Midterm Review Solutions

PC12

Answers

Chapter 1

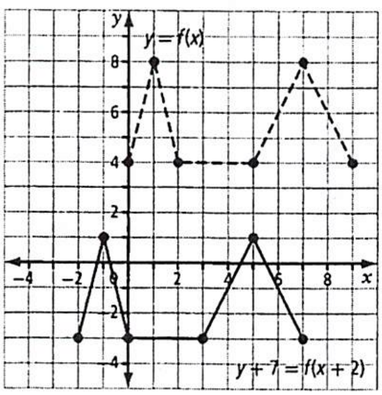
1.1 Horizontal and Vertical Translations, pages 1–8

1. **a)** $h = 10, k = 0$ **b)** $h = -2, k = 3$
c) $h = 17, k = 13$ **d)** $h = -1, k = -7$
e) $h = 0, k = 4$

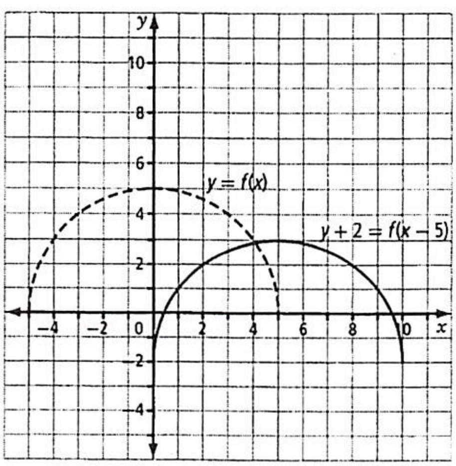
2. **a)** $y + 5 = (x - 2)^2$ **b)** $y + 5 = |x - 2|$
c) $y + 5 = \frac{1}{x - 2}, x \neq 2$

3. **a)** $(x, y) \rightarrow (x + 25, y)$; horizontal translation 25 units to the right
b) $(x, y) \rightarrow (x, y - 50)$; vertical translation 50 units down
c) $(x, y) \rightarrow (x - 20, y + 10)$; horizontal translation 20 units to the left and vertical translation 10 units up

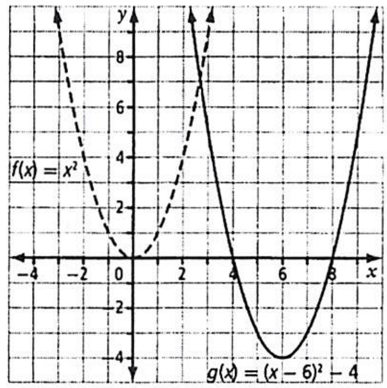
4. **a)** $(x, y) \rightarrow (x - 2, y - 7)$



b) $(x, y) \rightarrow (x + 5, y - 2)$

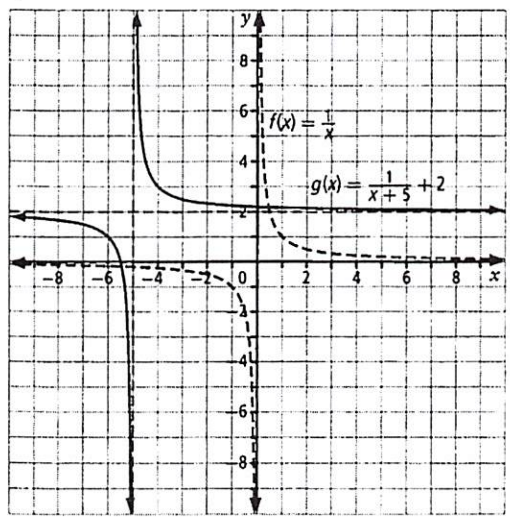


5. **a)** $h = 6, k = -4$ **b)** $(x, y) \rightarrow (x + 6, y - 4)$
c) $y = (x - 6)^2 - 4$
d)



e) $(0, 0)$, $(6, -4)$; vertex has coordinates (h, k)
f) domain of each function: $\{x \mid x \in \mathbb{R}\}$; range of $f(x)$: $\{y \mid y \geq 0, y \in \mathbb{R}\}$, range of $g(x)$: $\{y \mid y \geq -4, y \in \mathbb{R}\}$; in general, the range is $\{y \mid y \geq k, y \in \mathbb{R}\}$

