FINANCE

I – Simple Interest:

Interest: The amount of money earned on an investment or paid on a loan.
Principal: The original amount of money invested or loaned.

1. Simple Interest:

Simple Interest is determined only on the principal of an investment, as opposed to compound interest that are calculated each time on the current/updated amount of money at the time of calculation.

Example If you're investing \$1000 at an annua simple interest rate of 5%, then at the end of the year, you'll have:

$$A = 1000 + 0.05 \times 1000 = \$1050 \quad \underline{\text{or}} \quad 1000(1+0.05)$$

After 2 years, you'll have:

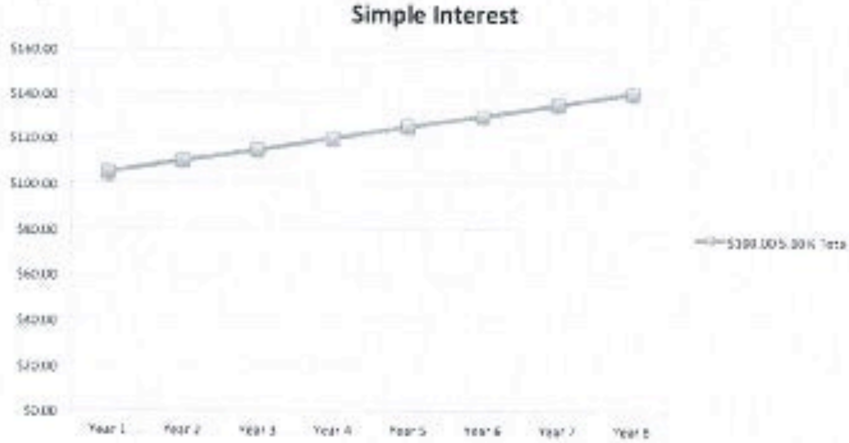
$$A = 1000 + 0.05 \times 1000 \times 2 = \$1100 \quad \underline{\text{or}} \quad 1000(1+0.05 \times 2)$$

Formula: $\boxed{A = P(1+rt)}$

where **A** is the future value (in \$), **t** the time in years, **r** the interest rate (in decimal form) and **P** the principal (in \$).

Note : The interest rates are always annual interest rates.

The value of an investment that earns simple interest over time is a linear function (because the interest added is always the same amount).
ex :



Examples :

1) Marty invested in a $2500 guaranteed investment certificate (GIC) at 2.5% simple interest, paid annually, with a term of 10 years.

a) How much interest will accumulate over the term if Marty's investment?

$$0.025 \times 2500 \times 10 = \$625$$

b) What is the future value of his investment at maturity?

$$2500 + 625 = \$3125 \quad \text{or} \quad M = 2500(1 + 0.025 \times 10) = \$3125$$

2) Sunni invested $15 000 in a savings account. Sunni earned a simple interest rate of 8%, paid semi-annually on her investment. She intends to hold the investment for 4.5 years, when she will withdraw all the money to buy a car. Determine the value of the investment at each half year until she withdraws the money.

$$A = 15000 \times (1 + 0.08 \times 4.5) = \$20400$$

$$\boxed{\textbf{Rate of Return: } \frac{\textit{money earned (or lost)}}{\textit{money invested}}}$$

The rate of return is usually expressed as a percentage or a decimal.

3) Ingrid invested her summer earnings of $5000 at 8% simple interest, paid annually. She intends to use the money in a few years to take a holiday with a girlfriend.

a) How long will it take for the future value of the investment to grow to $8000?

$$8000 = 5000(1 + 0.08t)$$
$$\frac{8}{5} = 1 + 0.08t$$
$$\frac{3}{5} = 0.08t$$
$$t = \frac{3/5}{0.08}$$
$$t = 7.5 \Rightarrow \boxed{8 \text{ years}}$$

b) What is Ingrid's rate of return?

