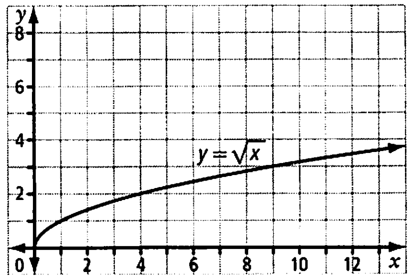
Chapter 2 Radical Functions

2.1 Radical Functions and Transformations

KEY IDEAS

Base Radical Function

- The base radical function $y = \sqrt{x}$ has the following graph and properties:
  - x-intercept of 0
  - y-intercept of 0
  - domain: $\{x \mid x \geq 0, x \in R\}$
  - range: $\{y \mid y \geq 0, y \in R\}$
  - The intercepts and domain and range suggest an endpoint at (0, 0), and no right endpoint.



- The graph is shaped like half of a parabola. The domain and range indicate that the half parabola is in the first quadrant.

Transforming Radical Functions

The base radical function $y = \sqrt{x}$ is transformed by changing the values of the parameters a, b, h, and k in the equation $y = a\sqrt{b(x - h)} + k$. The parameters have the following effects on the base function:

a	• vertical stretch by a factor of $\lvert a \rvert$ • if a is $a < 0$, the graph of $y = \sqrt{x}$ is reflected in the x-axis
b	• horizontal stretch by a factor of $\frac{1}{\lvert b \rvert}$ • if b is $b < 0$, the graph of $y = \sqrt{x}$ is reflected in the y-axis
h	• horizontal translation • $(x - h)$ means the graph of $y = \sqrt{x}$ moves h units right. For example, $y = \sqrt{x - 1}$ means that the graph of $y = \sqrt{x}$ moves 1 unit right. • $(x + h)$ means the graph of $y = \sqrt{x}$ moves h units left. For example, $y = \sqrt{x + 5}$ means that the graph of $y = \sqrt{x}$ moves 5 units left. This translation has the opposite effect than many people think. It is a common error to think that the + sign moves the graph to the right and the – sign moves the graph to the left. This is not the case.
k	• vertical translation • $+ k$ means the graph of $y = \sqrt{x}$ moves k units up • $- k$ means the graph of $y = \sqrt{x}$ moves k units down

Working Example 1: Explain How to Recognize Transformations

Explain how to transform the graph of $y = \sqrt{x}$ to obtain $y = -2\sqrt{4(x-3)} + 1$. Sketch the graph of each function. Then, identify the domain and range of each function.

Solution

Begin by identifying the parameters and the effect each has on the base function.

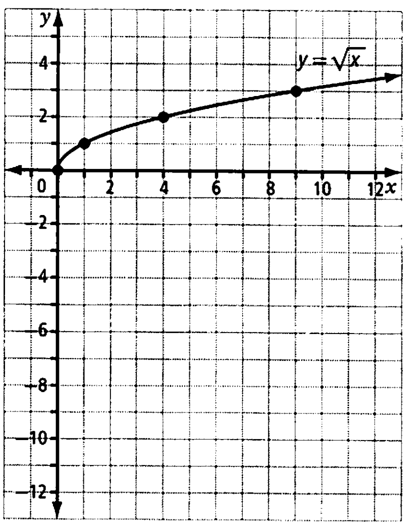
- Parameter a = ________, resulting in a ______________________ by a factor of ________.

  Since a is negative, the graph is reflected in the ______________.

- Parameter b = ________, resulting in a ______________________ by a factor of ________.
- Parameter h = ________, so the graph is translated ______________ by ________ units.
- Parameter k = ________, so the graph is translated ______________ by ________ units.

> Why is the graph not reflected in the *y*-axis?

Apply the transformations to sketch the graph of transformed function.



> Create a table of values and describe how the transformations are reflected in the values.

The domain of the base function is $\{x \mid x \geq 0, x \in R\}$ and its range is $\{y \mid y \geq 0, y \in R\}$. The domain and range of the transformed function are

domain: ______________

range: ______________

Working Example 2: Use Transformations to Sketch a Graph

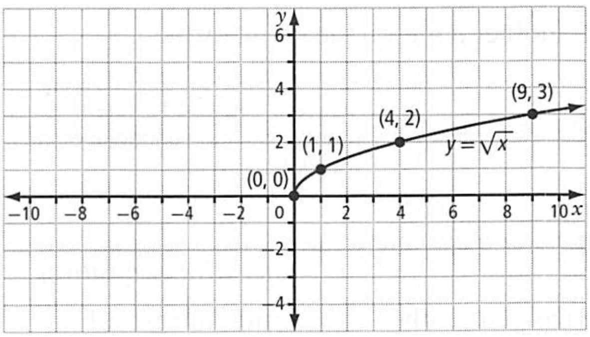
Use transformations to sketch the graph of $y = 4\sqrt{-2(x + 3)} - 5$.

Solution

The function $y = 4\sqrt{-2(x + 3)} - 5$ is expressed in the form $y = a\sqrt{b(x - h)} + k$. Identify each parameter and how it will transform the graph of $y = \sqrt{x}$.

- $a =$ ________ results in a ________________ stretch by a factor of ________.
- $b =$ ________ results in a ________________ stretch by a factor of ________, and a reflection in the ________-axis.
- $h =$ ________ results in a ________________ translation of ________ units ________________. (*left* or *right*)
- $k =$ ________ results in a ________________ translation ________ units ________________. (*up* or *down*)

Sketch the graph of $y = \sqrt{x}$ and plot four identifiable points.



Using the four points on the graph of the base function, $y = \sqrt{x}$, complete the table to determine the resulting coordinates on the graph of $y = 4\sqrt{-2(x + 3)} - 5$. One of the points has been done for you.

Point on $y = \sqrt{x}$	(0, 0)	(1, 1)	(4, 2)	(9, 3)
Vertical stretch			(4, 8)	
Horizontal stretch			(2, 8)	
Reflection in the ________-axis			(−2, 8)	
Horizontal translation			(−5, 8)	
Vertical translation			(−5, 3)	
Point on $y = 4\sqrt{-2(x + 3)} - 5$			(−5, 3)	

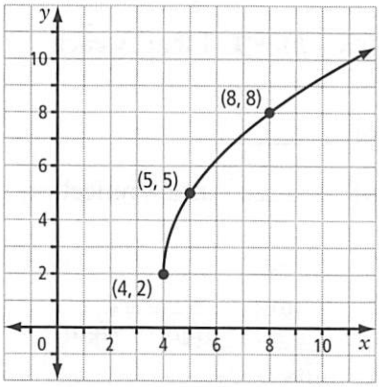
Plot the four points from the bottom row of the table to help you sketch the graph of $y = 4\sqrt{-2(x + 3)} - 5$ on the grid above.

What other method can you use for transforming the graph? Which method do you prefer?

To see a similar question, refer to Example 2 on pages 65–67 in *Pre-Calculus 12*.

Working Example 3: Determine a Radical Function From a Graph

Use the graph below to write the equation of the corresponding radical function in the form $y = a\sqrt{b(x - h)} + k$.



How could you compare this graph to the graph of $y = \sqrt{x}$ to determine the equation of the transformed function?

Solution

You can use the endpoint and the coordinates of another point on the graph of the transformed function to determine the equation.

Identify the endpoint of the transformed function: ________.

Why are you able to determine h and k from the endpoint of the translated graph?

The endpoint represents the parameters (h, k).

There is no reflection in the x-axis or y-axis, so the a or b parameters are ____________.
(*negative* or *positive*)

The graph is stretched. This stretch can be viewed as being either a horizontal or vertical stretch. To determine the factor by which the graph has been stretched, begin by identifying one other point that the graph passes through: ____________.

