

Precalc 11 – Practice Final Exam

CALCULATOR ALLOWED

1. Solve the following equations.

Give exact values when possible. Otherwise, round to the nearest hundredth.

$$\begin{aligned} \text{a) } 3x^2 + 5x &= 3 \\ 3x^2 + 5x - 3 &= 0 \\ \Delta &= 25 - 4(3)(-3) = 61 \end{aligned}$$

$$x = \frac{-5 \pm \sqrt{61}}{6}$$

$$\begin{aligned} \text{b) } 2(x+4)^2 &= 3 \\ (x+4)^2 &= \frac{3}{2} \\ x+4 &= \pm \sqrt{\frac{3}{2}} \end{aligned}$$

$$x = -4 \pm \sqrt{\frac{3}{2}}$$

$$\begin{aligned} \text{c) } 2x^2 + 3x - 4 &= 0 \\ \Delta &= 9 - 4(2)(-4) = 41 \end{aligned}$$

$$x = \frac{-3 \pm \sqrt{41}}{4}$$

$$\begin{aligned} \text{d) } \sin x &= -\frac{1}{2} \\ \begin{array}{c} \text{X} \quad \text{X} \\ \uparrow \\ \text{X} \end{array} \quad \theta_R = 30^\circ \\ \Rightarrow \quad \boxed{x = 210^\circ \text{ or } 330^\circ} \end{aligned}$$

$$\begin{aligned} \text{a') } 6x^2 + 11x + 3 &= 0 \\ \left. \begin{array}{l} \otimes 18 \\ \oplus 11 \end{array} \right\} 9 \times 2 \\ 6x^2 + 9x + 2x + 3 &= 0 \\ 3x(2x+3) + 1(2x+3) &= 0 \\ (3x+1)(2x+3) &= 0 \\ \boxed{x = -\frac{1}{3}} \quad \boxed{x = -\frac{3}{2}} \end{aligned}$$

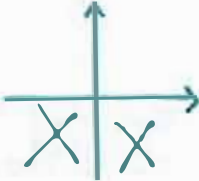
$$\begin{aligned} \text{b') } -3(x+2)^2 + 12 &= 0 \\ -3(x+2)^2 &= -12 \\ (x+2)^2 &= 4 \\ x+2 &= \pm 2 \\ x &= -2 \pm 2 \\ \boxed{x = -4} \quad \boxed{x = 0} \end{aligned}$$

$$\begin{aligned} \text{c') } -2x^2 - 4x + 1 &= 0 \\ \Delta &= 16 - 4(-2)(1) = 24 \\ x &= \frac{4 \pm \sqrt{24}}{-4} \quad \boxed{x = \frac{-2 \pm \sqrt{6}}{2}} \end{aligned}$$

$$\begin{aligned} \text{d') } \cos x &= -\frac{\sqrt{3}}{2} \\ \begin{array}{c} \text{X} \\ \text{X} \end{array} \quad \theta_R = 30^\circ \\ \Rightarrow \quad \boxed{x = 150^\circ \text{ or } 210^\circ} \end{aligned}$$

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e) $\sin \theta = \frac{1}{4}$