After 8 years, she'll have : $5000(1 + 0.08 \times 8) = \$8200$

$\Rightarrow$ money earned : $3200

Rate of Return : $\frac{3200}{5000} = 0.64$ or 64%

4) Grant invested \$25 000 in a simple interest Canada Savings Bond (CSB) that paid interest annually.

a) If the future value of the CSB is \$29 375 at the end of 5 years, what interest rate does the CSB earn?

$$29375 = 25000(1+5r)$$

$$\frac{29375}{25000} = 1+5r$$

$$5r = \frac{4375}{25000}$$

$$r = \frac{875}{25000}$$

$$\Rightarrow \boxed{3.5\%}$$

b) Grant cashed in the bond after 4.5 years because a house he had been admiring came up for sale and he needed a down payment. How much money did he have for the down payment?

Interest is paid annually $\Rightarrow t = 4$

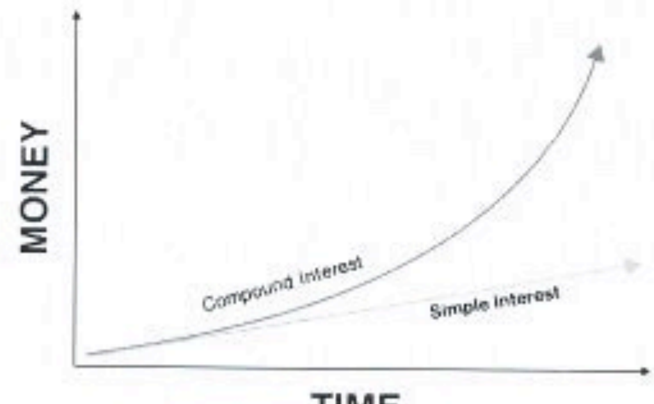
$$25000(1+4\times 0.035) = \$28500$$

Hwk : Worksheet Simple Interest (1.1 p 14) # 4, 5, 6, 7, 10, 11, 12

2. Compound Interest:

Compound Interest is determined by applying the interest rate to the sum of the principal and any accumulated interest. Previously earned interest is reinvested over the course of the investment.

Note: For an investment, with the same interest rate, compound interest is a better choice than simple interest.



Financial institutions pay compound interest on investments at regular equal intervals. If interest is paid annually, it is calculated at the end of the first year on the principal and then added to the principal. At the end of the second year, the interest is calculated on the balance at the end of the first year (principal + interest earned from the previous year). This pattern continues every year until the end of the investment term.

Interest can be compounded annually, semi-annually (twice a year), quarterly (4 times a year), monthly (12 times a year), weekly (52 times a year) or daily (365 times a year).

General formula : $\boxed{A = P(1+i)^{n}}$ number of times the interest is applied

Future Value

Principal

interest (actually) applied each time. } ⚠ not always the annual interest rate

you can also remember it as $A = P\left(1+\frac{i}{n}\right)^{nt}$

annual interest rate

time years

number of times the interest is compounded per year

Example 1:

Yvonne earned \$4 300 in overtime on a carpentry job. She invested the money in a 10-year Canada Savings Bond that will earn 3.8% compounded annually.
Determine the future value (after 10 years)

$A = 4300(1+0.038)^{10}$

$A = \$6243.70$

Example 2:

Matt has invested a \$23 000 inheritance in an account that earns 13.6%, compounded semi-annually. The interest rate is fixed for 10 years. Matt plans to use the money for a down payment on a house in 5 to 10 years.

a) What is the future value of the investment after 5 years? What is the future value after 10 years?

After 5 years: $A = 23000\left(1+\frac{0.136}{2}\right)^{10} = \$44405.87$

After 10 years: $A = 23000\left(1+\frac{0.136}{2}\right)^{20} = \$85733.96$

b) Compare the principal and the future values at 5 years and 10 years. What do you notice?

In the 1st 5 years, the principal increased by \$21405.87

In the last 5 years, it increased by \$41328.09

c) If the investment had earned simple interest, would the relationship between the principal and the future values have been the same? Explain.