6. **a)** $h = -5, k = 2$ **b)** $(x, y) \rightarrow (x - 5, y + 2)$
c) $y = \frac{1}{x + 5} + 2$
d)



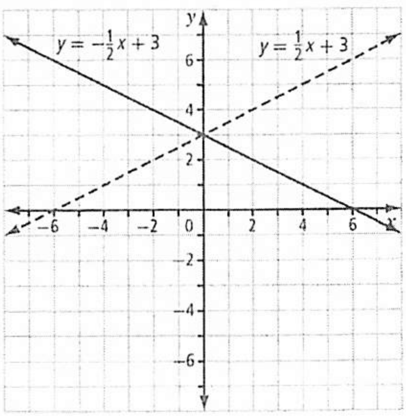
e) For $f(x)$: domain $\{x \mid x \neq 0, x \in \mathbb{R}\}$, range $\{y \mid y \neq 0, y \in \mathbb{R}\}$, asymptotes $y = 0$, $x = 0$; For $g(x)$: domain $\{x \mid x \neq -5, x \in \mathbb{R}\}$, range $\{y \mid y \neq 2, y \in \mathbb{R}\}$, asymptotes $y = 2$, $x = -5$; restriction on the domain of $g(x)$ is $x \neq h$, restriction on the range of $g(x)$ is $y \neq k$, asymptotes are at $x = h$ and $y = k$

7.

Function	Horizontal Translation		Vertical Translation	
	to the right 1 unit	to the left 3 units	up 2 units	down 4 units
Quadratic $y = x^2$	$y = (x - 1)^2$ $(x, y) \rightarrow (x + 1, y)$ vertex at (1, 0)	$y = (x + 3)^2$ $(x, y) \rightarrow (x - 3, y)$ vertex at (−3, 0)	$y - 2 = x^2$ $(x, y) \rightarrow (x, y + 2)$ vertex at (0, 2)	$y + 4 = x^2$ $(x, y) \rightarrow (x, y - 4)$ vertex at (0, −4)
Absolute value $y = \|x\|$	$y = \|x - 1\|$ $(x, y) \rightarrow (x + 1, y)$ vertex at (1, 0)	$y = \|x + 3\|$ $(x, y) \rightarrow (x - 3, y)$ vertex at (−3, 0)	$y - 2 = \|x\|$ $(x, y) \rightarrow (x, y + 2)$ vertex at (0, 2)	$y + 4 = \|x\|$ $(x, y) \rightarrow (x, y - 4)$ vertex at (0, −4)
Reciprocal $y = \frac{1}{x}$	$y = \frac{1}{x - 1}$ $(x, y) \rightarrow (x + 1, y)$ vertical asymptote: $x = 1$; horizontal asymptote: $y = 0$	$y = \frac{1}{x + 3}$ $(x, y) \rightarrow (x - 3, y)$ vertical asymptote: $x = -3$; horizontal asymptote: $y = 0$	$y - 2 = \frac{1}{x}$ $(x, y) \rightarrow (x, y + 2)$ vertical asymptote: $x = 0$; horizontal asymptote: $y = 2$	$y + 4 = \frac{1}{x}$ $(x, y) \rightarrow (x, y - 4)$ vertical asymptote: $x = 0$; horizontal asymptote: $y = -4$
Any function $y = f(x)$	$y = f(x - 1)$ $(x, y) \rightarrow (x + 1, y)$	$y = f(x + 3)$ $(x, y) \rightarrow (x - 3, y)$	$y - 2 = f(x)$ $(x, y) \rightarrow (x, y + 2)$	$y + 4 = f(x)$ $(x, y) \rightarrow (x, y - 4)$

