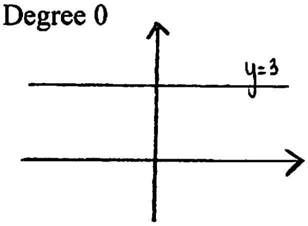
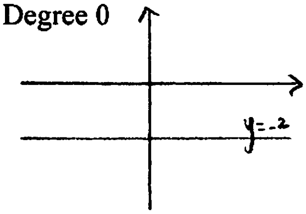
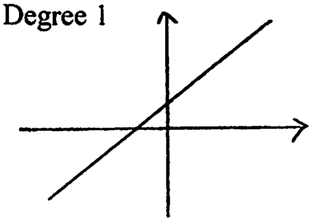
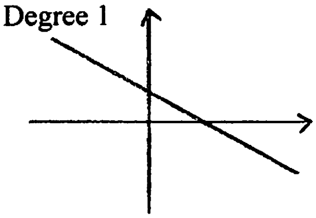
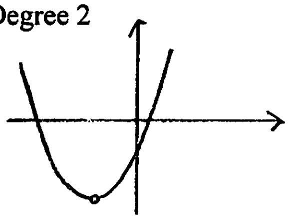
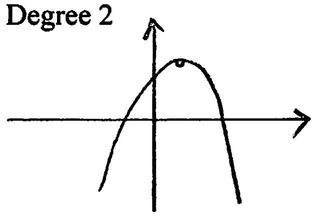
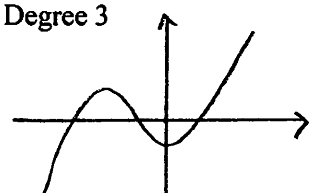
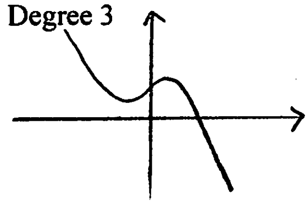
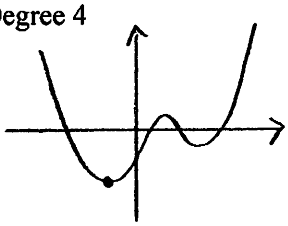
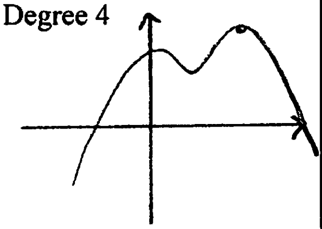
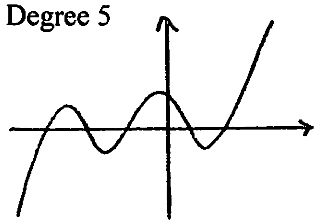
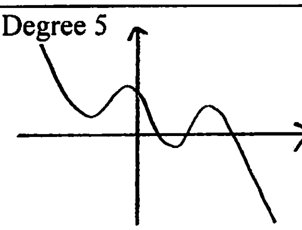
3.1 – CHARACTERISTICS OF POLYNOMIAL FUNCTIONS

Example of a polynomial function: $f(x) = 2x^4 - 5x^2 + 7$

- Degree: 4
- # terms: 3
- Constant term: 7
- Leading coefficient (LC): 2

Graphs of polynomial functions have specific characteristics depending on their degree and on the sign of their leading coefficient.
The number of "legs" tells us the degree.
For all of them: the y-intercept is the constant term, and the domain is all real numbers.

Even Degree Polynomials		**Odd Degree Polynomials**	
Positive LC	**Negative LC**	**Positive LC**	**Negative LC**
Degree 0 	Degree 0 	Degree 1 	Degree 1 
Degree 2 	Degree 2 	Degree 3 	Degree 3 
Degree 4 	Degree 4 	Degree 5 	Degree 5 
G^{al} direction: ↘ ↗ Q II → Q I MIN	G^{al} direction: ↗ ↘ Q III → Q IV MAX	G^{al} direction: ↗ ↗ Q III → Q I NO MIN NO MAX	G^{al} direction: ↘ ↘ Q II → Q IV NO MIN NO MAX
# possible zeros: 0 → degree		# possible zeros: 1 → degree	

<u>Hwk</u>: p 114 # 1 – 6, 9, 10

FH Collins – Fleur Marsella

3.2 – THE REMAINDER THEOREM

To divide polynomials, you can use long division or synthetic division.

