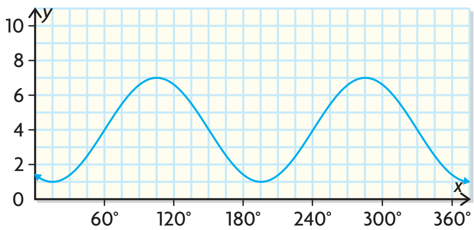
12. **a)** 86 ft
   b) about 1.8 min, or 1 min 48 s
   c) 4 ft
13. **a)** iii
   b) i
14. **a)** v
   b) ii
15. **a)** e.g., sinusoidal function, amplitude: 3, midline: $y = -1$, range: $\{y \mid -4 \le y \le 2, y \in \mathbf{R}\}$, period: 72°, horizontal translation from $y = \cos x$: 30° left
   b)



16. **a)**

Note	Period (s)	Frequency (Hz)
G	0.0051	196.0
D	0.0034	293.7
A	0.0023	440.0
E	0.0015	659.3

   b) e.g., Each string is tuned to almost 1.5 times the frequency of the previous string.
   c) e.g., About 2 periods of the lower note and 3 periods of the higher note. The frequency of the higher note is approximately 1.5 times the frequency of the lower note.
   d) e.g., 4, 6, and 9 periods
17. **a) i)** 3, $y = 4$, $\{y \mid 1 \le y \le 7, y \in \mathbf{R}\}$, 180°, 60° to the right
   ii) 3, $y = 4$, $\{y \mid 1 \le y \le 7, y \in \mathbf{R}\}$, 180°, 240° to the right
   iii) 3, $y = 4$, $\{y \mid 1 \le y \le 7, y \in \mathbf{R}\}$, 180°, 120° to the left
   b) e.g., The graphs are all the same.
   c)



   e.g., It is also the same because it is a cosine graph with the same attributes as the sine graphs, except that it is translated to the left by 45° (90° divided by b).
18. **a)** 10.5 cm, 2.5 cm
   b) 0.25 s, e.g., The apple completes 4 bounces per second.

19. **a)** 33 m, 3 m

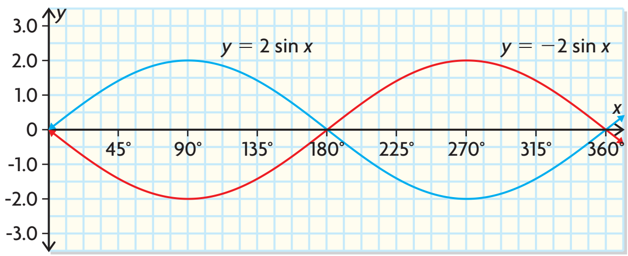
b) 2π min or about 6.3 min, e.g., The wheel completes a rotation every 6.3 min.

20. **a)** 0.75 s

b) e.g., A person's blood pressure makes a complete low-to-high cycle about every 0.75 s.

21. e.g., If the equation is in the form $y = a \sin b(x - c) + d$, the amplitude is a, the midline is $y = d$, it is translated to the right by c degrees or radians, and the period is $\frac{360°}{b}$ or $\frac{2\pi}{b}$. For example, the equation $y = 2 \sin 2(x - 45°) + 3$ has an amplitude of 2, a midline of $y = 3$, it is translated to the right 45°, and a period of 180°.

22. **a)**



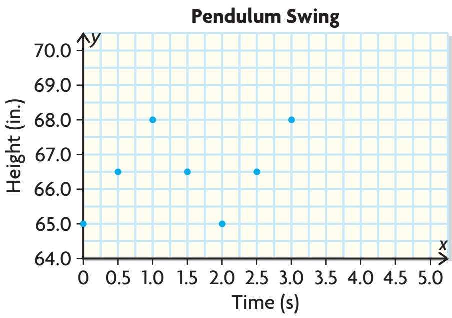
e.g., The period, amplitude and equation of midline are the same; they are reflected across the x-axis.

b) e.g., A translation of π or 180°.

c) Yes, e.g., graphs of functions of the form $y = a \sin x$ and $y = -a \sin x$ are horizontal translations of each other by 180° or π.