View as a Vertical Stretch	View as a Horizontal Stretch
Substitute h, k, x, and y into $y = a\sqrt{x - h} + k$ and solve for parameter ________.	Substitute h, k, x, and y into $y = \sqrt{b(x - h)} + k$ and solve for parameter ________.
Equation of the function: ______________	Equation of the function: ______________

To see a similar question, refer to Example 3 on pages 68–69 in *Pre-Calculus 12*.

Check Your Understanding

Practise

1. Explain how to transform the graph of $y = \sqrt{x}$ to obtain the graph of each function. State the domain and range in each case.

   a) $y = 3\sqrt{-(x + 4)} - 2$

   b) $y = -2\sqrt{4(x - 3)} + 5$

   c) $y = 4\sqrt{5(x + 1)} - 4$

   d) $y = -\sqrt{-3(x + 2)}$

2. Write the radical function that results from applying each set of transformations to the graph of $y = \sqrt{x}$.

   a) vertical stretch by a factor of 3, reflection in the x-axis, a translation of 4 units right and 2 units down

   b) horizontal stretch by a factor of $\frac{1}{4}$, reflection in the y-axis, a translation of 5 units left and 3 units up

   c) vertical stretch by a factor of 2, horizontal stretch by a factor of 3, translation of 4 units left and 1 unit up

   d) vertical stretch by a factor of 3, horizontal stretch by a factor of $\frac{1}{2}$, reflection in the x-axis and y-axis, and translation of 6 units left

3. Match each function with its graph.

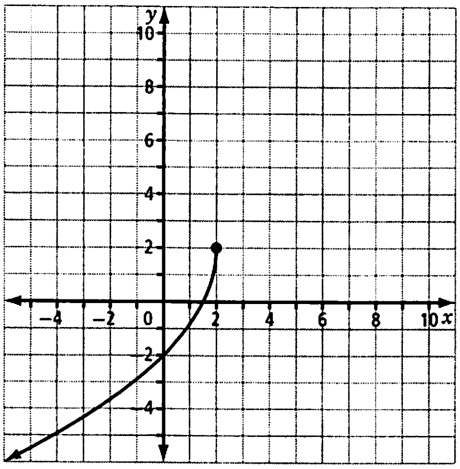
a) $y = 2\sqrt{2(x-2)} + 2$

b) $y = -2\sqrt{2(x-2)} + 2$

c) $y = 2\sqrt{-2(x-2)} + 2$

d) $y = -2\sqrt{-2(x-2)} + 2$

A



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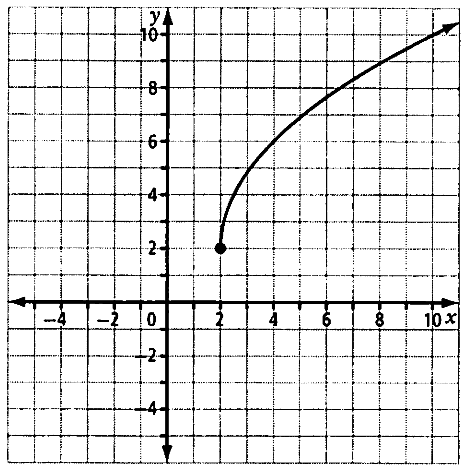

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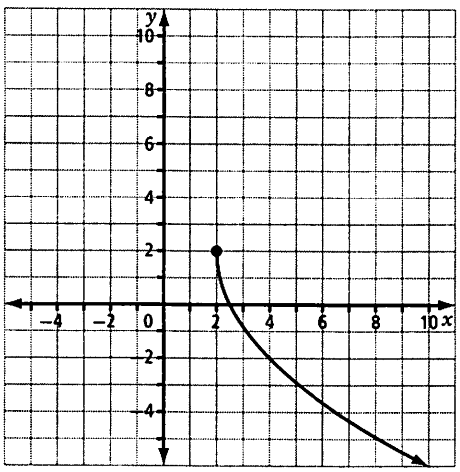
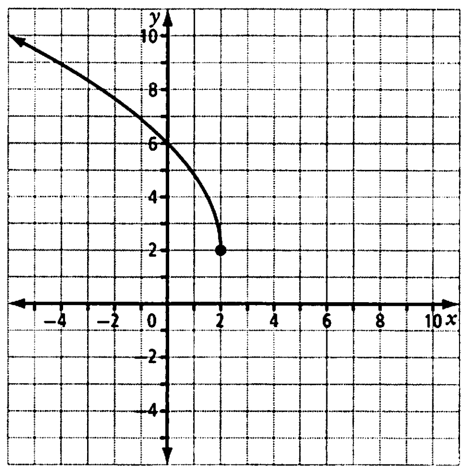


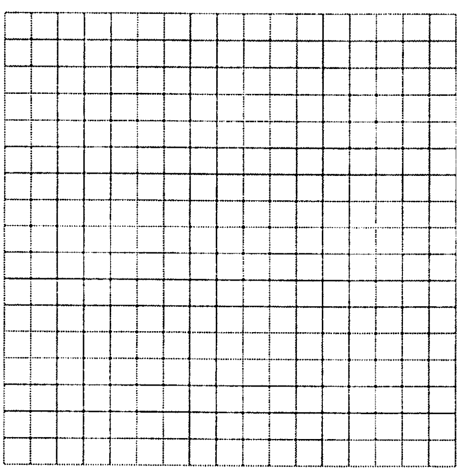
D



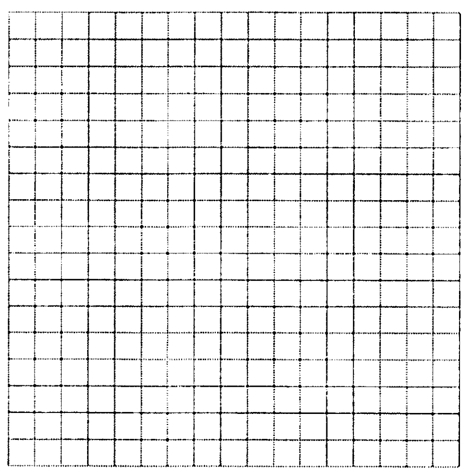
4. Sketch the graph of each function using transformations.

a) $y = 3\sqrt{x-1} + 4$

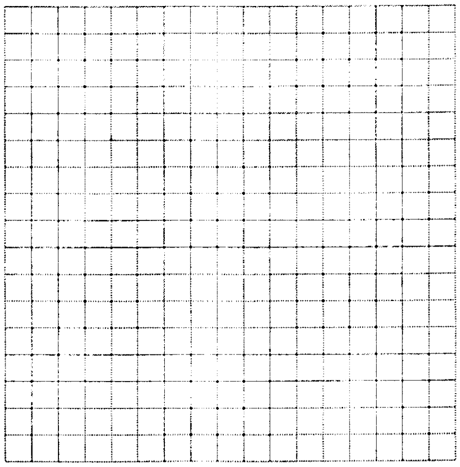
$a =$ ____, $b =$ ____, $h =$ ____, $k =$ ____