Examples of LONG DIVISION:

1. $\frac{5x^3-13x^2+10x-9}{x-2}$

$$
\begin{array}{r|l}
 & 5x^2 - 3x + 4 \\
x-2 & 5x^3 - 13x^2 + 10x - 9 \\
 & 5x^3 - 10x^2 \\
 & \quad -3x^2 + 10x - 9 \\
 & \quad -3x^2 + 6x \\
 & \qquad 4x - 9 \\
 & \qquad 4x - 8 \\
 & \qquad\quad -1
\end{array}
$$

order the terms by descending exponents

Remainder

$$\frac{5x^3-13x^2+10x-9}{x-2} = 5x^2-3x+4 - \frac{1}{x-2}$$

2. $\frac{x^3+2x^2-5x-6}{x+1}$

$$
\begin{array}{r|l}
 & x^2 + x - 6 \\
x+1 & x^3 + 2x^2 - 5x - 6 \\
 & x^3 + x^2 \\
 & \quad x^2 - 5x - 6 \\
 & \quad x^2 + x \\
 & \qquad -6x - 6 \\
 & \qquad -6x - 6 \\
 & \qquad\quad 0
\end{array}
$$

$$\frac{x^3+2x^2-5x-6}{x+1} = x^2+x-6$$

In other words:

$$x^3+2x^2-5x-6 = (x^2+x-6)(x+1)$$

Your turn p 121

> Divide the polynomial $P(x) = x^4 - 2x^3 + x^2 - 3x + 4$ by $x - 1$.
> Express the result in the form $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$.
> Identify any restrictions on the variable.

$\rightarrow x^3 - x^2 - 3 + \frac{1}{x-1}$

Examples of SYNTHETIC DIVISION: when you divide by $x - a$

1. $\frac{5x^3-13x^2+10x-9}{x-2}$

value of a

$$
\begin{array}{c|cccc}
2 & 5 & -13 & 10 & -9 \\
+ & \downarrow & 10 & -6 & 8 \\
\hline
\times & 5 & -3 & 4 & -1
\end{array}
$$

list all the coefficients (write 0 for any missing coeff)

mult by a

repeat

remainder

$$\frac{5x^3-13x^2+10x-9}{x-2} = 5x^2-3x+4 - \frac{1}{x-2}$$

Remainder

coeff you got on your table (1 less degree)

2. $\frac{2x^3+3x^2-4x+15}{x+3}$

-3	2	3	-4	15
$+$	$\downarrow$	-6	9	-15
$\times$	2	-3	5	0

$$\frac{2x^3+3x^2-4x+15}{x+3} = 2x^2-3x+5$$

In other words:

$$2x^3+3x^2-4x+15 = (2x^2-3x+5)(x+3)$$

3. $\frac{4x^3-5x+1}{x-2}$

2	4	0	-5	1
$+$	$\downarrow$	8	16	22
$\times$	4	8	11	23

$$\frac{4x^3-5x+1}{x-2} = 4x^2+8x+11+\frac{23}{x-2}$$

Your turn p 122:

$$\frac{x^3+7x^2-3x+4}{x-2} = x^2+9x+15+\frac{34}{x-2}$$

<u>The REMAINDER THEOREM</u>: If you divide a Polynomial $P(x)$ by $(x-a)$, then the remainder is $P(a)$.

Example 1: $P(x) = 5x^3 - 13x^2 + 10x - 9$ divided by $x - 2$

$$P(2) = 5(2)^3 - 13(2)^2 + 10(2) - 9$$
$$= -1$$

like we saw in example 1

$$\frac{P(x)}{x-2} = Q(x) - \frac{1}{x-2}$$

degree 2

Example 2: $P(x) = 2x^3 + 3x^2 - 4x + 15$ divided by $x + 3$

$$P(-3) = 2(-3)^3 + 3(-3)^2 - 4(-3) + 15$$
$$= 0$$

Remainder 0. It means that $P(x) = \boxed{Q(x)}(x+3)$

i.e it can be factored by $x+3$

$\boxed{2x^2-3x+5}$

Your turn: $\frac{x^3+2x^2-5x-6}{x-2}$ and $\frac{x^3-10x+6}{x+4}$

$$2^3+2(2)^2-5(2)-6 = 0$$
$$x^3+2x^2-5x-6 = (x-2)(x^2+4x+3)$$

$$(-4)^3-10(-4)+6 = -18$$
$$\frac{x^3-10x+6}{x+4} = Q(x) - \frac{18}{x+4}$$

<u>Hwk</u>: p 124 # 3 – 15 + 16

3.3 – THE FACTOR THEOREM

FACTOR THEOREM

$P(x)$ can be factored by $(x-a)$ if and only if $P(a)=0$

Indeed : if $P(a)=0$, then the remainder is 0, therefore $\frac{P(x)}{x-a}=Q(x)$ or $P(x)=(x-a)Q(x)$.