$A = 23000(1+0.136\times 10) = \$54280$ (after 10 years)

Example 3:

Céline wants to invest \$3 000 so that she can buy a new car in thenext 5 years. She has the following options:

A. 4,8% compounded annually
B. 4.8% compounded semi-annually
C. 4.8% compounded monthly
D. 4.8% compounded weekly
E. 4.8% compounded daily

Compare the interest earned by each of these options for terms of 1 to 5 years.

$$A: 3000 \times 1.048^5 = \$3792.52$$

$$B: 3000 \times 1.024^{10} = \$3802.95$$

$$C: 3000 \times 1.004^{60} = \$3811.92$$

$$D: 3000 \times \left(1 + \frac{0.048}{52}\right)^{260} = \$3813.33$$

$$E: 3000 \times \left(1 + \frac{0.048}{365}\right)^{1825} = \$3813.69$$

Rule of 72: A simple formula for estimating the doubling time of an investment; 72 is divided by the annual interest rate as a percent to estimate the doubling time of an investment in years. The Rule of 72 is most accurate when the interest is compounded annually.

Example 4:

Both Berta and Kris invested \$5 000 by purchasing Canada Savings Bonds. Berta's CSB earns 8%, compounded annually, while Kris's CSB earns 9%, compounded annually.

a) Estimate the doubling time for each CSB.

Berta: $72/8 = 9$ (around 9 years)

Kris: $72/9 = 8$ (around 8 years)

b) Verify your estimates by determining the doubling time for each CSB.

Berta: $10\,000 = 5000(1+0.08)^t$

$$2 = 1.08^t$$

$$\left.\begin{array}{l} 1.08^9 = 1.999 \\ 1.08^{9.1} = 2.014 \end{array}\right\} \text{around 9 years.}$$

Present Value: The amount that must be invested now to result in a specific future value in a certain time at a given interest rate.

Example 5:
Joanie is 18 years old. She has inherited some money from a relative. She wants to invest some money so that she can buy a home in Milk River, Alberta, when she turns 30. She estimates that she will need about \$170 000 to buy a home.
How much does she have to invest now, at 6.5% compounded annually?

$$170\,000 = C(1+0.065)^{12}$$

$$C = \frac{170\,000}{(1.065)^{12}}$$

$$C \simeq \$79\,846.09$$

Example 6:
Laura has invested \$15 500 in a Registered Education Savings Plan (RESP). She wants her investment to grow to at least \$50 000 by the time her newborn enters university, in 18 years.
What interest rate, compounded annually, will result in a future value of \$50 000?

$$50\,000 = 15\,500(1+i)^{18}$$

$$\frac{100}{31} = (1+i)^{18}$$

$$\sqrt[18]{\frac{100}{31}} = 1+i$$

$$i = \sqrt[18]{\frac{100}{31}} - 1$$

$$i \simeq 0.0672$$

$$\Rightarrow 6.72\%$$

Hwk : Compounded Interest A (1.3 p 30) # 2, 4a, 5, 7, 8, 10 & B (1.4 p 40) #1, 3, 5, 7, 10, 11, 12.

II – Investments Involving Regular Payments:

The are many different ways to invest money. We can, for example, binvest in real estate, buy stock options, deposit money in a savings account, …
Many investments are made buy making regular payments (because we don't necessarily have a large amount of money from the start).
Banks often propose (among other examples):

- RRSP (pension plans)
- RESP (savings towards kid's university)

For an investment that involves a series of equal deposits or payments made at regular intervals, the future value is the sum of all the regular payments plus the accumulated interest.
The future value of an investment involving regular payments can be found by determining the sum of all the future values of each regular payment:

$$A = R(1+i)^0 + R(1+i)^1 + R(1+i)^2 + R(1+i)^3 + \cdots + R(1+i)^{n-1}$$

because you pay at the e[illegible] of the 1st interva[illegible]

where A is the amount, or future value of the investment; R is the regular payment; i is the interest rate per compounding period, expressed as a decimal; and n is the number of compounding periods.