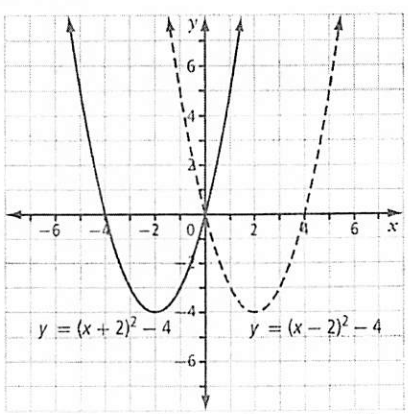
1.2 Reflections and Stretches, pages 9–17

1. a)



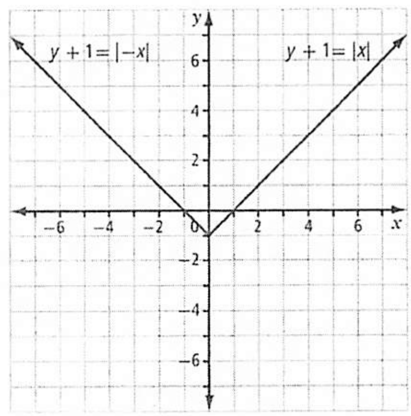
$y = -\frac{1}{2}x + 3$; same y-intercept, different x-intercepts, opposite slopes, same domain and range

b)



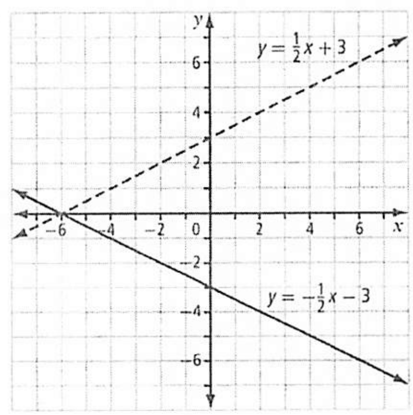
$y + 4 = (x + 2)^2$; same y-intercept, different x-intercepts, same domain and range, same shape, same orientation, vertex has opposite x-coordinate (h) but same y-coordinate (k)

c)

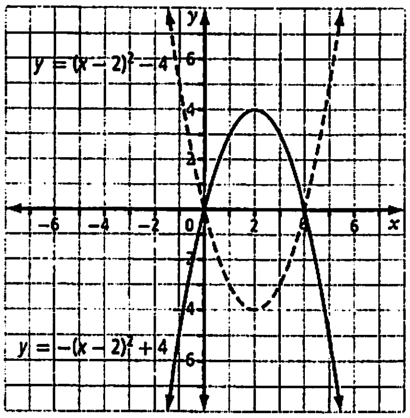


$y + 1 = |-x|$; reflection maps to the original graph

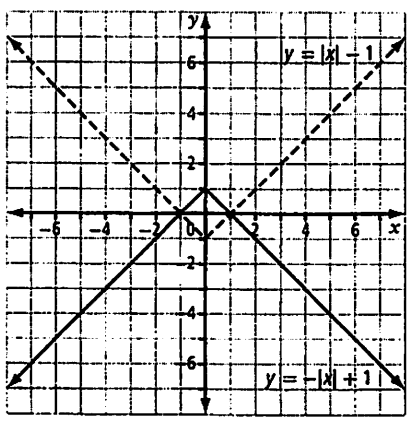
2. a)



$y = -\frac{1}{2}x - 3$; same x-intercept, different y-intercepts, opposite slopes, same domain and range

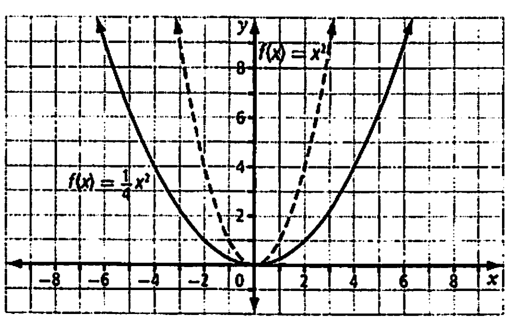
b) 