Example : $P(x)=x^3-x^2-5x+2$
$P(-2)=(-2)^3-(-2)^2-5(-2)+2$
$=0$

Therefore, $P(x)$ can be factored by $(x+2)$
Using division or observation, we get : $P(x)=(x+2)(x^2-3x+1)$

Example 1 p 128

Which binomials are factors of the polynomial $P(x)=x^3-3x^2-x+3$? Justify your answers.

a) $x-1$ yes $P(1)=0$
b) $x+1$ yes $P(-1)=0$
c) $x+3$ no $P(-3)\neq 0$
d) $x-3$ yes $P(3)=0$

$P(x)=(x-1)(x+1)(x-3)$

Your turn p 128

Determine which of the following binomials are factors of the polynomial $P(x)=x^3+2x^2-5x-6$.

~~$x-1$~~, $\boxed{x+1}$, $\boxed{x-2}$, ~~$x+2$~~, ~~$x-3$~~, $\boxed{x+3}$, ~~$x-6$~~, ~~$x+6$~~
−8, yes, yes, 4, 24, yes, −120

$P(x)=(x+1)(x-2)(x+3)$

INTEGRAL ZERO THEOREM

If $P(x)$ can be factored by $(x-a)$ [a being an integer], then its constant term can be divided by a.

This theorem tells us that if we're looking for potential factors, we should look at the constant term, and see by what it can be divided. These will be the only potential a values to be considered.

Example: $P(x)=2x^3-5x^2-4x+3$

can be divided by ± 1 and ± 3

$P(1)=-4$
$P(-1)=0$
$P(3)=0$
$P(-3)=-84$

$P(x)=(x+1)(x-3)\,Q(x)$

$\boxed{\text{degree } 1}$

$(2x-1)$

Note: When the constant term can be divided by many integers, it can be very time consuming to evaluate the polynomial by each of them. We can use the table of our graphing calculator to save time (when allowed…)

Example: Factor $x^4 - 5x^3 + 2x^2 + 20x - 24$ ← can be divided by ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24

X	Y_1
0	-24
1	-6
2	0
3	0

$x^4 - 5x^3 + 2x^2 + 20x - 24 = (x-2)(x-3)(x^2-4)$ ← from 2 divisions…

$= (x-2)(x-3)(x+2)(x-2)$

$= (x-2)^2(x-3)(x+2)$

Your turn p 131:

What is the fully factored form of $x^4 - 3x^3 - 7x^2 + 15x + 18$?

$(x-3)^2(x+1)(x+2)$

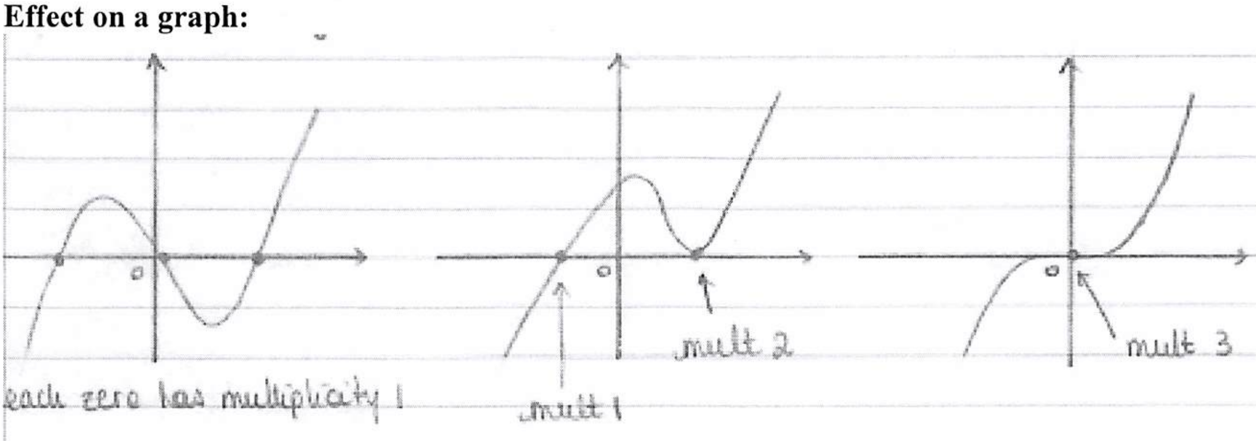
Hwk: **p 133 # 1, 2ab, 3ab, 4ab, 5 – 11, 14 – 16.**

3.4 – EQUATIONS AND GRAPHS OF POLYNOMIAL FUNCTIONS

Sometimes a polynomial has a factor $(x-a)$ repeated several times.
Example: $P(x) = (x-3)^2(x+1)$
The number of times the factor $(x-a)$ is repeated is called the **multiplicity** of the zero $x = a$.