Example 1:
Darva is saving for a trip to Australia in 5 years. She plans to work on a student visa while she is there, so she needs only enough money for a return flight and her expenses until she finds a job. She deposits $500 into her savings account at the end of each 6-month period from what she earns as a server. The account earns 3.8% compounded semi-annually. How much money will be in the account at the end of 5 years? How much of this money will be earned interest?

$$A = 500 \times 1.019^0 + 500 \times 1.019^1 + \cdots + 500 \times 1.019^9$$

$$= \$5449.90$$

Interest earned: $5449.90 - 500 \times 10 = \$449.90$

Example 2:
Adam made a $200 payment at the end of each year into an investment that earned 5%, compounded annually. Blake made a single investment at 5%, compounded annually. At the end of 5 years, their future values were equal.

a) What was their future value?

Adam: $200\left(1.05^0 + 1.05^1 + 1.05^2 + 1.05^3 + 1.05^4\right) = \$1105.13$

b) What principal amount did Blake invest 5 years ago?

$$1105.13 = P \times 1.05^5$$

$$P = \frac{1105.13}{1.05^5} \simeq \$865.90$$

c) Who earned more interest?

Adam: $1105.13 - 200 \times 5 = \$105.13$

Blake: $1105.13 - 865.90 = \$239.23$

As soon as the number of periods becomes large, it becomes very tedious to make all the calculations. We could use Excel, but it takes some effort and care to set up the formulas. Otherwise, we can use internet free calculators or the TVM solver on our graphing calculators.

TVM Solver on the TI 83 or 84

N= Total number of payments
I%= interest rate (as a percentage)
PV= Present Value
PMT= Payment amount
FV= Future Value
P/Y= Number of payments per year
C/Y= Number of times the interest is compounded per year.

Example : RESP :
If you invest $2000 per year, with interest rate 1.75%, compounded monthly when your child is born. How much money do you have when he/she turns 18?

N= 18
I%= 1.75
PV= 0
PMT= – 2000
FV= ?
P/Y= 1
C/Y= 12

$\Rightarrow FV = \$41941.37$

Interest: $41941.37 - 2000 \times 18$

$= \$5941.37$

Note: With RESP, the government actually will also participate to the investment with a given cap per year, as an incentive.

Example 2:

Jeremiah deposits $750 into an investment account at the end of every 3 months. Interest is compounded quarterly, the term is 3 years, and the future value is $10 059.07.
What annual rate of interest does Jeremiah's investment earn?

N= 12
I%= ?
PV= 0
PMT= −750
FV= 10059.07
P/Y= 4
C/Y= 4

=> I% = 8%

Notes:

- The future value of a single deposit has a greater future value than a series of regular payments of the same total amount.

- Small deposits over a long term can have a greater future value than a large deposit over a short term because there is more time for compound interest to be earned.

Hwk : Investing Money (1.5 p 55) # 1 – 11

III – Loans:

There are many situations when we have to or want to borrow money. There are usually several options to do so. You have to always keep in mind that it ends up costing more than a single payment at the start.

- The large majority of commercial loans are compound interest loans, although simple interest loans are also available.
- The cost of a loan is the interest charged over the term of the loan.
- A loan can involve regular payments over the term of the loan or a single payment at the end of the term.
- The interest that is charged on a loan will be less under any or all of these conditions: Les intérêts à payer sur un emprunt seront moins élevés si une ou toutes les conditions suivantes s'appliquent:
  - The interest rate is decreased.
  - The interest compounding frequency is decreased.
  - Regular payments are made.
  - The regular payment amount is increased.
  - The payment frequency is increased.
  - The term is decreased.

An Amortization Table: is a table that lists regular payments of a loan and shows how much of each payment goes toward the interest charged and the principal borrowed, as the balance of the loan is reduced to zero. We can do it with Excel, but it takes time to set it up. Tere are also some calculators online. We won't spend time on it this semester.

Payment Period (month)	Payment ($)	Interest Paid ($) Balance $\times \frac{I}{C/Y}$	Principal Paid ($) Payment − Interest paid	Balance ($) Principal borrowed − Principal paid
0 1 2 ⋮				

Example 1:

Lars borrowed $12000 at 5%, compounded monthly. He reimburses $350 at the end of each month.

a) In which month will Lars have at least half of the loan paid off?