$y - 4 = -(x - 2)^2$; same y-intercept, same x-intercepts (zeros), different orientation, one has a maximum value and one has a minimum value, same shape, vertex has same x-coordinate (h) and opposite y-coordinate (k)

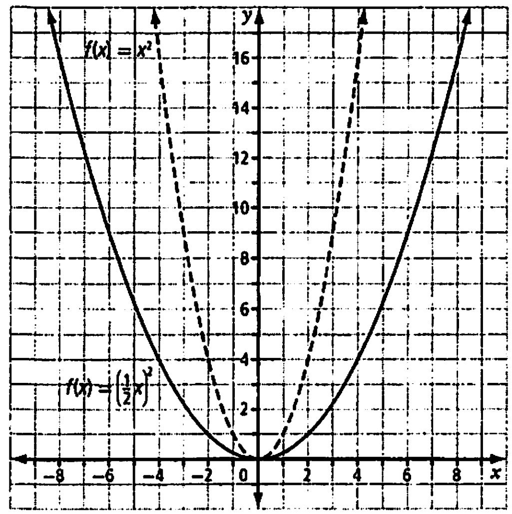
c) 

$y - 1 = -|x|$; same x-intercepts (zeros), different y-intercepts, different orientation, one has a maximum value and one has a minimum value, same shape, vertex has same x-coordinate (h) and opposite y-coordinate (k)

3. a) $(x, y) \rightarrow \left(x, \frac{1}{4}y\right)$; $f(x) = \frac{1}{4}x^2$



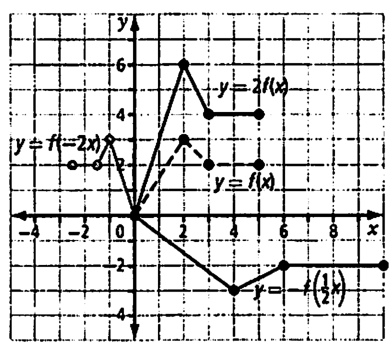
b) $(x, y) \rightarrow (2x, y)$; $f(x) = \left(\frac{1}{2}x\right)^2$



4. a) $\left(\frac{1}{2}x\right)^2 = \left(\frac{1}{2}\right)^2 (x)^2 = \frac{1}{4}x^2$

b) Example: Given $f(x) = x^2$, any horizontal stretch by a factor of p is equivalent to a vertical stretch by a factor of $\frac{1}{p^2}$.

5. a) $y = 2f(x)$ b) $y = -f\left(\frac{1}{2}x\right)$ c) $y = f(-2x)$



6. Answers may vary.

1.3 Combining Transformations, pages 18–25

1. Steps i) and ii) may be reversed and the answer will still be correct.

a) i) reflection in the y-axis, ii) vertical stretch by a factor of 4, iii) translation 5 units down

b) i) horizontal stretch by a factor of $\frac{1}{2}$, ii) reflection in the x-axis, iii) translation 7 units to the left

c) i) horizontal stretch by a factor of 4, ii) vertical stretch by a factor of 1.75, iii) translation 1.5 units to the right

d) i) horizontal stretch by a factor of $\frac{1}{3}$ and reflection in the y-axis, ii) vertical stretch by a factor of $\frac{1}{2}$ and reflection in the x-axis, iii) translation 3 units up and 1 unit to the left

2. **a)** $y + 7 = -f\left(\frac{1}{6}x\right)$

b) $y = \frac{1}{2}|-(x - 3)|$

c) $y + 4 = -\frac{1}{9}(x - 10)^2$ or $y + 4 = -\left[\frac{1}{3}(x - 10)\right]^2$

