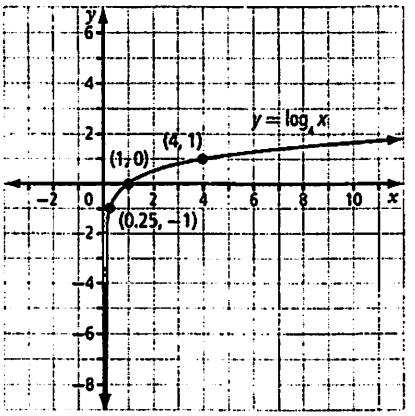
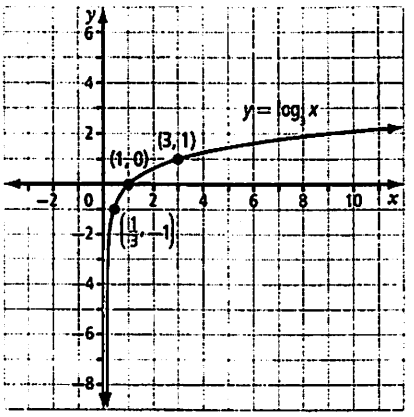
Chapter 8

8.1 Understanding Logarithms, pages 260–266

1. a) 

domain: $\{x \mid x > 0, x \in \mathrm{R}\}$;
range: $\{y \mid y \in \mathrm{R}\}$; x-intercept 1;
vertical asymptote $x = 0$; $y = \log_4 x$

b) 

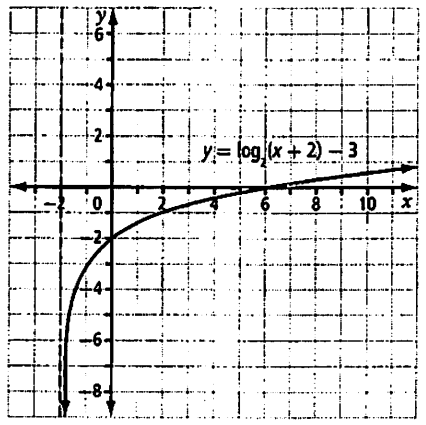
domain: $\{x \mid x > 0, x \in \mathrm{R}\}$;
range: $\{y \mid y \in \mathrm{R}\}$; x-intercept 1;
vertical asymptote $x = 0$; $y = \log_3 x$

2. a) $\log_3 243 = 5$ b) $\log 10\,000 = 4$
   c) $\log_{16} 4 = \frac{1}{2}$ d) $\log_8 \frac{1}{64} = -2$
   e) $\log 0.01 = -2$ f) $\log_{27} 9 = \frac{2}{3}$
   g) $\log_{12} 2y = x$ h) $\log_2 (y - 1) = 2x - 5$
3. a) $2^5 = 32$ b) $8^3 = 512$
   c) $5^4 = 625$ d) $10^3 = 1000$
   e) $10^{-4} = 0.0001$ f) $\left(\frac{1}{2}\right)^{-3} = 8$
   g) $3^y = x + 1$ h) $4^{y+1} = 2x$
4. a) 2 b) 6 c) 2
   d) −2 e) −3 f) 1
   g) 4 h) $\frac{11}{2}$ i) 0
   j) $\frac{1}{2}$
5. $\log_{10} 300$, $\log_6 400$, $\log_2 100$
6. a) 3 b) 3 c) 16
   d) $\frac{1}{25}$ e) 4 f) $\frac{1}{2}$
   g) 3 h) 36 i) 5
   j) 4
7. a) 216 b) 4 c) 12
   d) 7 e) −8 f) 10
8. a) 13 dB b) 1000 times
9. Examples:
   a) The graph will increase more slowly.
   b) The graph of $y = 5^x$ increases more quickly.
   c) Because the graph of $y = 5^x$ increases more quickly, the graph of $y = \log_5 x$ will increase more slowly.
   d) A larger base leads to a logarithmic graph that increases more slowly.
10. Examples:
   a) If $c < 0$, the function is discontinuous and is not defined for many real numbers.
   b) Since $1^x = 1$ for all values of x, the only domain of $y = \log_1 x$ is 1.

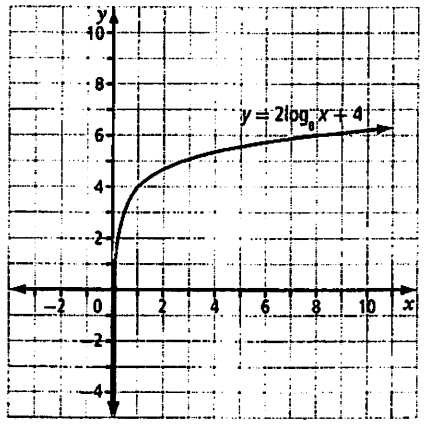
8.2 Transformations of Logarithmic Functions, pages 267–274