N= ?
I%= 5
PV= 12000
PMT= −350
FV = −6000
P/Y = 12
C/Y = 12

⇒ N = 19.2

⇒ 20 months

Important Notes:

- PV and FV always have opposite signs (either you receive money that you start owing right away, or you give money that you'll get back in the end.)
- If PV and PMT have the same sign, then it is an investment.
  If PV and PMT have opposite signs, then it is a loan.

b) How long will it take Lars to pay off the loan?

N= ?
I%= 5
PV= 12000
PMT= −350
FV = 0
P/Y = 12
C/Y = 12

⇒ 38 months.

c) How much interest will Lars have paid by the time he has paid off the loan?

$N \simeq 37.07$

$N \times 350 - 12000 = \$974.5$

- With each payment period, the interest paid decreases while the principal paid increases. This occurs because each payment decreases the balance of the loan, so the interest on the remainder of the balance will be less on the next payment. Also, because the payment amount stays the same, more of the payment goes toward paying off the principal, since less is being paid toward the interest.

Example 2:
Trina's employer loaned her \$10000 at a fixed interest rate of 6%, compounded annually, to pay for college tuition and textbooks. The loan is to be repaid in a single payment on the maturity date, which is at the end of 5 years. How much will Trina need to pay her employer on the maturity date? And what is the accumulated interest on the loan?

$$A = 10000 \times 1.06^5 = 13382.26$$

Interests : \$ 3382.26

Hwk : Analyzing loans (2.1 p 92) # 1, 2, 3, 4, 7, 8, 9, 11, 13, 14, 17

1. Credit Cards :

Credit Cards usually have a minimum amount that must be paid each month, based on a percent of the outstanding balance. If there is no outstanding balance from the previous month and the new balance is paid off in full by the payment due date, no interest is charged.

If a credit card does not have an outstanding balance and it is used for a single purchase, it can be treated as a loan. The purchase price is the principal borrowed, and regular payments can be made until the balance is paid off.

The cost of using credit is not just the amount of interest charged. There are incentives, such as cash rebates, that reduces the principal. This may end up costing more in interest but result in a lower total loan payment amount.

Example: Jayden saw the new sound system he wanted on sale for \$2623.95 (including taxes). He had to buy it on credit. He would like to pay \$110 per month. He has two possible options:

- Use his new bank credit card, which has an interest rate of 14.5%, compounded daily (with no outstanding balance from the previous month).

  or

- Apply for the store credit card, which offers an immediate rebate of \$100 on the price but has an interest rate of 19.3% compounded daily.

Which should he choose?

	New card	Store Card
N	$\boxed{?}$ 28.34	$\boxed{?}$ 28.92
I	14.5	19.3
PV	2623.95	2523.95
PMT	−110	−110
FV	0	0
P/Y	12	12
C/Y	365	365
Total cost : N×110	\$3117	\$3181

Hwk : Exploring Credit Card Use (2.2 p 100)

⇒ New Credit Card.

2. Other types of Credit:

Example 1: buying a home

Felix is buying a \$175000 apartment. He is paying \$25000 down payment and then borrows the rest over 25 years at a 6.75% interest rate compounded semi-annually (which is the law in Canada when buying real estate property).

→ Down payment: $0.25 \times 175000 = \$43750$

Remaining balance: $175000 - 43750 = 131250$

Using the TVM solver:
N= 25×12
I%= 6.75
PV= 131250
PMT= ?
FV= 0
P/Y= 12
C/Y= 2

We get : \$899.13 per month.
Total cost of the house : $43750 + 899.13 \times 25 \times 12 = \$313\ 489$

Interests: $313489 - 175000 = \$138489$ ← 154 payments

The interest represent 13 years of payment.

Line of credit: A pre-approved loan that offers immediate access to funds, up to a pre-defined limit, with a minimum monthly payment based on accumulated interest, a secure line of credit has a lower interest rate because it is guaranteed against the client's assets, usually property.