Lesson 8.5, page 571

1. **a)**



b) $y = 1.5 \sin(3.141...x - 1.570...) + 66.5$

2. **a)**

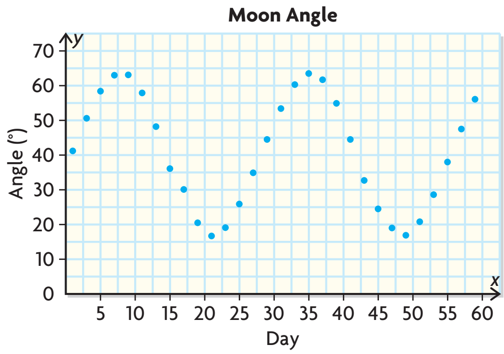
Revolution	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Time (s)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Height (ft)	11	6	1	6	11	6	1	6	11

b) $y = 5 \sin(3.141...x + 1.570...) + 6$

c) 1 ft 7 in.

d) 1 ft 11 in.

3. **a)**



e.g., about 28 days

b) e.g., Because the data follow a sinusoidal pattern.

c) $y = 22.954... \sin(0.230...x - 0.208...) + 40.284...$

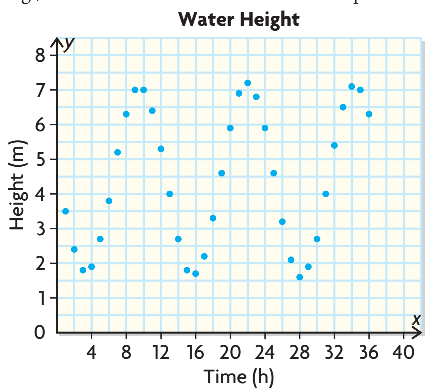
d) 63.2°

e) 23.3°

f) May 14

4. **a)** e.g., Because the data follows a sinusoidal pattern.

b)



$y = 2.726... \sin(0.505...x + 3.016...) + 4.426...$; yes

c) 7.2 m, 1.7 m

d) 12 h 25 min

e) 4.3 m

5. 6 h 50 min

6. **a)** $y = 19.581... \sin(0.462...x - 1.662...) + 6.238...$

b) $y = 16.992... \sin(0.470...x - 1.751...) - 4.738...$

c) $y = 18.388... \sin(0.464...x - 1.694...) + 0.686...$

d) e.g., The amplitudes, phase shifts and periods are similar; the equations of the midlines are different.

e) e.g., Yes, but it is more difficult to fit a curve to it.

f) 14.1 °C

7. **a)** $y = 20.341... \sin(0.536...x - 2.332...) - 2.706...$
b) 17.6 °C, −23.0 °C; e.g., the function is reasonably accurate.
c) 16.1 °C

8. **a)** $y = 4.207... \sin(0.017...x - 1.398...) + 12.411...$
b) 8 h 12 min to 16 h 37 min
c) day 171
d) 9 h 10 min
e) days 119 and 224

9. **a)** −12.9 °C, e.g., I determined a sinusoidal regression function that models the data, substituting numbers for months, and extrapolated the predicted value for the 13th month.
b) 39 mm, e.g., I determined a sinusoidal regression function that models the data, substituting numbers for months, and extrapolated the predicted value for the 13th month.
c) e.g., Temperature, since precipitation is more likely to vary.