1. a) translated 1 unit left and 8 units down
   b) vertically stretched by a factor of 2 and horizontally stretched by a factor of $\frac{1}{4}$
   c) reflected in the x-axis and horizontally stretched by a factor of $\frac{1}{3}$
   d) vertically stretched by a factor of 5, horizontally stretched by a factor of $\frac{1}{2}$, reflected in the y-axis, translated 4 units left
2. a) $y = 3 \log_5 (x - 2)$
   b) $y = -\log_5 (x + 4) - 1$
   c) $y = \frac{1}{2} \log_5 (2x)$
   d) $y = 4 \log_5 (-x) - 2.5$

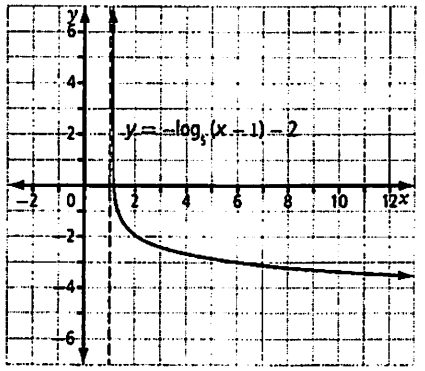
3. a)



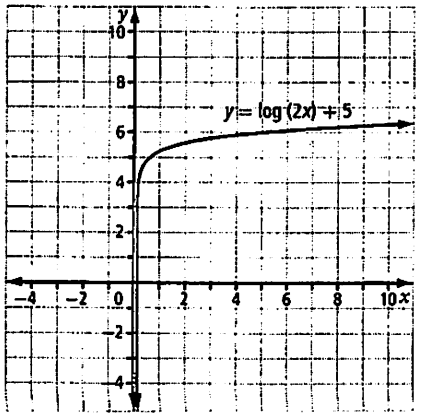
b)



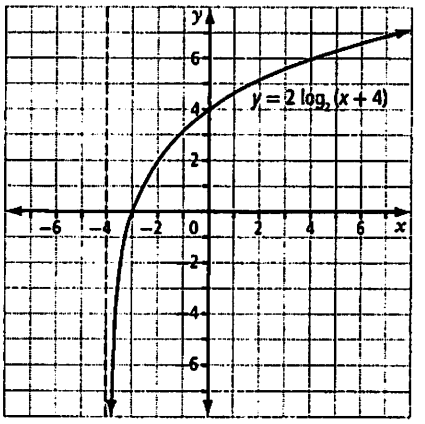
c)



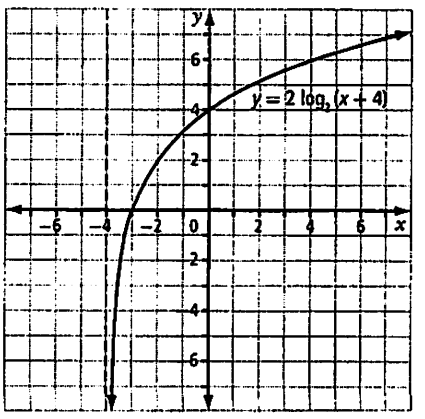
d)



4. a) horizontally stretched by a factor of $\frac{1}{2}$, translated 3 units to the left
   b) horizontally stretched by a factor of $\frac{1}{3}$, translated 4 units to the right
   c) horizontally stretched by a factor of 2, translated 6 units to the right
   d) horizontally stretched by a factor of 3, translated 18 units to the left
5. a) vertically stretched by a factor of B and translated 1 unit left
   b) 50 000 bits per second
6. a) domain: $\{x \mid x > 8, x \in \mathbb{R}\}$; range: $\{y \mid y \in \mathbb{R}\}$; x-intercept: 244 140 633; no y-intercept; vertical asymptote $x = 8$
   b) domain: $\{x \mid x > 1, x \in \mathbb{R}\}$; range: $\{y \mid y \in \mathbb{R}\}$; x-intercept: 2.1; no y-intercept; vertical asymptote $x = 1$
   c) domain: $\{x \mid x > 0, x \in \mathbb{R}\}$; range: $\{y \mid y \in \mathbb{R}\}$; x-intercept: 71 663 616; no y-intercept; vertical asymptote $x = 0$
   d) domain: $\{x \mid x > -3, x \in \mathbb{R}\}$; range: $\{y \mid y \in \mathbb{R}\}$; x-intercept 125; y-intercept −5.4; vertical asymptote $x = -3$
7. a) translated 2 units right and 5 units down
   b) translated 1 unit left and 2 units down
8. a) vertically stretched by a factor of 3
   b) vertically stretched by a factor of $\frac{1}{2}$
9. a)



b)



c) The graphs are identical. The order of the transformations does not matter when performing a vertical stretch and a horizontal translation.