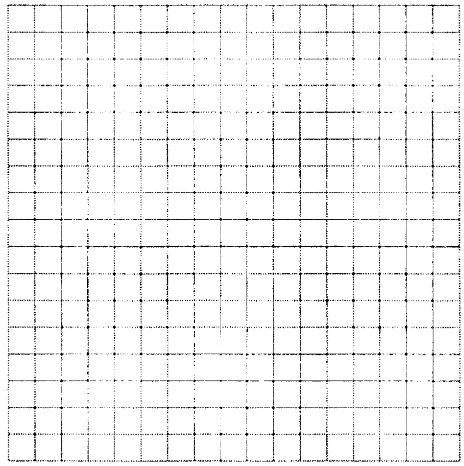
b) $y = -4\sqrt{x+3} - 2$



c) $y = 2\sqrt{4(x - 1)} + 3$



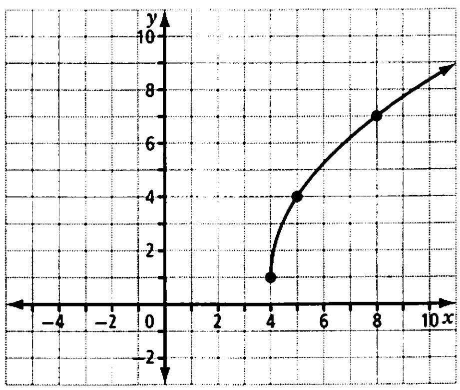
d) $y = -3\sqrt{-2(x + 1)} - 4$



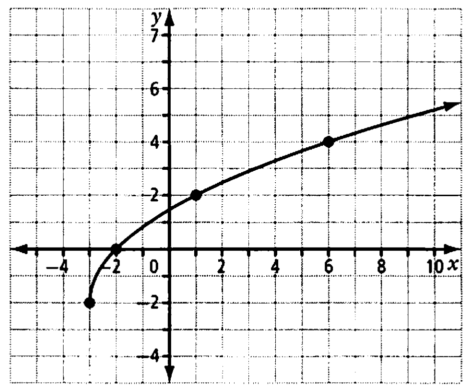
Apply

5. For each graph, write the equation of a radical function of the form $y = a\sqrt{b(x - h)} + k$.

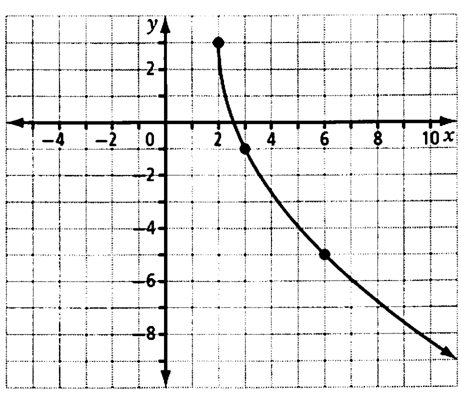
a)



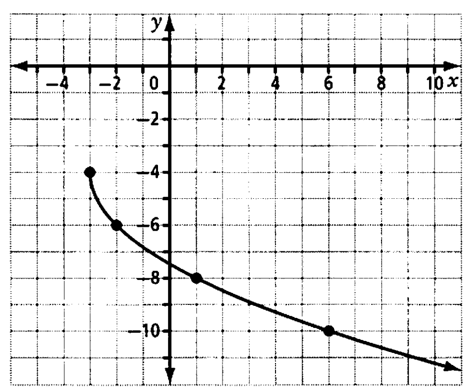
b)



c)



d)



6. Consider the function $y = \frac{1}{2}\sqrt{6x}$.

a) Describe the transformations that were applied to $y = \sqrt{x}$ to obtain this function.

b) Write a function equivalent to $y = \frac{1}{2}\sqrt{6x}$ in the form $y = a\sqrt{x}$. Describe the transformation applied to $y = \sqrt{x}$ to obtain this new function.

c) Write a function equivalent to $y = \frac{1}{2}\sqrt{6x}$ in the form $y = \sqrt{bx}$. Describe the transformation applied to $y = \sqrt{x}$ to obtain this new function.

Connect

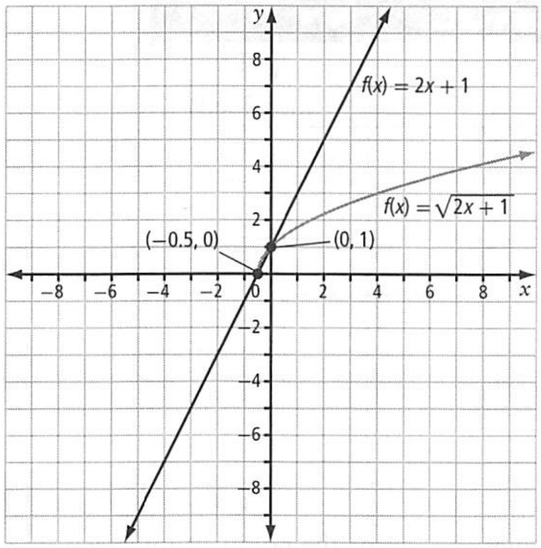
7. Joanne claims that when writing the equation of a radical function given its graph, you only need to find three parameters: a, h, and k, or b, h, and k. Do you agree? Explain using examples.

2.2 Square Root of a Function

KEY IDEAS

Graphing $y = f(x)$ and $y = \sqrt{f(x)}$

- To graph $y = \sqrt{f(x)}$, you can set up a table of values for the graph of $y = f(x)$. Then, take the square root of the elements in the range, while keeping the elements in the domain the same.
- When graphing $y = \sqrt{f(x)}$, pay special attention to the invariant points, which are points that are the same for $y = f(x)$ as they are for $y = \sqrt{f(x)}$. The invariant points are $(x, 0)$ and $(x, 1)$ because when $f(x) = 0$, $\sqrt{f(x)} = 0$, and when $f(x) = 1$, $\sqrt{f(x)} = 1$.



Domain and Range of $y = \sqrt{f(x)}$

- You cannot take the square root of a negative number, so the domain of $y = \sqrt{f(x)}$ is any value for which $f(x) \geq 0$.
- The range is the square root of any value in $y = f(x)$ for which $y = \sqrt{f(x)}$ is defined.

The Graph of $y = \sqrt{f(x)}$

$f(x) < 0$	$f(x) = 0$	$0 < f(x) < 1$	$f(x) = 1$	$f(x) > 1$
$y = \sqrt{f(x)}$ is ***undefined*** because you cannot take the square root of a negative number.	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ ***intersect*** at $x = 0$.	The graph of $y = \sqrt{f(x)}$ is ***above*** the graph of $y = f(x)$.	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ ***intersect*** at $x = 1$.	The graph of $y = \sqrt{f(x)}$ is ***below*** the graph of $y = f(x)$.

Working Example 1: Compare Graphs of a Linear Function and the Square Root of the Function

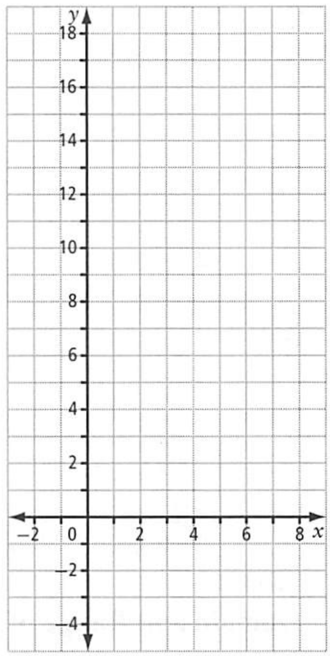
a) Given $f(x) = 4x - 3$, graph the functions $y = f(x)$ and $y = \sqrt{f(x)}$.

b) Compare the graphs.

Solution

a) Determine the y-value in the second column of the table. Then, complete the third column by taking the square root of the second column. Use the table of values to sketch the graphs of $y = f(x)$ and $y = \sqrt{f(x)}$. (Hint: You could graph $y = f(x)$ on your graphing calculator and then use the table function to complete the second column of the table.)

x	$y = 4x - 3$	$y = \sqrt{4x - 3}$
0	−3	$\sqrt{-3}$ (undefined)
0.75		
0.8		
1		
2		
3		
5		



b) From your table of values, determine the points of intersection:

(________, 0); (________, 1)

How is the x-intercept of the graph of $y = 4x - 3$ related to the graph of the function $y = \sqrt{4x - 3}$?