10. **a)**

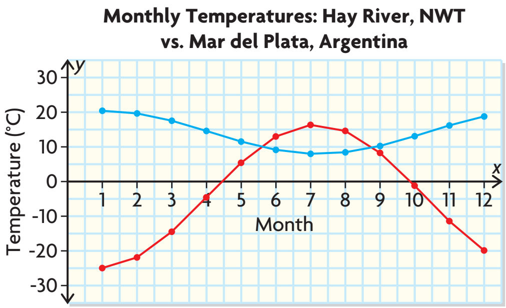
Time (s)	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
Height (cm)	70	55	40	55	70	55	40	55	70	55	40	55	70

b) $y = 15 \sin(2\pi x + 1.570...) + 55$
c) 55 cm

11. **a)** $y = 6 \sin(\pi x - 1.570...) + 11$
b) 15.2 in., 14.5 in.

12. e.g., No, because the regression function takes into account the effects of other months.

13. **a)** Hay River: $y = 20.681... \sin(0.501...x - 2.018...) - 4.284...$, yes;
Mar del Plata: $y = 6.221... \sin(0.504...x + 1.059...) + 14.188...$, yes
b)



e.g., The graphs have very similar periods; the amplitude is much less for Mar del Plata, but the midline is much greater; the maximum is only slightly greater for Mar del Plata; the maximums and minimums of the graphs are almost exactly opposite.
c) e.g., Their locations (north or south of the equator) affect when the temperature is high or low. Their distance from the equator affect the magnitude of the temperature changes.
d) e.g., Late May and late August, since that is when the graphs intersect.
e) e.g., south, as January and February temperatures are warmer than July and August temperatures.

14. **a)** e.g. Because the Earth is moving along its orbit, so the Moon must "catch up."
b) e.g., The period of the two graphs will be different (with the synodic being longer), but all other attributes will be the same.

Chapter Self-Test, page 579

1. **a)** 3.2
b) 8.8
c) 0.3
d) 6.0

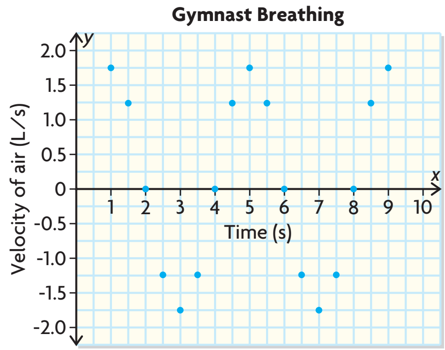
2. **a)** 315°
b) 270°
c) 330°
d) 540°

3. **a)** 0°, 60°, 120°, 180°, 240°, 300°, 360°, 420°, 480°, 540°, 600°, 660°, 720°
b) 0, 1.047, 2.094, 3.141, 4.189, 5.236, 6.283, 7.330, 8.378, 9.425, 10.472, 11.519, 12.566

4. **a)** $\{y \mid -1 \le y \le 7, y \in \mathrm{R}\}$, 4, 120°, 20° to the right, $y = 3$
b) $\{y \mid -6 \le y \le -2, y \in \mathrm{R}\}$, 2, 180°, 60° to the right, $y = -4$

5. $y = 19.557... \sin(0.476...x - 1.762...) + 6.141...$

6. **a)**



$y = 1.751... \sin(1.570...x)$
b) 4 s
c) e.g., Positive and negative velocities correspond to exhalations and inhalations (or vice versa).
d) 10 s, 12 s, 14 s, 16 s, 18 s

Chapter Review, page 581

1. **a)** 0.3
b) 2.0
c) 4.0
d) 9.6

2. **a)** 540°
b) 300°
c) 30°
d) 630°

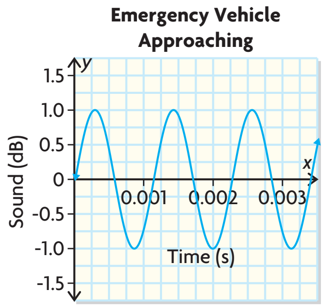
3. **a)** $180° < x < 360°$, $540° < x < 720°$
b) $0° \le x < 90°$, $270° < x < 450°$, and $630° < x \le 720°$

