

Worksheet - FACTORISATION

Définition:

1. Détermine si les expressions suivantes sont des sommes ou des produits, et fais la liste des termes ou facteurs selon le cas.

a) $5x^3 - 3x^2 + 1$ somme 3 termes: $5x^3$, $-3x^2$ et 1

b) $(x - 3)(x + 2)$ produit 2 facteurs: $x - 3$ et $x + 2$

c) $(x - 5)(2x + 1)^2$ produit 3 facteurs: $x - 5$, $2x + 1$ et $2x + 1$

d) $3x(x + 5) - 6$ somme 2 termes: $3x(x + 5)$ et -6

e) $(x - 3)^2 + 5(x - 3)$ somme 2 termes: $(x - 3)^2$ et $5(x - 3)$

2. Développe les expressions suivantes :

a) $5x(x - 3) = 5x^2 - 15x$

b) $(x + 2)(3x - 5) = 3x^2 - 5x + 6x - 10 = 3x^2 + x - 10$

c) $(3x - 2)^2 = 9x^2 - 12x + 4$

d) $(x + 1)^2 - 5(x - 3) = x^2 + 2x + 1 - 5x + 15 = x^2 - 3x + 16$

e) $(x - 1)(x + 2) - (x - 3)(x + 4) = x^2 + 2x - x - 2 - (x^2 + 4x - 3x - 12)$
 $= x^2 + x - 2 - x^2 - x + 12$
 $= 10$

f) $(x + 2)(2x - 3)(3x + 4) = (x + 2)(6x^2 + 8x - 9x - 12)$
 $= (x + 2)(6x^2 - x - 12)$
 $= 6x^3 - x^2 - 12x + 12x^2 - 2x - 24$
 $= 6x^3 + 11x^2 - 14x - 24$

La factorisation par facteur commun:

3. Factorise les expressions suivantes :

a) $5x^2 - 25x = 5x(x-5)$

b) $21x^2 - 14x + 28 = 7(3x^2 - 2x + 4)$

c) $(x-3)(2x+1) - 3(2x+1) = (2x+1)(x-3-3) = (2x+1)(x-6)$

d) $(x-5)^2 - (x-5)(2x+1) = (x-5)(x-5-(2x+1))$
 $= (x-5)(x-5-2x-1) = (x-5)(-x-6)$
 $= -(x-5)(x+6)$

La factorisation de trinômes de la forme $ax^2 + bx + c$:

4. Factorise les expressions suivantes :

a) $x^2 - 2x - 3 = (x-3)(x+1)$

⊗ -3

⊕ -2

b) $3x^2 + 5x - 2 = 3x^2 + 6x - x - 2 = 3x(x+2) - 1(x+2) = (3x-1)(x+2)$

⊗ -6

⊕ 5

c) $x^2 + 5x + 4 = (x+4)(x+1)$

⊗ 4

⊕ 5

d) $2x^2 + 7x - 15 = 2x^2 + 10x - 3x - 15 = 2x(x+5) - 3(x+5) = (2x-3)(x+5)$

⊗ -30

⊕ 7

e) $x^2 - 5x - 6 = (x-6)(x+1)$

⊗ -6

⊕ -5

f) $6x^2 + x - 2 = 6x^2 + 4x - 3x - 2 = 2x(3x+2) - 1(3x+2) = (2x-1)(3x+2)$

⊗ -12

⊕ 1

g) $5x^2 + 5x - 60 = 5(x^2 + x - 12) = 5(x+4)(x-3)$

⊗ -12

⊕ 1

La factorisation des trinômes particuliers :

5. Factorise les expressions suivantes :

a) $x^2 - 16 = (x+4)(x-4)$

b) $25x^2 - 1 = (5x+1)(5x-1)$

c) $x^2 - 6x + 9 = (x-3)^2$

d) $4x^2 + 4x + 1 = (2x+1)^2$

e) $2x^2 - 72 = 2(x^2 - 36) = 2(x+6)(x-6)$

f) $3x^2 - 6x + 3 = 3(x^2 - 2x + 1) = 3(x-1)^2$

g) $\frac{1}{4}x^2 + \frac{1}{2}x - 6 = \frac{1}{4}(x^2 + 2x - 24) = \frac{1}{4}(x+6)(x-4)$

h) $0.1x^2 - 0.1x - 3 = 0.1(x^2 - x - 30) = 0.1(x-6)(x+5)$

i) $\frac{1}{25}x^2 - \frac{1}{49}y^2 = \left(\frac{1}{5}x + \frac{1}{7}y\right)\left(\frac{1}{5}x - \frac{1}{7}y\right)$

j) $0.81x^2 - 0.25 = (0.9x + 0.5)(0.9x - 0.5)$

Autres techniques de factorisation :