Why are these points of intersection referred to as *invariant points*?

For which values of x is the graph of $y = \sqrt{4x - 3}$ above the graph of $y = 4x - 3$? How are these values related to the invariant points?

For which values of x is the graph of $y = \sqrt{4x - 3}$ below the graph of $y = 4x - 3$?

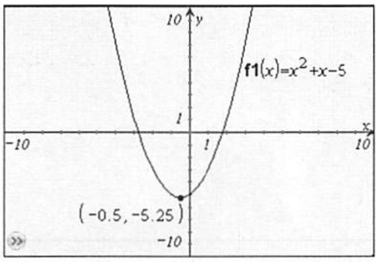
To see a similar question, refer to Example 1 on pages 80–81 in *Pre-Calculus 12*.

Working Example 2: Explore the Domains and Ranges of Functions and Their Square Roots

Find the domain and range for $y = x^2 + x - 5$ and $y = \sqrt{x^2 + x - 5}$.

Solution

To find the domain and range of the quadratic function $y = x^2 + x - 5$, graph the function using graphing technology and identify the minimum value.



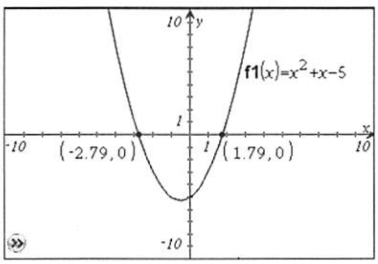
The graph shows that the function extends to the left and right infinitely. So, the domain of the function is ______________________.

The function opens upward and has a minimum value of ________, so the range of this function is ______________.

When considering the domain and range of $y = \sqrt{x^2 + x - 5}$, remember that you cannot take the square root of a ______________ number. So, you can only find the square root of this
(*negative* or *positive*)

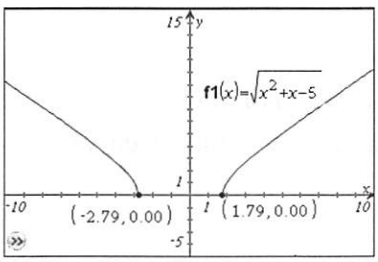
function when $x^2 + x - 5$ ________ 0.
($\le$ or $\ge$)

To find the values for which $y = x^2 + x - 5$ are positive, determine the values of the zeros of the function or the x-intercepts of the graph.



The x-intercepts are approximately ________ and ________. Therefore, the domain of $y = \sqrt{x^2 + x - 5}$ is ______________. The range is ______________.

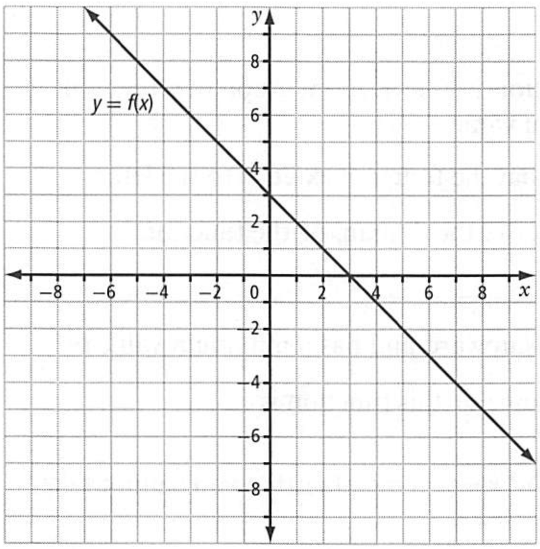
Check the domain and range of $y = \sqrt{x^2 + x - 5}$ by graphing the equation on a graphing calculator and examining the graph. How does this graphing screen confirm the domain and range you determined?



To see a similar question, refer to Example 2 on pages 82–83 in *Pre-Calculus 12*.

Working Example 3: Graph the Square Root for a Function From the Graph of the Function

Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.



When graphing the square root of a function, what is the significance of the points where $f(x) = 0$ and $f(x) = 1$?

Solution

Create a table of values using the graph of the function.

- First consider the invariant points on $y = f(x)$. Add these to the table.
- Locate any key points on $f(x)$ that are greater than $y = 1$. Add these values to your table.
- Complete the third column of the table by taking the square root of the y-values.

x	y	$\sqrt{y}$

On the same grid as the graph of $y = f(x)$ above,

- plot the invariant points of $y = f(x)$
- draw a smooth curve between the invariant points, and above the graph of $f(x)$
- plot $\sqrt{y}$ for the key points you identified in your table of values, and draw a smooth curve between these points

To see a similar question, refer to Example 3 on pages 84–85 in *Pre-Calculus 12*.

Check Your Understanding

Practise

1. For each point on the graph of $y = f(x)$, determine the corresponding point on the graph $y = \sqrt{f(x)}$. Round answers to the nearest tenth, if necessary.

   a) (3, 0)

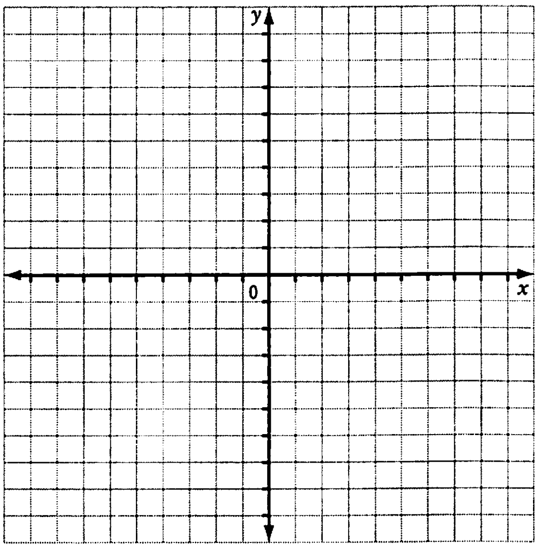
   b) (−5, 25)

   c) (9, 15)

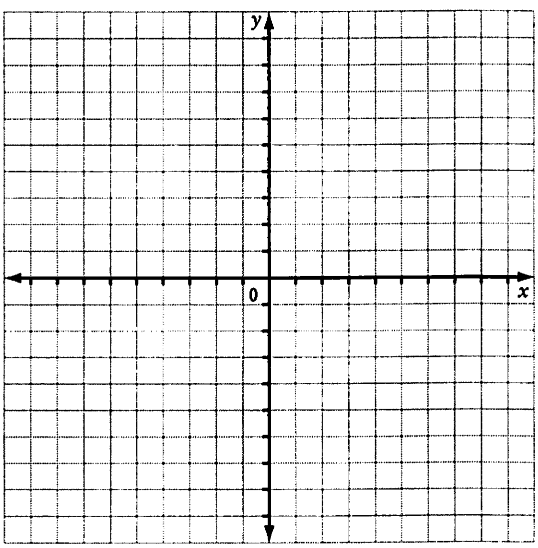
   d) (4, −16)

2. Graph $y = f(x)$ and $y = \sqrt{f(x)}$ for the given function.

   a) $f(x) = 3 - 2x$

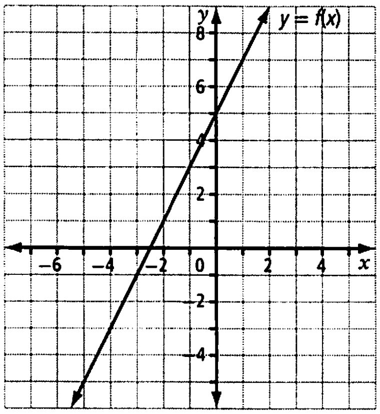


   b) $f(x) = x^2 - 5$

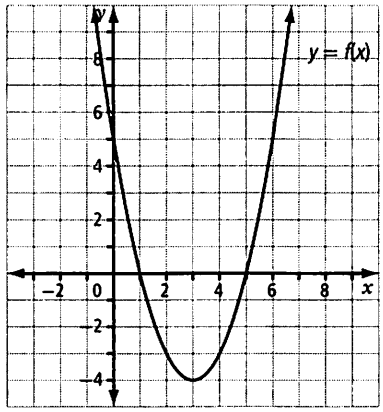


3. Match each graph of $y = f(x)$ with the corresponding graph, $y = \sqrt{f(x)}$.

a)



b)