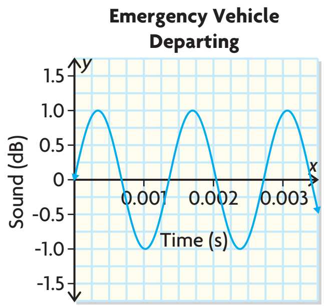
4. **a)** $\{y \mid -4 \le y \le 0, y \in \mathrm{R}\}$, 2, 360°, $y = -2$
b) $\{y \mid -2.5 \le y \le -0.5, y \in \mathrm{R}\}$, 1, 90°, $y = -1.5$

5. **a)** $y = 8$; e.g., She started at 8 m from the sensor.
b) 6 m
c) 4 s; this is the amount of time to complete one swing.
d) 2 m
e) e.g., Yes, since she is at the her furthest distance from the detector.

6. **a)** A: 3 m, 2 m, about 12 s; B: 4 m, 3 m, about 16 s
b) e.g., They are travelling at about the same speed (75 m in 48 s).

7. **a)**





e.g., When it's driving toward you.

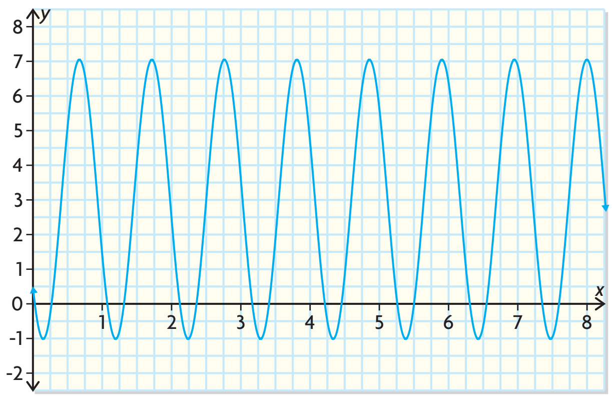
b) 877 Hz, 735 Hz

8. **a)** iv

b) ii

9. **a)** 4, $y = 3$, $\{y \mid -1 \leq y \leq 7, y \in \mathrm{R}\}$, $\frac{\pi}{3}$, 2.5 right

b)



10. about 2.4 cycles per nanosecond; about 5.0 cycles per nanosecond

11. about -12.2 °C

12. about 9.6 °C

Cumulative Review, page 586

1. **a)** x-intercepts: -3, -2, 2; y-intercept: -12; end behaviour: QIII to QI; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \in \mathrm{R}\}$

b) x-intercepts: -3, 1, 3; y-intercept: -9; end behaviour: QII to QIV; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \in \mathrm{R}\}$

2. **a)** 3, positive, -12

b) 3, negative, -9

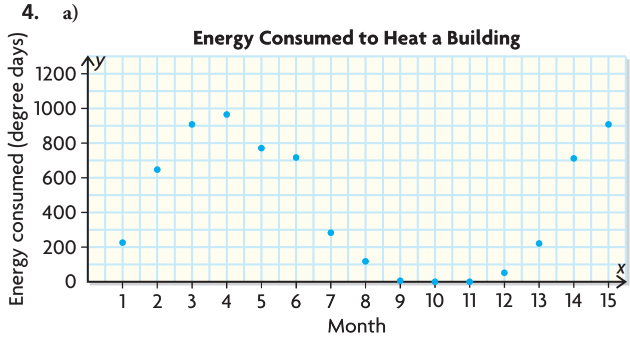
3. **a)** x-intercepts: 1; y-intercept: -2; end behaviour: QIII to QI; turning points: 0; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \in \mathrm{R}\}$

b) x-intercepts: 0, 1 or 2; y-intercept: -8; end behaviour: QIII to QIV; turning points: 1; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \leq -4, y \in \mathrm{R}\}$

c) x-intercepts: 1, 2 or 3; y-intercept: -2; end behaviour: QIII to QI; turning points: 2; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \in \mathrm{R}\}$

d) x-intercepts: 3; y-intercept: 0; end behaviour: QII to QIV; turning points: 2; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \in \mathrm{R}\}$