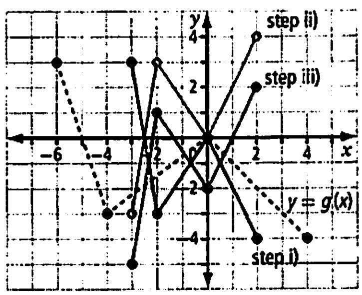
3. **a)** (6, 6)

b) (−11, −10)

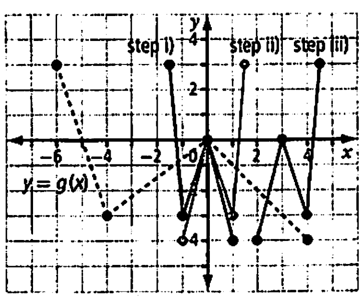
c) (18, 30)

4. (3, −12), (−14, 8), and (24, −24)

5. **a)** i) horizontal stretch by a factor of $\frac{1}{2}$, ii) reflection in the x-axis, iii) translation 2 units down

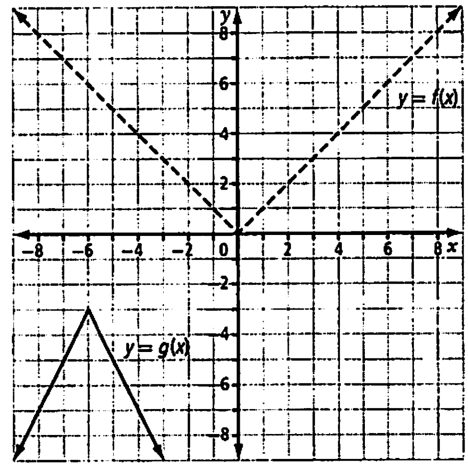


b) i) horizontal stretch by a factor of $\frac{1}{4}$, ii) reflection in the y-axis, iii) translation 3 units to the right



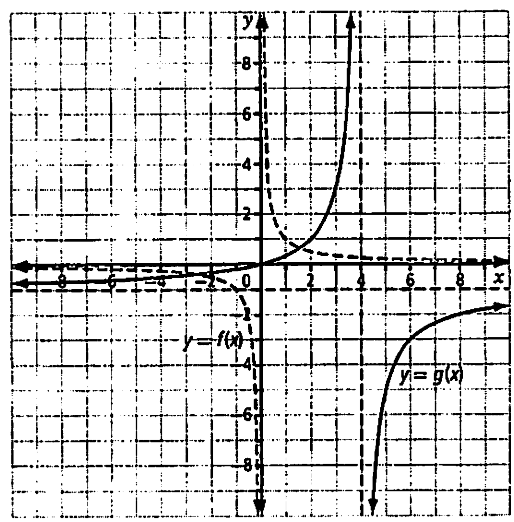
6. **a)** $y = -2|x + 6| - 3$

b)



7. **a)** $y = -\frac{1}{\frac{1}{4}(x - 4)} - 1$ or $y = -\frac{4}{x - 4} - 1$

b)



8. $y - 7 = -2f(x + 5)$

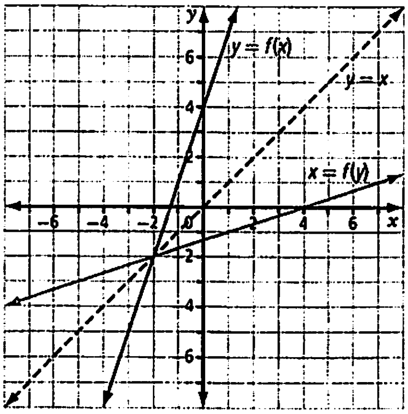
9. $y = 2f\left(-\frac{1}{2}x\right)$

10. $y = f(-2x) + 3$

11. Answers may vary.

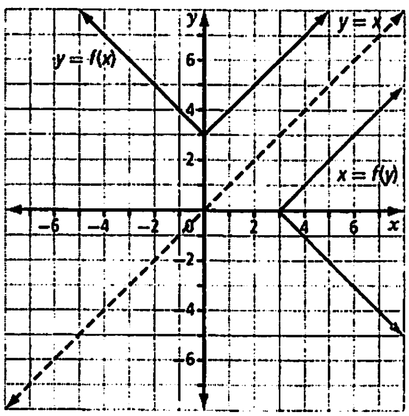
1.4 Inverse of a Relation, pages 26–34

1. a)