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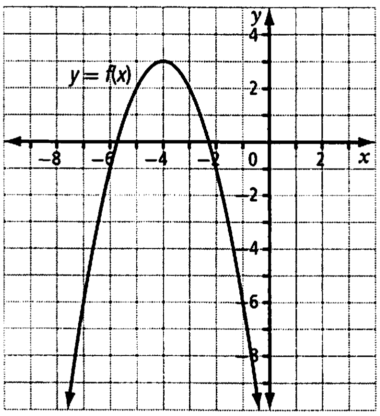

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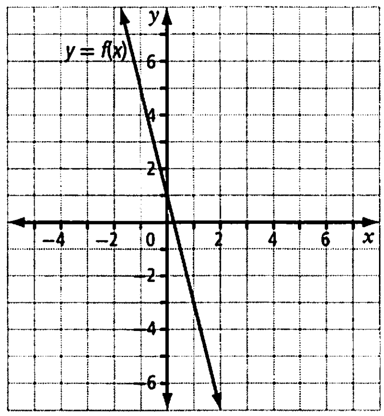
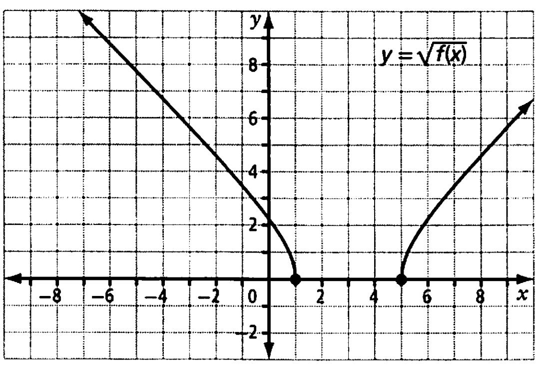
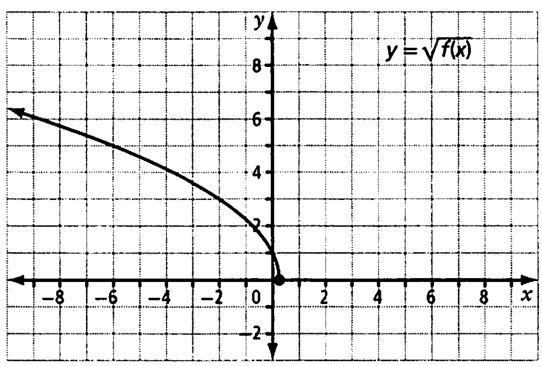


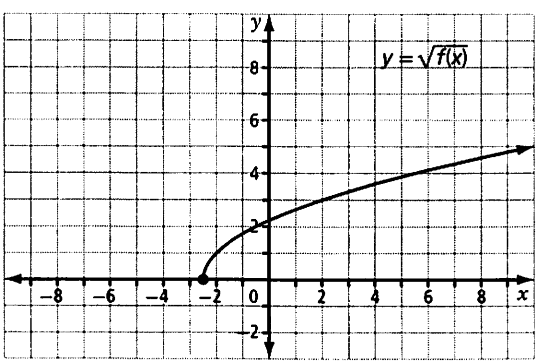
A



B



C



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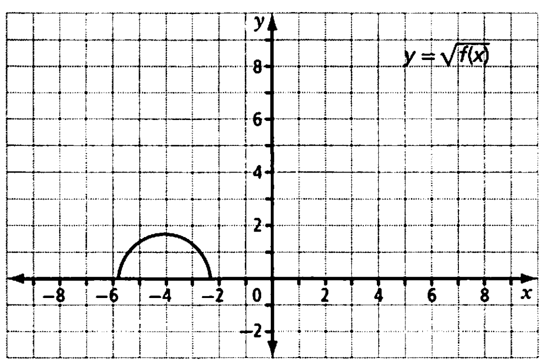


4. State the domain and range of $y = f(x)$ and $y = \sqrt{f(x)}$.

a) $f(x) = 2x - 4$

b) $f(x) = x^2 + 2$

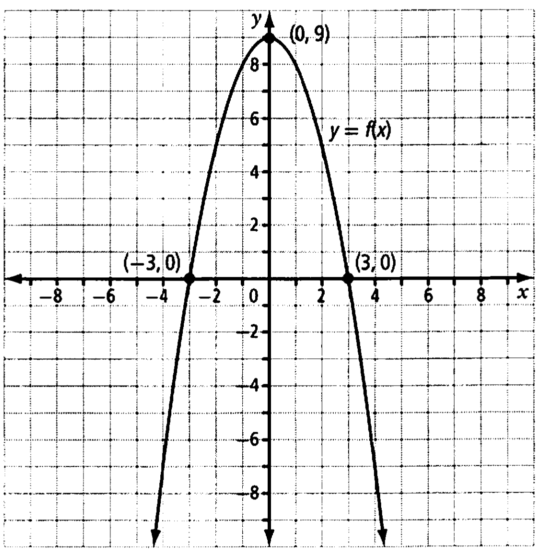
c) $f(x) = x^2 - 4$

d) $f(x) = -x^2 + 3$

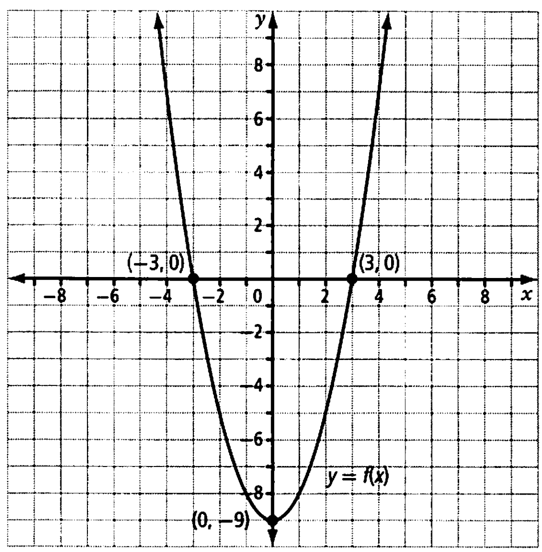
Apply

5. Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$ on the same grid.

a)



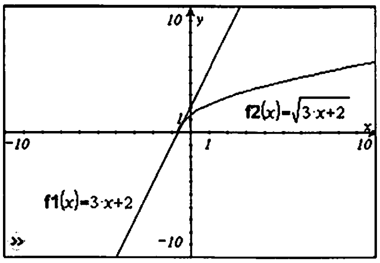
b)



Connect

6. Explain why the points where $f(x) = 0$ and $f(x) = 1$ are always invariant points when graphing the square root of a function.

7. Conal uses his graphing calculator to graph the functions $f(x) = 3x + 2$ and $f(x) = \sqrt{3x + 2}$. He produces the graph shown here.