Example 2:

Ed wants to buy a car and needs to use credit to finance it. The cost, with taxes and shipping, is \$24738. Ed wants to repay his loan in 4 years using monthly payments and has two credit options:

- His secured line of credit at 1.7% compounded monthly, above the Bank of Canada rate (we'll use 0.5% here).
- The dealership's financing plan at 2.5% compounded daily.

Line of Credit
N = 4×12
I = $1.7 + 0.5$
PV = 24738
PMT = ? → \$538.88
FV = 0
P/Y = 12
C/Y = 12

Dealership's plan
N = 4×12
I = 2.5
PV = 24738
PMT = ? → \$542.14
FV = 0
P/Y = 12
C/Y = 365

Example 3:

Nicki wants to be debt-free in 5 years. She has two credit cards on which she makes monthly payments.

- Card A has a balance of \$2 436.98 and an interest rate of 18,5% compounded daily.
- Card B has a balance of \$3 043.26 and an interest rate of 19%, compounded daily.

Nicki has qualified for a line of credit at her bank with an interest rate of 9.6% compounded monthly, and a credit limit of \$6 000. She plans to pay off noth credit card balances by borrowing the money from her line of credit. How much interest will she save?

Present Value: 2436.98 + 3043.26 = \$5480.24

Card A	Card B	Line of Credit
N = 5×12	N = 5×12	N = 5×12
I = 18.5	I = 19	I = 9.6
PV = 2436.98	PV = 3043.26	PV = 5480.24
PMT = ? → \$62.73	PMT = ? → \$79.19	PMT = ? → \$115.36 / month
FV = 0	FV = 0	FV = 0
P/Y = 12	P/Y = 12	P/Y = 12
C/Y = 365	C/Y = 365	C/Y = 12

Total cost: (62.73 + 79.19) × 12 × 5 = \$8515.20 vs. 115.36 × 5 × 12 = \$6921.60

Interest saved: \$1593.60

Example 4:

Freda signed up for a special credit offer when she bought her living-room furniture. There were no payments and no interest for 12 months, as long as she paid the balance of \$2 643.65 in full by the end of the first year. Otherwise, a penalty equal to an interest rate of 19.95% compounded monthly, on the full balance would be charged, starting from when she first borrowed the money.

a) If Freda missed the deadline by one day, what would she have to pay? What would the penalty be?

$$A = 2643.65\left(1+\frac{19.95}{12}\right)^{13}$$

$$= \$3275.62$$

penalty: 3275.62 − 2643.65
= \$631.97

b) Suppose that she made monthly payments of \$150 during the first year. What would her 12th and last payment need to be to avoid an interest penalty?

150 × 11 = 1650

2643.65 − 1650 = \$993.65

- Credit cards have a credit limit, which is the maximum amount you can borrow. The credit limit varies from person to person, based on credit history.

- A line of credit has a lower interest rate than most loans and credit cards. Because of this, a line of credit can be useful for consolidating debt.

Hwk : Solving Problems involving credit (2.3 p 114) # 1 – 5.

3. Buy, Rent or Lease?

Lease: A contract for purchasing the use of property, such as a building or vehicle, from another, the lessor, for a specified period.

Equity: The difference between the value of an item and the amount still owing on it; can be thought of as the portion owned.

Asset: An item or a portion of an item owned; also known as property. Assets include items as real estate, investment portfolios, vehicles, art, gems…

- Since each situation is unique, it is impossible to generalize about whether renting, leasing, or buying is best. A cost and benefit analysis should take everything into account.
  - Costs include initial costs and fees, short-term costs, long-term costs, disposable income, the cost of financing, depreciation and appreciation, penalties for breaking contracts, and equity.
  - Benefits include convenience, commitments, flexibility, and personal needs or wants, such as how often you want to buy a car.

- Appreciation and depreciation affect the value of a piece of property and should be considered when making decisions about renting, buying or leasing, based on the particular situation.

Hwk (optional) Buy Rent or Lease? (2.4 p 129)