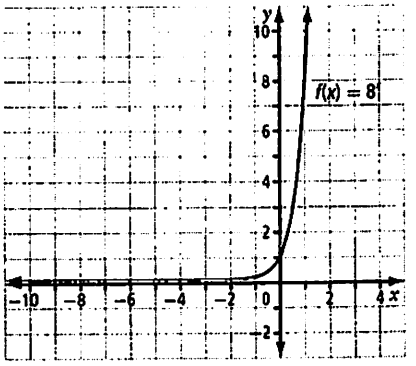
Chapter 7

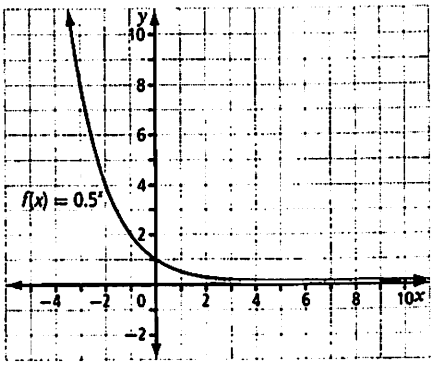
7.1 Characteristics of Exponential Functions, pages 229–237

1. a) No, the variable is not the exponent.
   b) Yes, the base is greater than 0 and the variable is the exponent.
   c) Yes, the base is greater than 0 and the variable is the exponent.
   d) No, the variable is not the exponent.
   e) No, the variable is not the exponent.
2. a) C b) A c) D d) B
3. a) $y = 10^x$ b) $y = 5^x$ c) $y = \left(\frac{1}{4}\right)^x$
4. a)



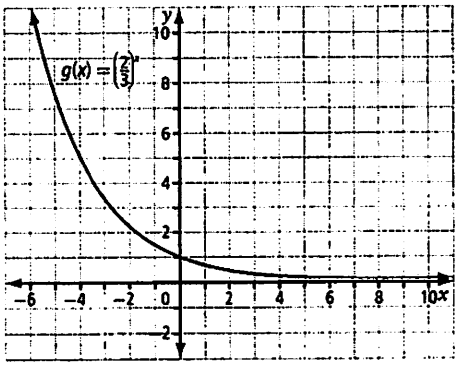
domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$; y-intercept 1; function increasing; horizontal asymptote $y = 0$

b)



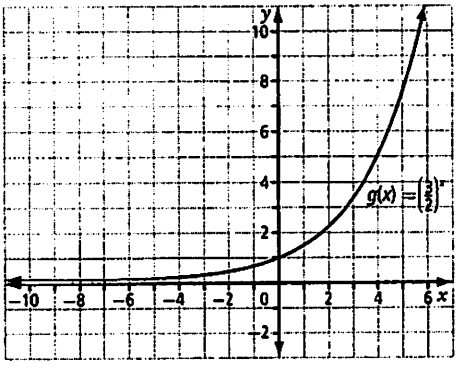
domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$; y-intercept 1; function decreasing; horizontal asymptote $y = 0$

c)



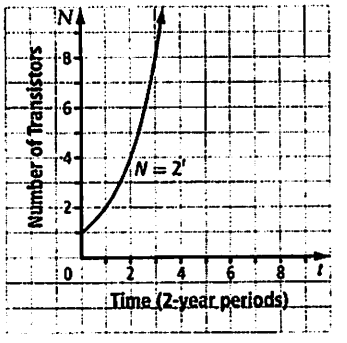
domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$; y-intercept 1; function decreasing; horizontal asymptote $y = 0$

d)



domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$; y-intercept 1; function increasing; horizontal asymptote $y = 0$

5. a)

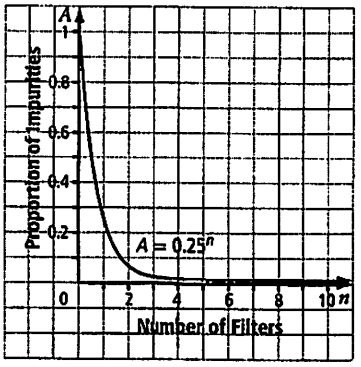


function increasing

b) domain: $\{t \mid t \geq 0, t \in R\}$; range: $\{N \mid N \geq 1, N \in N\}$

c) 2 transistors; 32 transistors; 1024 transistors

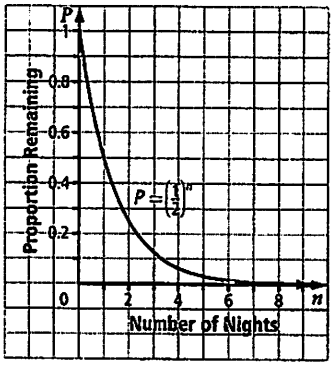
6. a) Example: Since 75% of impurities are removed, 25% remain, or as a decimal, 0.25.