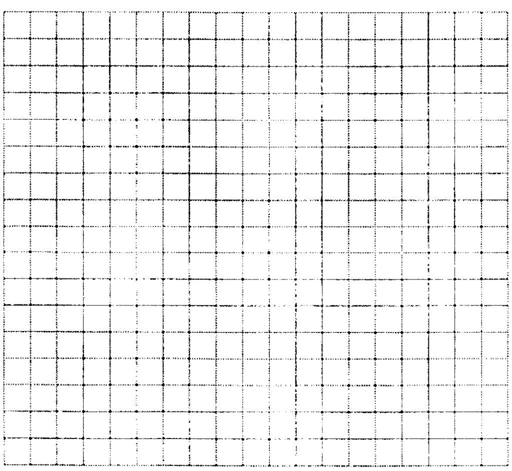


From his graph, Conal claims that $3x + 2 \geq \sqrt{3x + 2}$.

a) Using your knowledge of radical functions, explain why Conal is mistaken.

b) Describe two graphing strategies that Conal could use to redraw the functions so that it is clear that $3x + 2$ is not greater than $\sqrt{3x + 2}$ for all x-values.

c) Using one of your strategies described in part b), graph the functions so that the x-values for which $3x + 2$ is not greater than $\sqrt{3x + 2}$ are clear. Sketch the graphs below.



2.3 Solving Radical Equations Graphically

KEY IDEAS

Strategy for Solving Algebraically

Step 1: List any restrictions for the variable. You cannot take the square root of a negative number, so the value of the variable must be such that any operations under the radical sign result in a positive value.

Step 2: Isolate the radical and square both sides of the equation to eliminate the radical. Then, solve for x.

Step 3: Find the roots of the equation (that is, the value(s) of x that make the equation have a value of zero).

Step 4: Check the solution, ensuring that it does not contain *extraneous roots* (solutions that do not satisfy the original equation or restrictions when substituted in the original equation).

Example:

$7 = \sqrt{12 - x} + 4, x \leq 12$	Identify restrictions.	Check:
$3 = \sqrt{12 - x}$	Isolate the radical.	Solution meets the restrictions.
$3^2 = (\sqrt{12 - x})^2$	Square both sides.	$7 = \sqrt{12 - 3} + 4$
$9 = 12 - x$	Solve for x.	$7 = \sqrt{9} + 4$
$3 = x$		$7 = 7$

Strategies for Solving Graphically

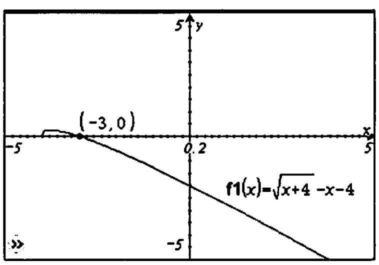
- **Method 1: Graph a Single Equation**

  Graph the corresponding function and find the zero(s) of the function.

  Example:

  $$2 + \sqrt{x + 4} = x + 6$$
  $$\sqrt{x + 4} - x - 4 = 0$$

  Graph $y = \sqrt{x + 4} - x - 4$.



  $x = -3$

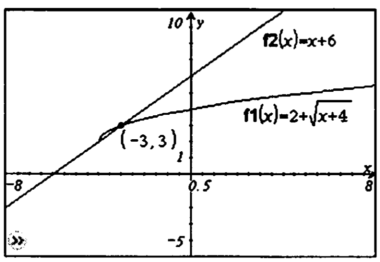
- **Method 2: Graph Two Equations**

  Graph each side of the equation on the same grid, and find the point(s) of intersection.

  Example:

  $2 + \sqrt{x + 4} = x + 6$

  Graph $y = 2 + \sqrt{x + 4}$ and $y = x + 6$.



  $x = -3$

Working Example 1: Relate Roots and *x*-Intercepts

Consider $2\sqrt{x - 4} - 3 = 0$.

a) Determine the roots algebraically.

b) Graph the corresponding function and determine the *x*-intercepts.

c) Compare the roots of the equation to the *x*-intercepts of the graph of the corresponding function.

Solution

a) Begin by stating the restrictions for the variable.

You cannot take the square root of a negative number, so in the equation $2\sqrt{x - 4} - 3 = 0$,

(________) ≥ 0. Therefore, $x \geq$ ________.

$2\sqrt{x - 4} - 3 = 0$

$2\sqrt{x - 4} - 3 +$ ________ $=$ ________

$2\sqrt{x - 4} =$ ________ Isolate the radical.

$4($________$) = 3^2$ Square both sides.

________ $-$ ________ $= 9$

$4x = 9 +$ ________ Solve.

$x = \frac{\square}{\square}$

How can you check your solution?

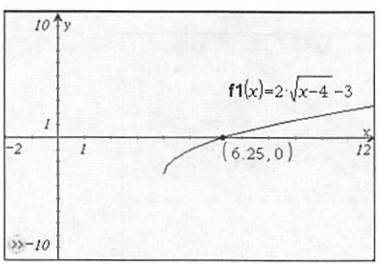
The solution is $x =$ ________.

b) Rewrite the equation as $y = 2\sqrt{x - 4} - 3$.

Then, enter the equation into your graphing calculator.

Graph the function and determine the root, or ________________.

The function has a single *x*-intercept at $x =$ ________.



c) The root that was determined algebraically is equal to the ________________ of the graph of the function $y = 2\sqrt{x - 4} - 3$.

To see a similar question, refer to Example 1 on page 91 in *Pre-Calculus 12*.

Working Example 2: Explore a Radical Equation Involving an Extraneous Root

Solve the equation $3 + \sqrt{x - 1} = x$ algebraically.

Solution

> What restriction have you identified? Why are there only restrictions for the variable under the radical?

a) State the restrictions for the variable: ______________.

$$3 + \sqrt{x - 1} = x$$

$$3 - \_\_\_\_ + \sqrt{x - 1} = x - \_\_\_\_ \quad \text{Isolate the radical.}$$

$$\sqrt{x - 1} = \_\_\_\_\_\_\_\_$$

$$(\sqrt{x - 1})^2 = (x - \_\_\_\_)^2 \quad \text{Square both sides.}$$

$$x - 1 = (x - \_\_\_\_)(x - \_\_\_\_)$$

$$0 = x^2 - \_\_\_\_ x + \_\_\_\_ - x + 1 \quad \text{Equate to 0.}$$

$$0 = x^2 - \_\_\_\_ x + \_\_\_\_ \quad \text{Combine like terms.}$$

$$0 = (x - \_\_\_\_)(x - \_\_\_\_) \quad \text{Factor and solve.}$$

$(x - \_\_\_\_) = 0$ or $(x - \_\_\_\_) = 0$

$x = \_\_\_$ $\quad$ $x = \_\_\_$

Check against the restriction and by substituting the value(s) in the original equation.

For $x =$ ________

Left Side	Right Side

Left Side ________ Right Side
(= or ≠)

For $x =$ ________

Left Side	Right Side

Left Side ________ Right Side
(= or ≠)

The root $x =$ ________ does not satisfy the original equation, so it is an ______________ root.

To see a similar question, refer to Example 2 on pages 92 in *Pre-Calculus 12*.

Working Example 3: Approximate Solutions to Radical Equations

a) Solve $4 + \sqrt{x + 4} = x - 4$ graphically.

b) Verify the solution algebraically.

Solution

a) There are two methods to solving a radical equation graphically.

Method 1: Use a Single Function and Find the *x*-Intercept(s)

Begin by stating the restrictions: $x \geq$ ________.
Then, equate the function to 0.

$$4 + \sqrt{x + 4} = x - 4$$

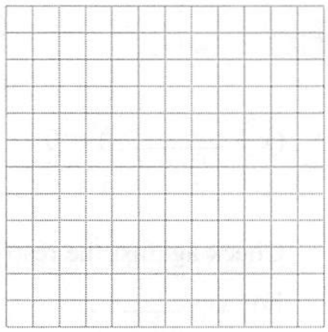
$$\_\_\_\_ - \_\_\_\_ + 4 + \sqrt{x + 4} = x - 4 + \_\_\_\_ - \_\_\_\_$$

$$\_\_\_\_ - \_\_\_\_ + \sqrt{x + 4} = 0$$

Enter the left-hand side of the equation into a graphing calculator. Then determine the ________________.