4. **a)**



$y = 4.808\ldots x^3 - 106.412\ldots x^2 + 597.474\ldots x - 163.803\ldots$

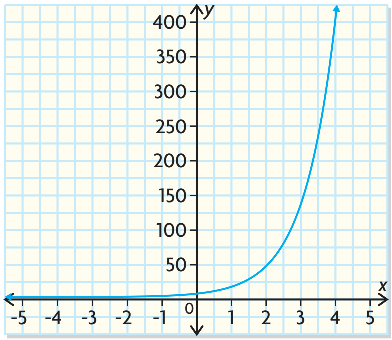
b) e.g., from halfway through month 8 to three quarters of the way through month 13

c) e.g., 274 degree days; there are 31 days in the seventh month, so $x = 8 - \frac{1}{31}$

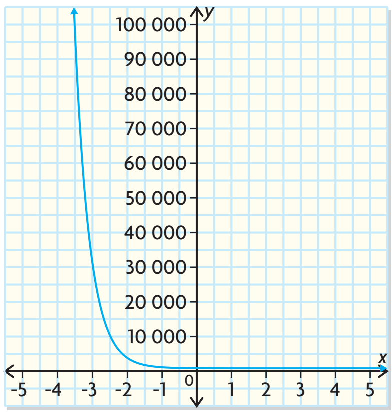
5.

Number of x-Intercepts	y-Intercept	End Behaviour	Domain	Range	Increasing or Decreasing?
0	5	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$	increasing
0	30	QII to QI	$\{x \mid x \in R\}$	$\{y \mid y > 0, y \in R\}$	decreasing

a)



b)

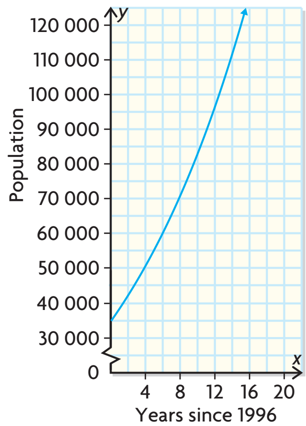


6. a)

Year	Population
1996	35 000
1997	37 800
1998	40 824
1999	44 090
2000	47 617
2001	51 426
2002	55 541
2003	59 984
2004	64 783
2005	69 965
2006	75 562

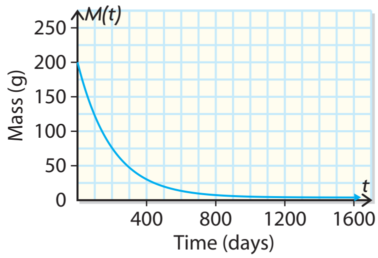
b) e.g., The rate of growth is constant, and growth occurs rapidly.
c) 35 000; The y-intercept represents the initial population (in 1996).