$\theta_R = \sin^{-1}\left(\frac{1}{4}\right)$
 $\approx 14.5^\circ$

$\Rightarrow \theta = 14.5^\circ \text{ or } 165.5^\circ$


e') $\cos x = -0.2$



$\theta_R = \cos^{-1}(0.2)$
 $\approx 78.5^\circ$

$\Rightarrow x \approx 101.5^\circ \text{ or } 258.5^\circ$


f) $\tan x = 1$



$\theta_R = 45^\circ$

$\Rightarrow x \approx 45^\circ \text{ or } 225^\circ$

f') $\tan x = -\frac{1}{\sqrt{3}}$



$\theta_R = 30^\circ$

$\Rightarrow x = 150^\circ \text{ or } 330^\circ$

g) $-3(x+2) > 4x-1$

$$-3x-6 > 4x-1$$

$$-7x > 5$$

$x < -\frac{5}{7}$

g') $(x-3)(x+2) \leq (x+1)^2$

$$x^2+2x-3x-6 \leq x^2+2x+1$$

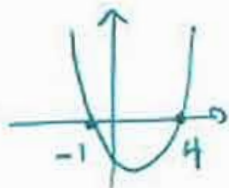
$$-3x \leq 7$$

$x \geq -\frac{7}{3}$

h) $x^2 - 3x \geq 4$

$$x^2 - 3x - 4 \geq 0$$

$$(x-4)(x+1) \geq 0$$



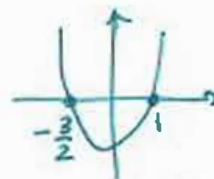
$x \leq -1 \text{ or } x \geq 4$

h') $2x^2 + x - 3 < 0$

$$2x^2 + 3x - 2x - 3 < 0$$

$$x(2x+3) - 1(2x+3) < 0$$

$$(x-1)(2x+3) < 0$$



$-\frac{3}{2} < x < 1$

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i) $x^2 < 16$

$$x^2 - 16 < 0$$

$$(x+4)(x-4) < 0$$



$$-4 < x < 4$$

ii) $x^2 \geq 9$

$$x^2 - 9 \geq 0$$



$$x \leq -3 \text{ or } x \geq 3$$

j) $x + \sqrt{x+2} = 4$

• restrictions: $x \geq -2$

• $\sqrt{x+2} = 4 - x$

$$x+2 = x^2 - 8x + 16$$

$$x^2 - 9x + 14 = 0$$

$$(x-7)(x-2) = 0$$

$$x = 7 \text{ or } x = 2$$

TESTS:

$$\begin{array}{r|l} x + \sqrt{x+2} = 4 & \\ \hline 7 + \sqrt{9} & 4 \\ 7 + 3 & \\ 10 & \times \end{array}$$

$$\begin{array}{r|l} x + \sqrt{x+2} = 4 & \\ \hline 2 + \sqrt{4} & 4 \\ 2 + 2 & \\ 4 & \checkmark \end{array}$$

$$\Rightarrow \boxed{x = 2}$$

j) $\sqrt{-2x+30} + 3 = x$

• restrictions $-2x+3 \geq 0$
 $-2x \geq -3$
 $x \leq \frac{3}{2}$

• restrictions

$$\sqrt{-2x+30} = x - 3$$

$$-2x+30 = x^2 - 6x + 9$$

$$x^2 - 4x - 21 = 0$$

$$(x-7)(x+3) = 0$$

$$x = 7 \text{ or } x = -3$$

TESTS

$$\begin{array}{r|l} \sqrt{-2x+30} + 3 = x & \\ \hline \sqrt{-14+30} + 3 & 7 \\ \sqrt{16} + 3 & \\ 4 + 3 & \\ 7 & \checkmark \end{array}$$

$$\begin{array}{r|l} \sqrt{-2x+30} + 3 = x & \\ \hline \sqrt{6+30} + 3 & -3 \\ \sqrt{36} + 3 & \\ 6 + 3 & \\ 9 & \times \end{array}$$

$$\Rightarrow \boxed{x = 7}$$

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$$k) \frac{4}{x^2-1} - \frac{x-1}{x+1} = \frac{x+7}{x-1}$$

• restrictions: $x \neq \pm 1$

$$\bullet \frac{4}{(x+1)(x-1)} - \frac{(x-1)^2}{(x+1)(x-1)} = \frac{(x+7)(x+1)}{(x+1)(x-1)}$$

$$4 - (x^2 - 2x + 1) = x^2 + 8x + 7$$

$$4 - x^2 + 2x - 1 = x^2 + 8x + 7$$

$$2x^2 + 6x + 4 = 0$$

$$2x^2 + 4x + 2x + 4 = 0$$

$$2x(x+2) + 2(x+2) = 0$$

$$k') 2 - \frac{1}{x^2+x} = \frac{3}{x+1}$$

• restrictions: $x \neq -1$ $x \neq 0$

$$\bullet \frac{2x(x+1)}{x(x+1)} - \frac{1}{x(x+1)} = \frac{3x}{x(x+1)}$$

$$2x^2 + 2x - 1 = 3x$$

$$2x^2 - x - 1 = 0$$

$$2x^2 - 2x + x - 1 = 0$$

$$2x(x-1) + 1(x-1) = 0$$

$$k'') \frac{x}{x-5} + \frac{3}{x+2} = \frac{7x}{x^2-3x-10}$$

• restrictions: $x \neq -2$ $x \neq 5$

$$\bullet \frac{x(x+2)}{(x-5)(x+2)} + \frac{3(x-5)}{(x+2)(x-5)} = \frac{7x}{(x-5)(x+2)}$$