Sketch the resulting graph on the grid.

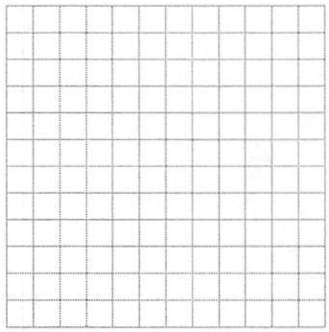
The solution to the equation $4 + \sqrt{x + 4} = x - 4$ is ________.



Method 2: Use a System of Two Functions and Find the Point of Intersection

Enter the left-hand side of the equation, $4 + \sqrt{x + 4}$, into your graphing calculator. Then, enter the right-hand side of the equation, $x - 4$. Graph the equations on the same axes and determine the ________. Sketch your graph on the grid.

The x-value of the ________________ is $x =$ ________.



b) Check the solution algebraically by substituting it into the original equation:

$$4 + \sqrt{\_\_\_\_ + 4} = \_\_\_\_ - 4$$

$$4 + \sqrt{\_\_\_\_} = \_\_\_\_$$

$$4 + \_\_\_\_ = \_\_\_\_$$

The solution, $x =$ ________, is correct.

To see a similar question, refer to Example 3 on pages 93–94 in *Pre-Calculus 12*.

Check Your Understanding

Practise

1. Determine the root(s) of each equation algebraically.

a) $y = \sqrt{x + 3} - 5$

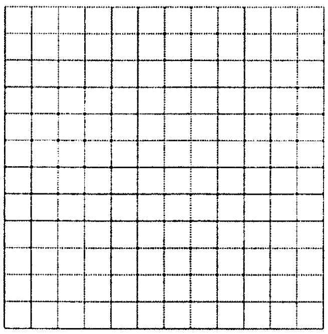
b) $y = -\sqrt{x + 6} + 7$

c) $2\sqrt{x - 4} - 1 = 7$

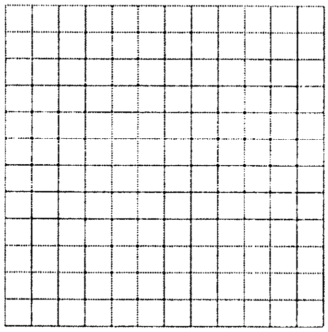
d) $\sqrt{x + 1} + 3 = 5$

2. Find the x-intercepts of each equation graphically. Include a sketch for each.

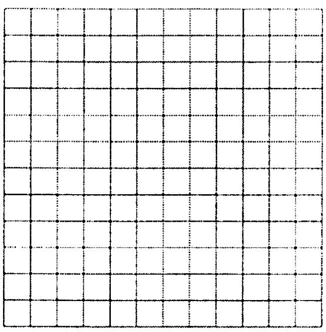
a) $y = \sqrt{x - 2} - 1$



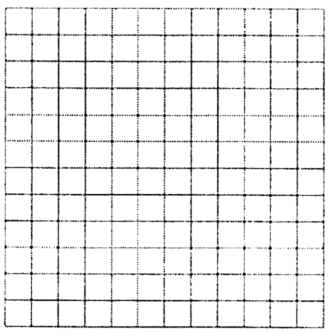
b) $y = -\sqrt{x + 3} + 2$



c) $y = \sqrt{x + 5} - 2$



d) $y = -\sqrt{x + 2} - 2$



3. Identify any restrictions on the variables. Then, use technology to solve each equation graphically. Sketch the graph on the grid.

a) $\sqrt{x + 2} - 4 = -2$

b) $\sqrt{x - 5} = 3$

c) $3\sqrt{1 - x} = 12$

d) $-2\sqrt{1 - 4x} = -6$

4. Identify any restrictions on the variables. Then, solve each equation algebraically.

a) $x = \sqrt{x + 10} + 2$

b) $x + 2 = \sqrt{-6x - 12}$

c) $x - 4 = \sqrt{-x + 4}$

d) $x = \sqrt{-5x + 26} + 4$

Apply

5. The equation $\sqrt{2x - 5} + 4 = 1$ has no solution.

a) Verify algebraically.

b) Verify graphically.

6. Use what you know about the graph of the base function $f(x) = \sqrt{x}$ and transformations. For each of the following, use a graphical method to solve the equation $f(x) = 0$.

a) $f(x) = \sqrt{x - 4}$

b) $f(x) = \sqrt{x} - 3$

c) $f(x) = -\sqrt{x} + 1$

d) $f(x) = \sqrt{-x - 3}$

Connect

7. Create a radical function of the form $y = \sqrt{bx} + k$ with a zero of $x = 8$ that passes through (2, −2).

8. Amber solved the equation $\sqrt{3x - 1} - 4 = 1$ as follows:

$$\sqrt{3x - 1} - 4 = 1$$
$$(\sqrt{3x - 1})^2 - (4)^2 = (1)^2$$
$$3x - 1 - 16 = 1$$
$$3x - 17 = 1$$
$$3x = 18$$
$$x = 6$$

Identify and correct her error.

9. The equation $3 + \sqrt{2x + 7} = 1$ has no solution.

a) Explain why it has no solution.

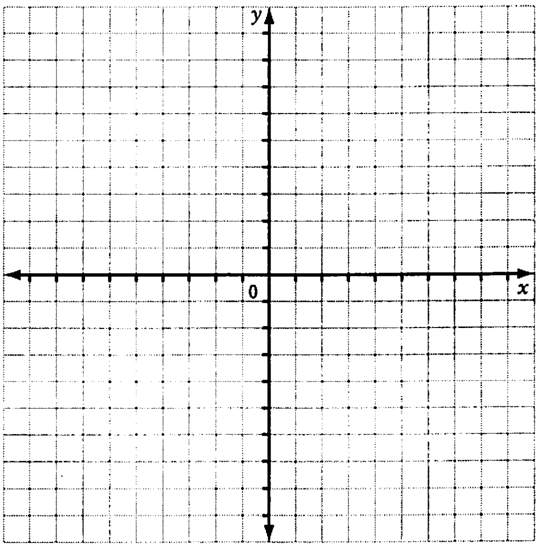
b) Create another equation with no solution.

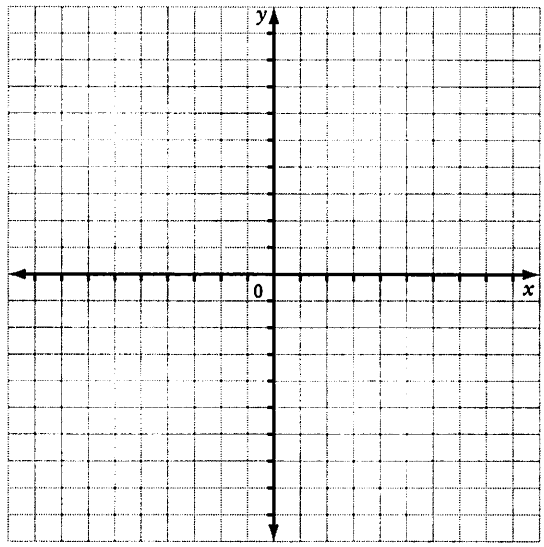
Chapter 2 Review

2.1 Radical Functions and Transformations, pages 39–46

1. Explain how to transform the graph of $y = \sqrt{x}$ to obtain the graph of each transformed function. Then, draw a sketch of the new function.