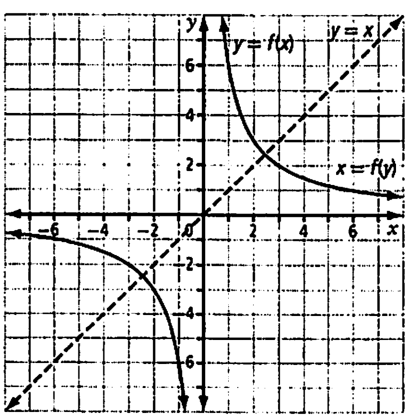
The inverse of $f(x)$ is a function; invariant point at (−2, −2).

b)



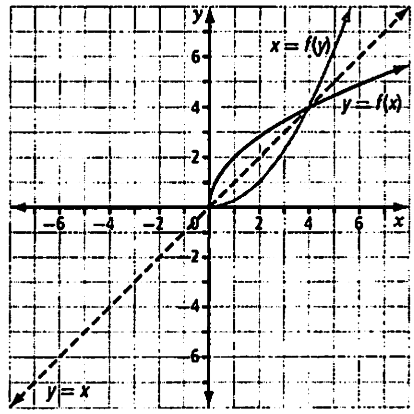
The inverse of $f(x)$ is not a function.

c)



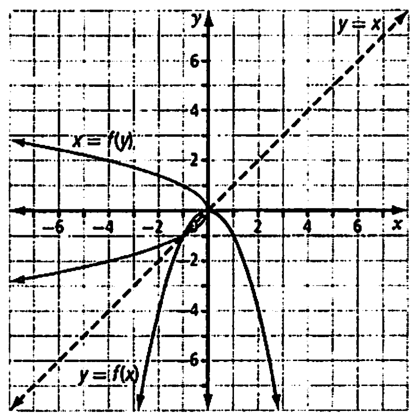
The inverse of $f(x)$ is a function; invariant points at approximately (2.5, 2.5) and (−2.5, −2.5).

d)



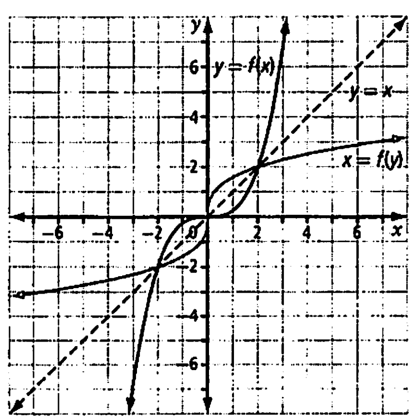
The inverse of $f(x)$ is a function; invariant points at (0, 0) and (4, 4).

e)



The inverse of $f(x)$ is not a function; invariant points at (−1, −1) and (0, 0).

f)

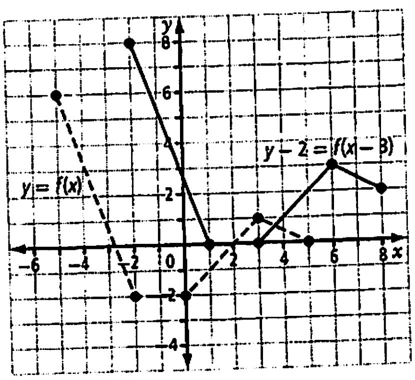


The inverse of $f(x)$ is a function; invariant points at (−2, −2), (0, 0), and (2, 2).