$$x^2 + 2x + 3x - 15 = 7x$$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

$$x \neq 5 \quad \text{or} \quad \boxed{x = -3}$$

Restr x

$$(2x+2)(x+2) = 0$$

$$2x+2=0$$

$$x = -1$$

Restr x

$$x+2=0$$

$$\boxed{x = -2}$$

$$(2x+1)(x-1) = 0$$

$$\boxed{x = -\frac{1}{2} \quad \text{or} \quad x = 1}$$

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2. A flea jumps from a stone. Its height above ground, in centimetres, is given by $h(x) = -x^2 + 3x + 10$, where x is the horizontal distance from the stone, in centimetres.

a) From what height did the flea jump?

$$\boxed{10 \text{ cm}} \quad (\text{y-intercept})$$

b) How far from the stone does the flea land on the ground?

$$-x^2 + 3x + 10 = 0$$

$$-x^2 + 5x - 2x + 10 = 0$$

$$-x(x-5) - 2(x-5) = 0$$

$$(-x-2)(x-5) = 0$$

$$x = -2 \text{ or } x = 5$$

not on the domain

$$\Rightarrow \boxed{5 \text{ cm}}$$



c) What maximum height did the flea reach?

$$\text{vertex: } -\frac{b}{2a} = -\frac{3}{-2} = \frac{3}{2}$$

$$h\left(\frac{3}{2}\right) = -\left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right) + 10 = 12.25$$

$$\Rightarrow \boxed{12.25 \text{ cm}}$$

d) What are the Domain and Range for the function?

$$D = [0, 5]$$

$$R = [0, 12.25]$$

3. A football player throws a ball. The height, in metres, of the ball is modeled by the function $h(t) = -\frac{5}{3}t^2 + \frac{10}{3}t + 5$, where t is the time in seconds after the throw.

a) From what height did the player throw the ball?

$$\boxed{5 \text{ m}}$$

b) For how long will the ball be in the air if nobody catches it?

$$-\frac{5}{3}t^2 + \frac{10}{3}t + 5 = 0$$

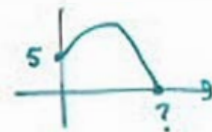
$$t^2 - 2t - 3 = 0$$

$$(t-3)(t+1) = 0$$

$$t = 3 \text{ or } t = -1$$

out of the domain

$$\Rightarrow \boxed{3 \text{ s}}$$



c) What maximum height did the ball reach?

$$-\frac{b}{2a} = \frac{-\frac{10}{3}}{-\frac{10}{3}} = 1$$

$$h(1) = -\frac{5}{3} + \frac{10}{3} + 5$$

$$= 6.\bar{6} \text{ or } \frac{20}{3}$$

$$\Rightarrow \boxed{6.67 \text{ m}}$$

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4. Simplify the following expressions:

$$\begin{aligned} \text{a) } \sqrt{75x^3} + 2x\sqrt{12x} &= \sqrt{25x^2 \cdot 3x} + 2x\sqrt{4 \cdot 3x} \\ &= 5x\sqrt{3x} + 4x\sqrt{3x} \\ &= 9x\sqrt{3x} \end{aligned}$$

$$\begin{aligned} \text{a) } \frac{3x\sqrt{14} + 4\sqrt{63x^2}}{3\sqrt{7}} &= \frac{3x\sqrt{14}}{3\sqrt{7}} + \frac{4\sqrt{9x^2 \cdot 7}}{3\sqrt{7}} \\ &= x\sqrt{2} + 4x \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{(2x^2y^3)^{-3}}{4x^{-2}y^5} &= \frac{2^{-3}x^{-6}y^{-9}}{4x^{-2}y^5} \\ &= \frac{1}{32x^4y^{14}} \end{aligned}$$