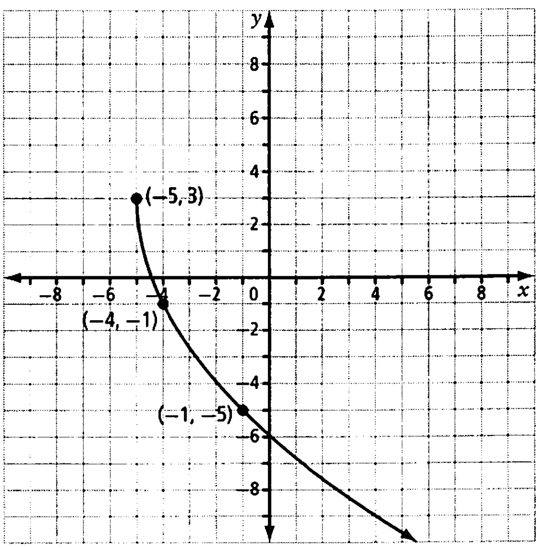
a) $y = 4\sqrt{-(x - 5)} + 1$ **b)** $y = -3\sqrt{2(x + 1)} - 3$



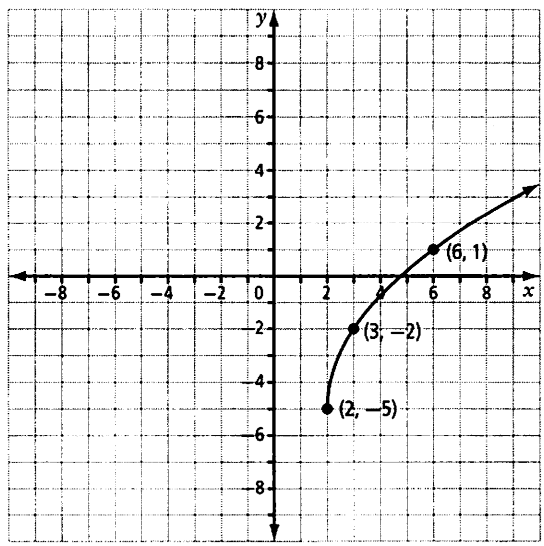


2. For each graph, write the equation of a radical function in the form $y = a\sqrt{b(x - h)} + k$. State the domain and range.

a)



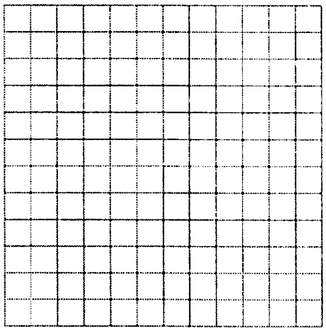
b)



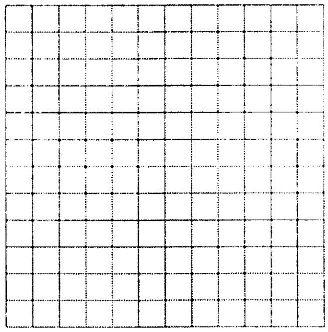
2.2 Square Root of a Function, pages 47–54

3. Use technology to graph $y = \sqrt{f(x)}$ given the following functions. Sketch the graph on the grid. State the domain and range.

   a) $f(x) = 4x - 1$



   b) $f(x) = x^2 - 9$



2.3 Solving Radical Equations Graphically, pages 55–62

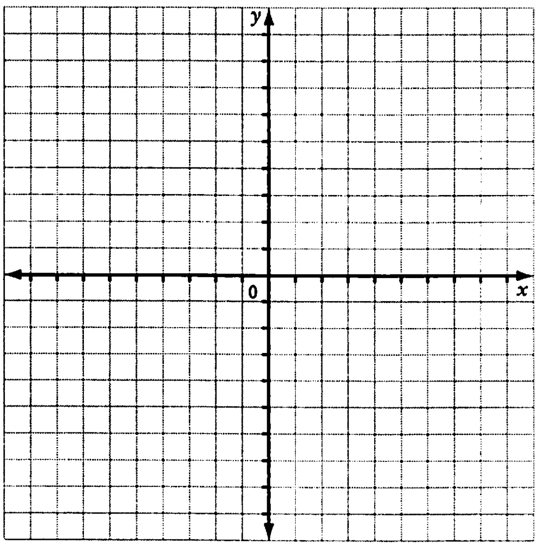
4. Determine the root(s) of each radical equation algebraically.

   a) $0 = \sqrt{x - 2} - 3$

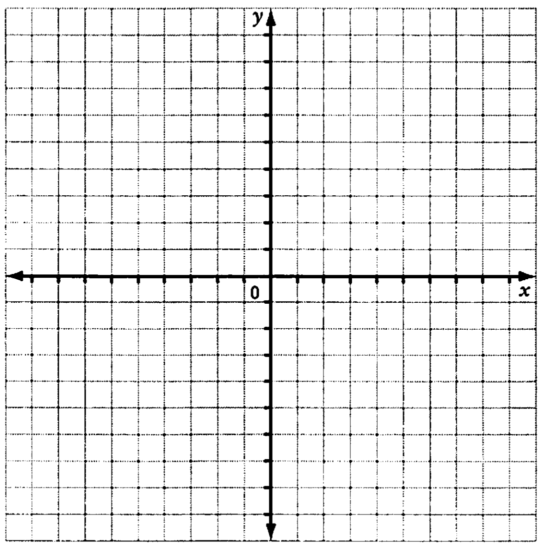
   b) $x = \sqrt{x - 2} + 4$

5. Identify any restrictions on the variables. Then, solve each radical equation graphically.

   a) $\sqrt{x - 1} - 5 = -2$



   b) $\sqrt{x + 3} = -1$



Chapter 2 Skills Organizer

Complete the graphic organizer for the concepts learned in Chapter 2. Fill in each box with notes and examples.

Definition	Example	Base Function: Graph and Characteristics

How to Solve Algebraically	How to Solve Graphically

Extraneous Roots	Restrictions	Graphing Using Transformations

Chapter 3 Polynomial Functions

3.1 Characteristics of Polynomial Functions

KEY IDEAS

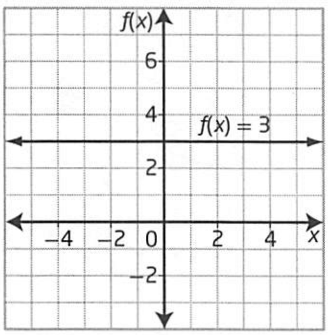
What Is a Polynomial Function?

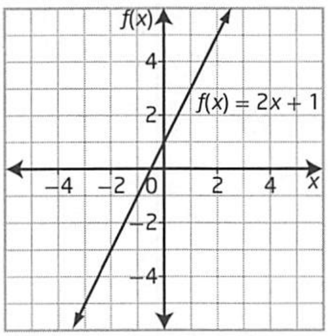
A polynomial function has the form $f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0$ where

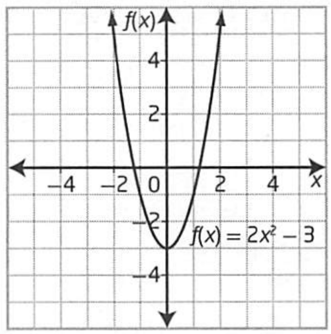
- n is a whole number
- x is a variable
- the coefficients a_n to a_0 are real numbers
- the degree of the polynomial function is n, the exponent of the greatest power of x
- the leading coefficient is a_n, the coefficient of the greatest power of x
- the constant term is a_0

Types of Polynomial Functions

Constant Function	Linear Function	Quadratic Function
Degree 0	Degree 1	Degree 2







Cubic Function	Quartic Function	Quintic Function
Degree 3	Degree 4	Degree 5