6. Factorise les expressions suivantes :

$$\begin{aligned}
 \text{a) } 4(x+3)^2 + 8(x+3) - 5 &= 4t^2 + 8t - 5 = 4t^2 + 10t - 2t - 5 = 2t(2t+5) - 1(2t+5) \\
 \text{soit } t &= x+3 & \begin{matrix} \otimes -20 \\ \oplus 8 \end{matrix} & = (2t-1)(2t+5) = (2x+6-1)(2x+6+5) \\
 & & & = (2x+5)(2x+11)
 \end{aligned}$$

b) $9(x-1)^2 - 100(x+1)^2$

$$\begin{aligned}
 \text{b) } 9(x-1)^2 - 100(x+1)^2 &= (3(x-1) + 10(x+1))(3(x-1) - 10(x+1)) \\
 &= (3x - 3 + 10x + 10)(3x - 3 - 10x - 10) \\
 &= (13x + 7)(-7x - 13) \\
 &= -(13x + 7)(7x + 13)
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } (2x-1)^2 + 16(2x-1) + 63 &= t^2 + 16t + 63 = (t+7)(t+9) \\
 \text{soit } t &= 2x-1 & &= (2x-1+7)(2x-1+9) \\
 & & &= (2x+6)(2x+8) = 4(x+3)(x+4)
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } 25(x^2-9)^2 - 16(x-3)^2 &= 25(x+3)^2(x-3)^2 - 16(x-3)^2 \\
 &= (x-3)^2(25(x+3)^2 - 16) \\
 &= (x-3)^2(5(x+3)+4)(5(x+3)-4) \\
 &= (x-3)^2(5x+19)(5x+11)
 \end{aligned}$$

Hwk: p 229 # 5, 6

Pot pourri :

7. Factorise les expressions suivantes :

$$\text{a) } 14x^3y^2 - 21z^3x^2 = 7x^2(2xy^2 - 3z^3)$$

$$\text{b) } x^2 + 8x + 12 = (x+2)(x+6)$$

$$\begin{aligned}
 \text{c) } 2x^2 + 5x + 2 &= 2x^2 + 4x + x + 2 \\
 &= 2x(x+2) + 1(x+2) \\
 &= (2x+1)(x+2)
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } 3x^2 + 24x + 45 &= 3(x^2 + 8x + 15) \\
 &= 3(x+5)(x+3)
 \end{aligned}$$

$$\begin{aligned}
 \text{e) } 8x^2 - 14x + 6 &= 2(4x^2 - 7x + 3) & &= 2(4x-3)(x-1) \\
 &= 2(4x^2 - 4x - 3x + 3) \\
 &= 2(4x(x-1) - 3(x-1))
 \end{aligned}$$

$$f) 81 - 4x^2 = (9+2x)(9-2x)$$

$$g) x^2 + 10x + 25 = (x+5)^2$$

$$h) \frac{9x^2}{16} - \frac{100y^2}{81} = \left(\frac{3}{4}x + \frac{10}{9}y\right)\left(\frac{3}{4}x - \frac{10}{9}y\right)$$

$$i) (x-6)^2 + 10(x-6) + 9 = t^2 + 10t + 9 = (t+9)(t+1)$$

$\text{set } t = x-6$

$$= (x-6+9)(x-6+1)$$

$$= (x+3)(x-5)$$

$$j) -8x^3 + 20x^2 - 4x = -4x(2x^2 - 5x + 1)$$

$$k) x^2 + 7x + 10 = (x+5)(x+2)$$

$$l) 7x^2 + 8x + 1 = 7x^2 + 7x + x + 1$$

$$= 7x(x+1) + 1(x+1)$$

$$= (7x+1)(x+1)$$

$$m) 5x^2 - 15x - 20 = 5(x^2 - 3x - 4) = 5(x-4)(x+1)$$

$$\begin{aligned}
 \text{n) } -18x^2 + 39x - 15 &= -3(6x^2 - 13x + 5) = -3(6x^2 - 10x - 3x + 5) \\
 &= -3(2x(3x - 5) - 1(3x - 5)) \\
 &= -3(2x - 1)(3x - 5)
 \end{aligned}$$

$$\begin{aligned}
 \text{o) } 18x^2 - 2y^2 &= 2(9x^2 - y^2) \\
 &= 2(3x + y)(3x - y)
 \end{aligned}$$

$$\text{p) } 16 - 40x + 25x^2 = (5x - 4)^2$$

$$\begin{aligned}
 \text{q) } (x + 5)^2 - 49(x - 9)^2 &= (x + 5 + 7(x - 9))(x + 5 - 7(x - 9)) \\
 &= (x + 5 + 7x - 63)(x + 5 - 7x + 63) \\
 &= (8x - 58)(-6x + 68) \\
 &= -4(4x - 29)(3x - 34)
 \end{aligned}$$

$$\begin{aligned}
 \text{r) } 4(5x - 1)^2 - 12(5x - 1) + 5 &= 4t^2 - 12t + 5 \\
 \text{Seit } t = 5x - 1 &= 4t^2 - 10t - 2t + 5 \\
 &= 2t(2t - 5) - 1(2t - 5) \\
 &= (2t - 1)(2t - 5) \\
 &= (10x - 2 - 1)(10x - 2 - 5) \\
 &= (10x - 3)(10x - 7)
 \end{aligned}$$