d) **Population of Wood Buffalo, Alberta**



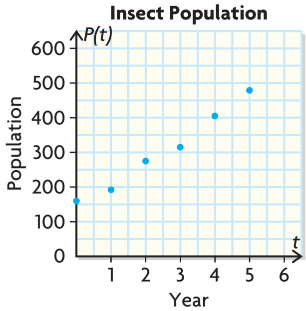
i) 111 026
ii) during 2009

7. a) **Polonium-210 Half-Life**



b) 0.02 g
c) 276 days

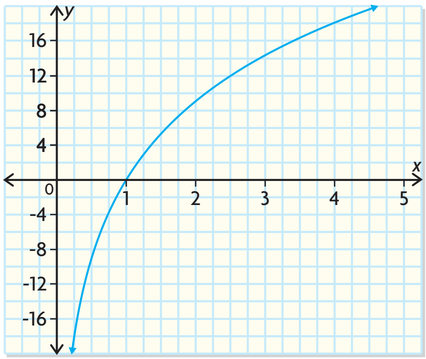
8. a) **Insect Population**



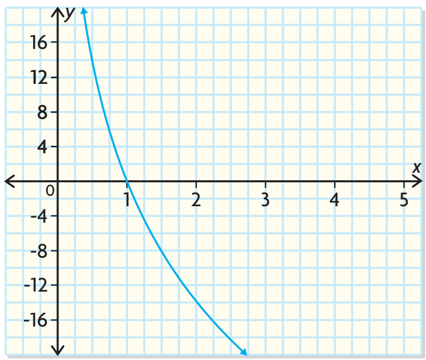
b) $P(t) = 161.581...\,(1.251...)^t$
c) e.g., a represents the initial population, 161.581..., b represents the growth factor, 1.251....
d) about 8 years

9. x-intercept: 1, y-intercept: none, end behaviour: QIV to QI, domain$\{x \mid x > 0, x \in R\}$, range: $\{y \mid y \in R\}$

10. **a)** x-intercept: 1, y-intercept: none, end behaviour: QIV to QI, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$, function: increasing

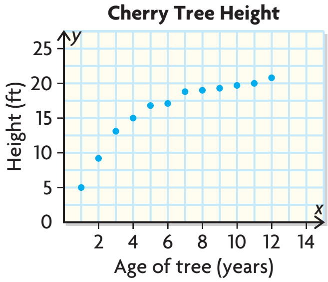


b) x-intercept: 1, y-intercept: none, end behaviour: QI to QIV, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$, function: decreasing



11. **a)**

Cherry Tree Height



b) $y = 6.357... \ln x + 5.561...$

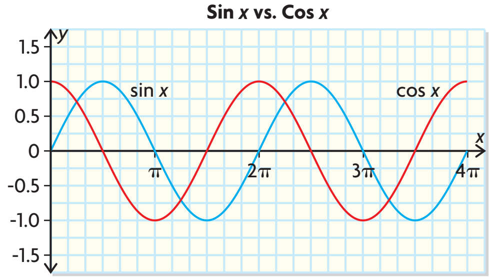
c) about 22.8 ft

d) about 2.8 years old

12. **a)** number of x-intercepts: multiple; y-intercept: $\sin x$, 0; $\cos x$, 1; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid -1 \leq y \leq 1, y \in \mathrm{R}\}$, period: 2π; amplitude: 1; equation of the midline: $y = 0$

b)

Sin *x* vs. Cos *x*



13. **a)** range: $\{y \mid 1 \leq y \leq 11, y \in \mathrm{R}\}$, midline: $y = 6$, amplitude: 5 m, period: 90 s

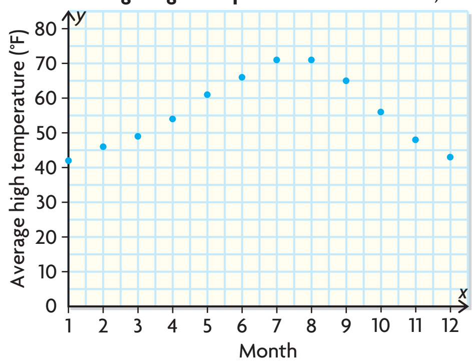
b) e.g., The Ferris wheel starts 1 m off the ground and over 90 s rotates to 11 m and back. The axis of the wheel is 6 m off the ground and the wheel has a radius of 5 m.

14.

	Amplitude	Period	Equation of the Midline	Domain	Range	Horizontal Translation
a)	3	90°	$y = 2$	$\{x \mid x \in \mathrm{R}\}$	$\{y \mid -1 \leq y \leq 5, y \in \mathrm{R}\}$	30° right
b)	5	6.28	$y = -2$	$\{x \mid x \in \mathrm{R}\}$	$\{y \mid -7 \leq y \leq 3, y \in \mathrm{R}\}$	4.00 left

15. **a)**

Average High Temperature in Vancouver, BC



e.g., The average monthly temperatures should be relatively consistent from year to year.

b) $y = 13.983... \sin(0.551...x - 2.363...) + 56.676...$

c) 70.6°F

d) e.g., from early May to early October