d) Example: a horizontal stretch by a factor of 2 and a translation 3 units up; a vertical stretch by a factor of 3 and a translation 1 unit right

e) Example: when a stretch and a translation are in the same direction

8.3 Laws of Logarithms, pages 275–281

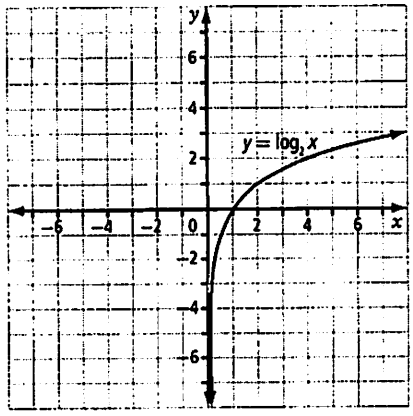
1. a) 2 b) 3 c) 5 d) 2 e) 2
   f) 3 g) 5 h) 11 i) 16 j) 1000
2. a) $4\log_7 x + \frac{3}{2}\log_7 y$
   b) $3\log_{12} x + 6\log_{12} y + 15\log_{12} z$
   c) $3\log_8 x - \frac{1}{2}\log_8 y - \frac{5}{2}\log_8 z$
   d) $\frac{1}{2}\log x - \frac{3}{2}\log y$
3. a) $2 + \frac{5}{3}\log_7 x$ b) $2 - 2\log x - 2\log y$
   c) $\frac{7}{3}\log_5 y - 3 - \log_5 x$ d) $6\log_2 x - 5 - 2\log_2 y$
4. a) $\log_6 54x^4$ b) $\log_2 \frac{y^8}{2x}$ c) $\log_4 x^{16}y^{20}$
   d) $\log_3 (xy)^{\frac{11}{3}}$ e) $\log \frac{2}{25x^{\frac{3}{4}}y^{\frac{1}{4}}}$ f) $\log_7 \frac{x^{\frac{9}{2}}}{\sqrt[6]{5}}$
   g) $\log \frac{2x^{\frac{5}{3}}}{3}$ h) $\log_9 x^6y^9$
5. a) $L = \log I^{10} - \log I_0^{10}$ b) $I = 10^{0.1L + \log I_0}$
6. a) $[H_3O]^+ = 10^{-pH}$
   b) $pH = \log([H_3O]^+)^{-1} = \log \frac{1}{[H_3O^+]}$
7. a) False; it must be a multiplication inside the logarithm.
   b) False; the division must take place inside the logarithm.
   c) True d) True
   e) False; the exponent must apply to the entire argument of the logarithm.
8. a) $\frac{1}{6}$ b) 3 c) 8
9. a) $7P$ b) $P + 1$ c) $2P$
   d) $P - 1$ e) $-P$ f) $\frac{P}{2}$
10. a) The function $y = \log_2 x^2$ can be written as $y = 2\log_2 x$, which is a vertical stretch by a factor of 2 of $y = \log_2 x$.
    b) The function $y = \log_2 3x$ is of the form $y = \log_2 bx$. This is a horizontal stretch by a factor of $\frac{1}{3}$ of the function $y = \log_2 x$.
    c) The function $y = \log_2 3x$ can be written as $y = \log_2 x + \log_2 3$, which is a translation of $\log_2 3$ units up.
    d) No. Example: $y = \log_2 \frac{1}{x}$ can be written as $y = -\log_2 x$, which is a reflection in the x-axis, not the reciprocal transformation.

8.4 Logarithmic and Exponential Equations, pages 282–291

1. a) 1024 b) 25 c) 32
   d) 213 e) 5 f) 1005
2. a) 0.93 b) 1.13 c) −3.64 d) 8.00
3. a) $\frac{\log 205}{\log 5}$ b) $\frac{\log 311}{\log 4} + 3$
   c) $\frac{\log 7539 - 1}{2}$ d) $\frac{\log 40}{\log 4} - 2$
   e) $\frac{2\log 85}{\log 6}$
4. a) 6 b) 10 or −10
   c) 16 d) 9
5. a) $x > 0$ b) $x > 2$
   c) undefined for all x
6. a) $\frac{3\log 5}{\log 5 - 1} \approx -6.97$
   b) $\frac{-3\log 8}{2\log \frac{2}{3}} \approx 7.69$
   c) $\frac{2\log 6 + 5\log 2}{2\log 2 - \log 6} \approx -17.39$
   d) $\frac{3\log 3 + 2\log 6 + \log 2}{2\log 3 - \log 6} \approx 18.68$
7. a) $\frac{5}{2}$ b) $\frac{17}{4}$ c) 2 d) no soluion
8. a) 2 b) 6 c) 6 d) 3
9. a) $m = 65\left(\frac{1}{2}\right)^{\frac{t}{88}}$ b) 43.84 g c) 149.6 years
10. a) $p = 974(1.015)^t$ b) 1049 c) 14 years
11. a) $\frac{3}{2}$ or 1 b) −3 or $\frac{3}{2}$

Chapter 8 Review, pages 292–295

1. a)



domain: $\{x \mid x > 0, x \in \mathbb{R}\}$;
range: $\{y \mid y \in \mathbb{R}\}$; x-intercept 1;
vertical asymptote $x = 0$