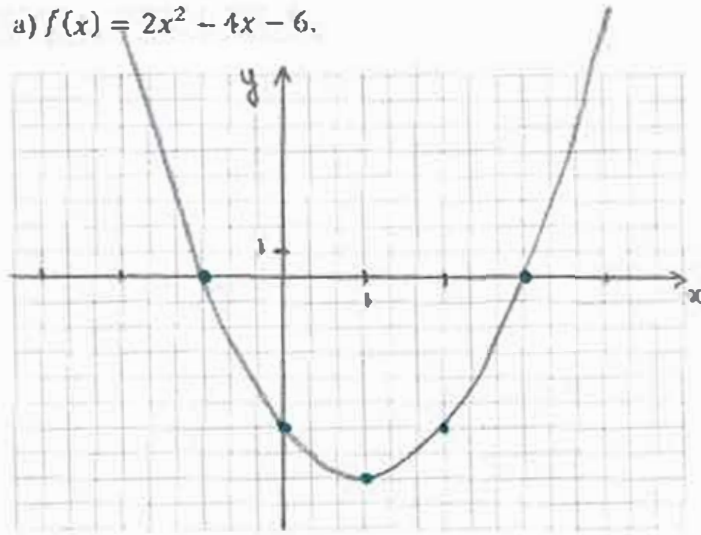
$$\begin{aligned} \text{b) } \frac{(3x^{-3}y^2)^{-3}x^2y^3}{3x^{-2}y} &= \frac{3^{-1}x^3y^{-2}x^2y^3}{3x^{-2}y} \\ &= \frac{x^5}{9} \end{aligned}$$

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CALCULATORS NOT ALLOWED

5. Graph the following functions. Show your work. .

a) $f(x) = 2x^2 - 4x - 6$.

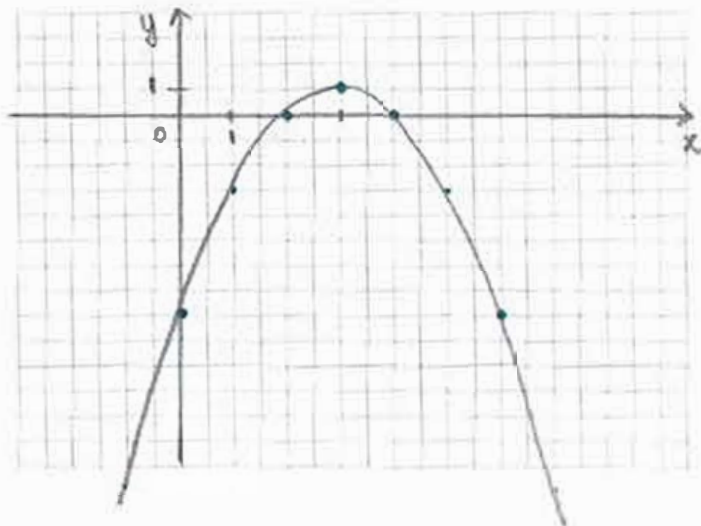


vertex: $(1, -8)$

y-int: -6

zeros: $-1 \text{ \& } 3$

a) $y = -x^2 + 6x - 8$.



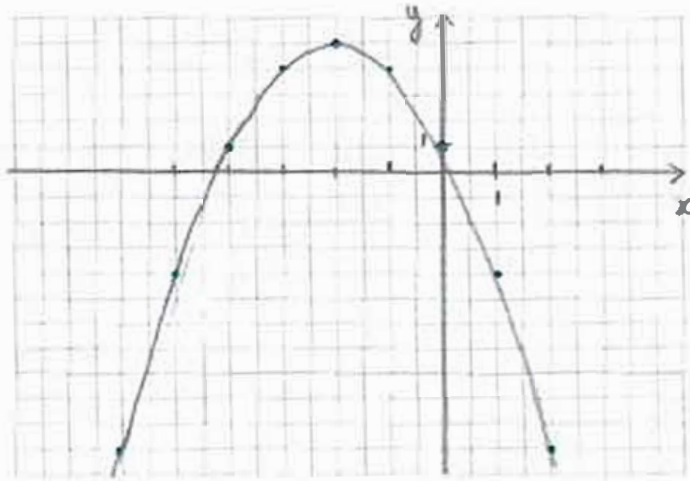
vertex: $(3, 1)$

y-int: -8

zeros: $2 \text{ \& } 4$

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b) $g(x) = -(x + 2)^2 + 5$.