b) 

c) domain: $\{n \mid n \geq 0, n \in W\}$;
range: $\{A \mid 0 < A \leq 1, A \in R\}$

d) 0.0039 of original impurities remain

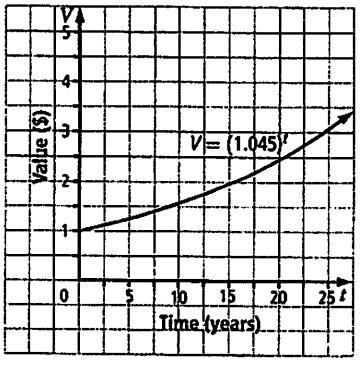
7. a) $P = \left(\frac{1}{2}\right)^n$

b) 

c) 3.13% after 5 nights;
0.39% after 8 nights

d) No; $y = 0$ is a horizontal asymptote.

8. a) $V = (1.045)^t$

b) 

c) $1.94

d) 25 years

9. a) Example: All exponential functions are defined for all real numbers, and all powers of positive bases are positive.

b) Example: All exponential functions have a y-intercept of 1 because $c^0 = 1$ for all $c > 0$.

c) Example: whether the function is increasing or decreasing

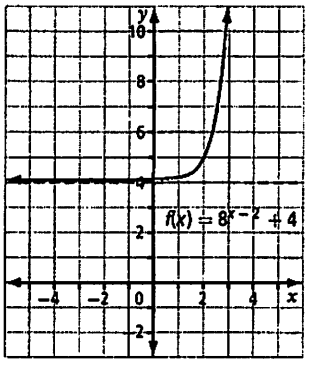
7.2 Transformations of Exponential Functions, pages 238–248

1. a) No b) Yes
   c) No d) No
2. a) No b) No
   c) No d) Yes
3. a) No b) No
   c) Yes d) No
4. a) horizontally stretched by a factor of $\frac{1}{2}$, translated 5 units right and 6 units down

b) vertically stretched by a factor of $\frac{2}{3}$, reflected in the y-axis, translated 9 units up

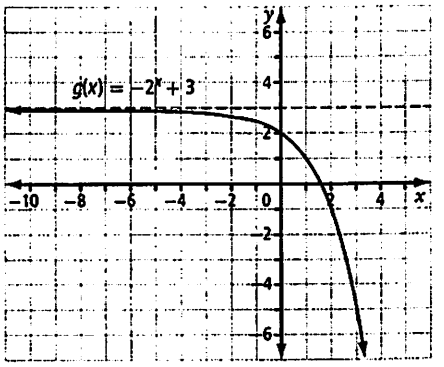
c) vertically stretched by a factor of 2, reflected in the x-axis, horizontally stretched by a factor of 4

d) vertically stretched by a factor of 500, horizontally stretched by a factor of $\frac{1}{2}$, translated 3 units left and 8 units down

5. a) 

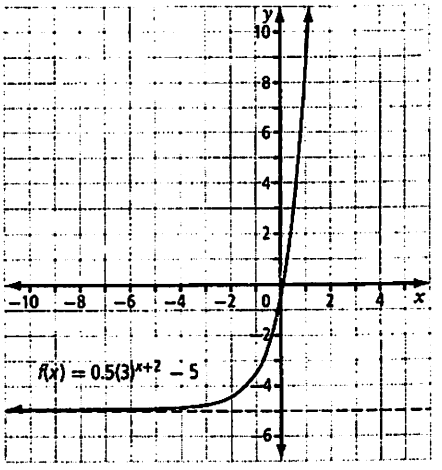
domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 4, y \in R\}$; y-intercept $\frac{257}{64}$ or ≈ 4.02; function increasing; horizontal asymptote $y = 4$

b)



domain: $\{x \mid x \in \mathbb{R}\}$; range: $\{y \mid y < 3, y \in \mathbb{R}\}$; y-intercept 2; function decreasing; horizontal asymptote $y = 3$

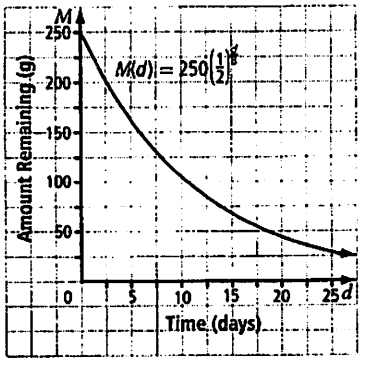
c)



domain: $\{x \mid x \in \mathbb{R}\}$; range $\{y \mid y > -5, y \in \mathbb{R}\}$; y-intercept $-\frac{1}{2}$; function increasing; horizontal asymptote $y = -5$

6. a) $M(d) = 250\left(\frac{1}{2}\right)^{\frac{d}{8}}$; vertical stretch by a factor of 250; horizontal stretch by a factor of 8

b)