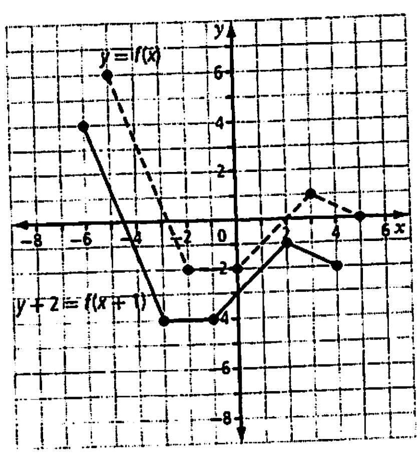
2. a) $f^{-1}(x) = x + 4$ b) $f^{-1}(x) = -\frac{1}{6}x - \frac{1}{3}$
   c) $f^{-1}(x) = \frac{5}{3}x + 5$ d) $f^{-1}(x) = 2x - 6$
3. Examples: a) $\{x \mid x \geq 2, x \in R\}$ or $\{x \mid x \leq 2, x \in R\}$
   b) $\{x \mid x \geq -4, x \in R\}$ or $\{x \mid x \leq -4, x \in R\}$
4. a) For $f(x) = -x^2 + 6, x \geq 0$, the inverse is $f^{-1}(x) = \sqrt{-(x-6)}$. For $f(x) = -x^2 + 6, x \leq 0$, the inverse is $f^{-1}(x) = -\sqrt{-(x-6)}$.
   b) For $f(x) = \frac{1}{2}x^2 + 4, x \geq 0$, the inverse is $f^{-1}(x) = \sqrt{2(x-4)}$. For $f(x) = \frac{1}{2}x^2 + 4, x \leq 0$, the inverse is $f^{-1}(x) = -\sqrt{2(x-4)}$.
5. $y = \pm\sqrt{x+2} - 3$
6. a) $42 < x < 105$
   b) $f^{-1}(x) = \sqrt{\frac{x}{0.01634}} + 26.643$, where x = CRL, in millimetres
   c) 14.3 weeks
7. Answers may vary.

Chapter 1 Review, pages 35–37

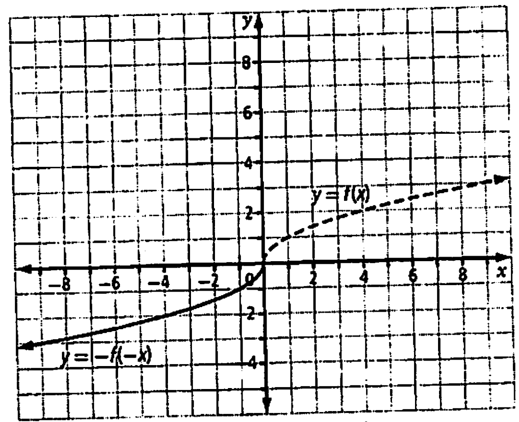
1. a) $y + 3 = |x - 5|$ b) $y - 1 = |x + 4|$
2. a)



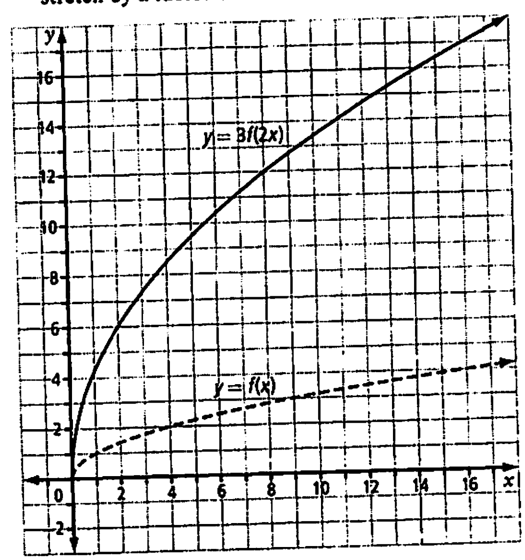
   b)



3. a) (12, 5) b) (−3, −5) c) (36, −10)
4. a) reflection in the y-axis and reflection in the x-axis



   b) horizontal stretch by a factor of $\frac{1}{2}$, vertical stretch by a factor of 3



5. a)