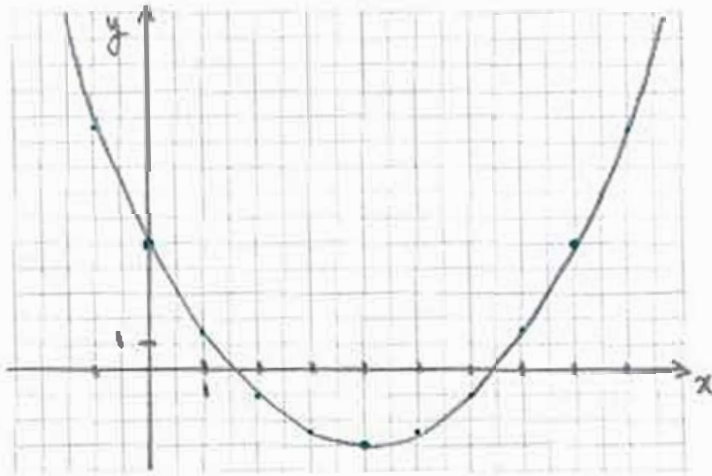


vertex : $(-2, 5)$

y-int: 1

zeros : $-2 \pm \sqrt{5}$ (not me)

b) $y = \frac{1}{2}(x - 4)^2 - 3$.



vertex : $(4, -3)$

y-int: 5

zeros : $4 \pm \sqrt{6}$

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6. Expand the following expressions and simplify:

a) $(3x - 5)(3x + 5) - (2x - 1)(3x + 2)$

$$= 9x^2 - 25 - (6x^2 + 4x - 3x - 2)$$

$$= 9x^2 - 25 - 6x^2 - x + 2$$

$$= 3x^2 - x - 23$$

b) $(4x - 3)^2 - (2x + 1)(x - 3)$

$$= 16x^2 - 24x + 9 - (2x^2 - 6x + x - 3)$$

$$= 16x^2 - 24x + 9 - 2x^2 + 5x + 3$$

$$= 14x^2 - 19x + 12$$

c) $(x + 3)(2x - 5) - (3x - 1)^2$

$$= 2x^2 - 5x + 6x - 15 - (9x^2 - 6x + 1)$$

$$= 2x^2 + x - 15 - 9x^2 + 6x - 1$$

$$= -7x^2 + 7x - 16$$

d) $(x + 5)(3x - 2) - (2x + 1)(2x - 1)$

$$= 3x^2 - 2x + 15x - 10 - (4x^2 - 1)$$

$$= 3x^2 + 13x - 10 - 4x^2 + 1$$

$$= -x^2 + 13x - 9$$

7. Factor as much as possible.

a) $x^2 - 4x - 21$

$$= (x - 7)(x + 3)$$

a') $x^2 + 7x + 10$

$$= (x + 2)(x + 5)$$

b) $2x^2 + 5x - 12$

$$= (x + 4)(2x - 3)$$

b') $3x^2 + x - 4$

$$= (x - 1)(3x + 4)$$

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$$c) 16x^2 - 72x + 81$$

$$= (4x - 9)^2$$

$$c') 25x^2 - 20x + 4$$

$$= (5x - 2)^2$$

$$d) 25x^2 - 36$$

$$= (5x + 6)(5x - 6)$$

$$d') 16x^2 - 9$$

$$= (4x + 3)(4x - 3)$$

$$e) 25x^3 - 15x^2$$

$$= 5x^2(5x - 3)$$

$$e') 27x^2 - 18x$$

$$= 9x(3x - 2)$$

$$f) 5x^2 - 20$$

$$= 5(x^2 - 4)$$

$$= 5(x + 2)(x - 2)$$

$$f') 7x^2 - 7$$

$$= 7(x^2 - 1)$$

$$= 7(x + 1)(x - 1)$$

$$g) 5x^2 + 20$$

$$= 5(x^2 + 4)$$

$$g') 8x^2 - 32$$

$$= 8(x^2 - 4)$$

$$= 8(x + 2)(x - 2)$$