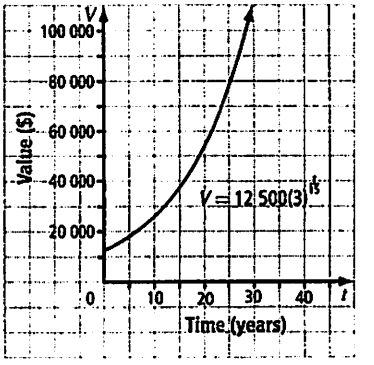


domain: $\{d \mid d \geq 0, d \in \mathbb{R}\}$; range: $\{M \mid 0 < M \leq 250, M \in \mathbb{R}\}$; $M = 0$; M-intercept 250

c) approximately 192.8 g; Example: substitute $x = 3$ and check against the graph

7. a) $V = 12\,500(3)^{\frac{t}{15}}$; vertically stretched by a factor of 12 500; horizontally stretched by a factor of 15

b)

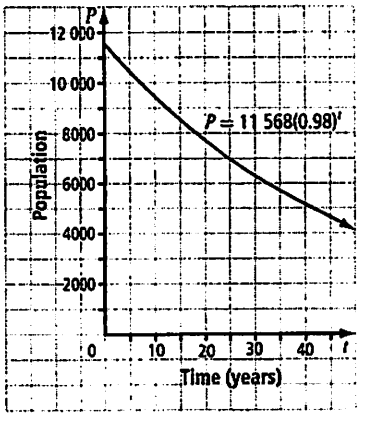


domain: $\{t \mid t \geq 0, t \in \mathbb{R}\}$; range: $\{V \mid V \geq 12\,500, V \in \mathbb{R}\}$

c) \$234 009.43; assumes that the growth rate does not change

d) approximately 19 years

8. a) $P = 11\,568(0.98)^t$



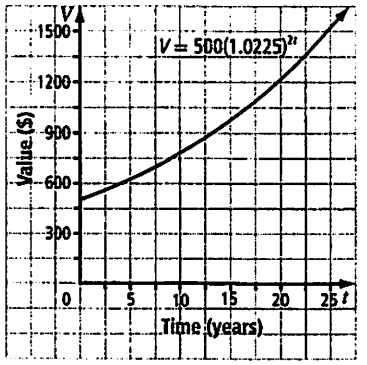
b) vertically stretched by a factor of 11 568

c) approximately 10 456

d) approximately 7722

e) Example: According to the equation, no, but since fractions of people are not possible, the population may reach zero in the future.

9. a) vertically stretched by a factor of 500, horizontally stretched by a factor of $\frac{1}{2}$



b) $500; This is the initial investment.

c) $974.70

d) approximately 16 years

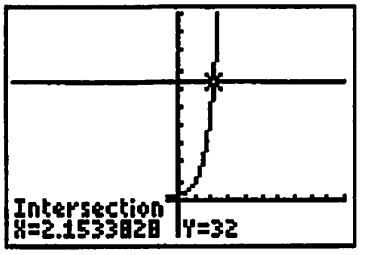
10. a) Example: They are both correct. David is correct because the y-intercepts of the graphs are 1 and 2, which suggests a vertical stretch of 1. But Jodi is also correct because the points on the transformed graph are 1 unit left of the same points on the base function, suggesting a horizontal translation by 1 unit left.

b) Example: The laws of exponents mean that $2(2)^x = 2^1 \times 2^x = 2^{x+1}$.

c) Example: $y = 3^{-x}$ and $y = \left(\frac{1}{3}\right)^x$

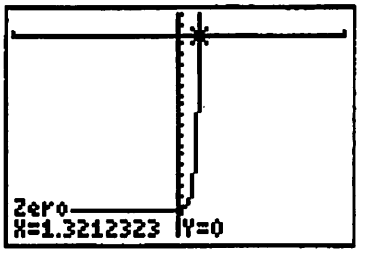
7.3 Solving Exponential Equations, pages 249–255

1. a) 3^4 b) 3^{15}
   c) $3^{\frac{1}{2}}$ d) $3^{\frac{5}{3}}$
   e) $3^{\frac{14}{3}}$ f) 3^{-6}
   g) $3^{\frac{21}{2}}$
2. a) $2^3, 2^6$ b) $3^2, 3^6$
   c) $5^{x+6}, 5^3$ d) $2^{3x}, 2^{6x+12}$
   e) $3^{15x+12}, 3^{-2x-6}$ f) $2^{-2x-14}, 2^{-9x}$
3. a) $x \approx 3.2$ b) $x \approx 3.4$
4. a) 3 b) 4
   c) −6 d) −2
5. a) 1 b) $\frac{13}{4}$
   c) −2 d) $-\frac{12}{5}$
   e) 3
6. a) Example: Graph $y = 5^x$ and $y = 32$, and calculate the point of intersection.



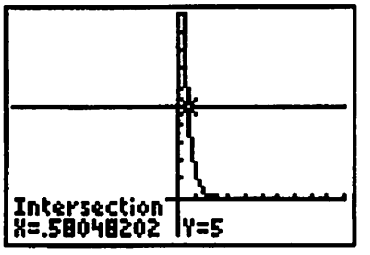
$x \approx 2.15$

b) Example: Graph $y = 10^{2x} - 439$ and calculate the x-intercept.



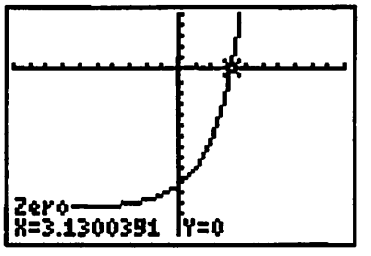
$x \approx 1.32$

c) Example: Graph $y = 25\left(\frac{1}{2}\right)^{4x}$ and $y = 5$ and calculate the point of intersection.



$x \approx 0.58$

d) Example: Graph $y = 200(1.05)^{12x} - 1250$ and calculate the x-intercept.



$x \approx 3.13$

7. a) $N = 4(2)^t$, where N is the number of bacteria, and t is the time, in hours.

   b) 10 h

8. a) $V = 1000(2)^{\frac{t}{8}}$, where V is the value of the painting and t is the time, in years.

   b) approximately 13.5 years

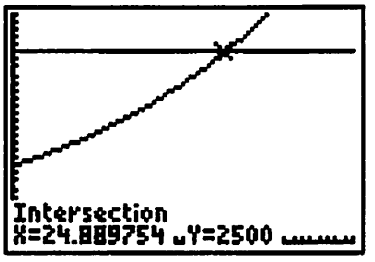
9. a) $M = 350(1.03)^t$, where M is the number of members and t is the time, in years.

   b) 16 years

10. a) $V = 1000(1.0375)^t$, where V is the value of the investment and t is the time, in years.

    b) systematic trial or use technology to graph either one or two functions

    c)



approximately 25 years

d) Example: There is no common base, so once I reach the point in the equation $1.0375^t = 2.5$, I do not have an algebraic method to calculate t other than systematic trial.

Chapter 7 Review, pages 256–258

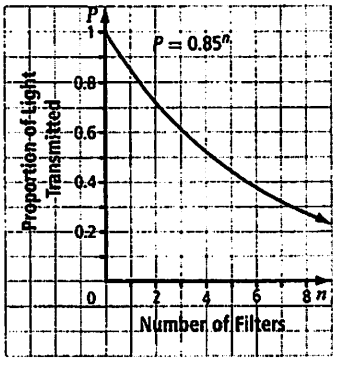
1. a) domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$; y-intercept 1; horizontal asymptote $y = 0$; function increasing

   b) domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$; y-intercept 1; horizontal asymptote $y = 0$; function decreasing

2. a) $y = 5^x$ b) $y = \left(\frac{1}{3}\right)^x$

3. a) $P = 0.85^n$

   b)



approximately 4 filters

4. a) vertically stretched by a factor of 2, translated 3 units down

   b) translated 3 units left

   c) horizontally stretched by a factor of $\frac{1}{2}$, translated 4 units right and 1 unit up

   d) vertically stretched by a factor of 5, horizontally stretched by a factor of $\frac{1}{6}$, translated 2 units left

5. a) $y = \frac{1}{2}(4)^{x+2} - 6$; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > -6, y \in \mathrm{R}\}$

   b) $y = 5(4)^{3x}$; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$

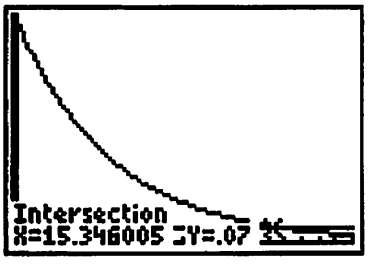
   c) $y = (4)^{\frac{1}{2}(x-3)} - 1$; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > -1, y \in \mathrm{R}\}$

6. a) 3 b) -2

   c) $\frac{15}{8}$ d) $-\frac{20}{7}$

7. a) $P = \left(\frac{1}{2}\right)^{\frac{t}{4}}$ b) 8 days

   c)



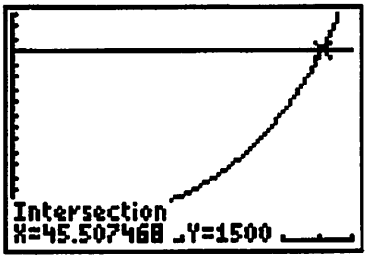
15.3 days

8. a) $N = 64(2)^{\frac{t}{10}}$

   b) 337 colonies

   c) 40 h

   d)



46 h