Example: $P(x) = (x-3)^2(x+1)$ has a zero multiplicity 2 at $x = 3$ and a zero multiplicity 1 at $x = -1$.

Effect on a graph:



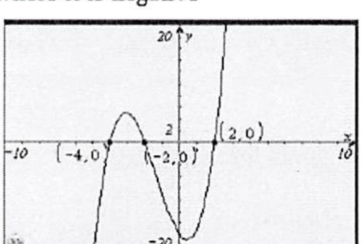
Note: When the multiplicity is odd, the sign of the function changes there.
When the multiplicity is even, it doesn't.

Getting Info from a graph:

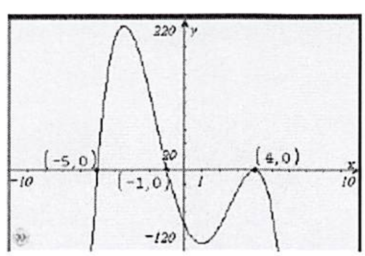
Example 1 p 138

For each graph of a polynomial function, determine
- the least possible degree
- the sign of the leading coefficient
- the x-intercepts and the factors of the function with least possible degree
- the intervals where the function is positive and the intervals where it is negative

a)



b)



coeff to be determined using a point.

$y = a(x+4)(x+2)(x-2)$

y-int: -16

$-16 = a(4)(2)(-2)$

$a = 1$

$y = (x+4)(x+2)(x-2)$

$y = a(x+5)(x+1)(x-4)^2$

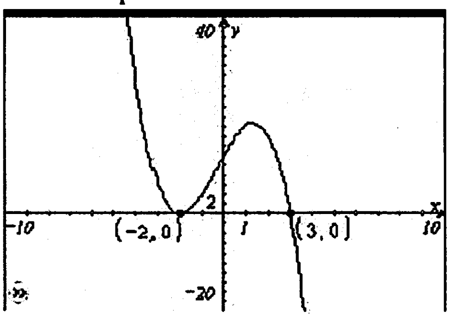
y-int: -80

$-80 = a(5)(1)(-4)^2$

$a = -1$

$y = -(x+5)(x+1)(x-4)^2$

Your turn p 139



$y = -(x+2)^2(x-3)$

Getting Info from Equations:

Example 2 p 140

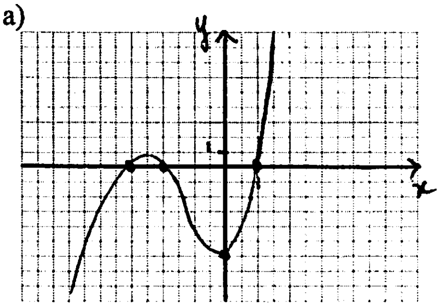
Sketch the graph of each polynomial function.

a) $y = (x-1)(x+2)(x+3)$

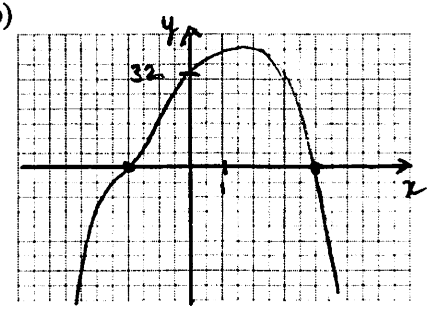
b) $f(x) = -(x+2)^3(x-4)$

c) $y = -2x^3 + 6x - 4$

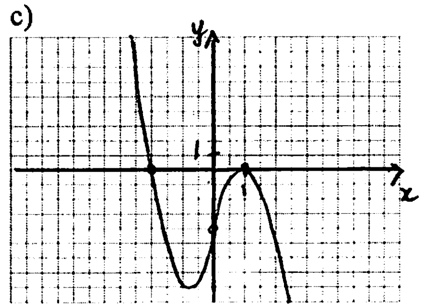
a)



b)



c)



$y = -2x^3 + 6x - 4$

$= -2(x-1)^2(x+2)$

Your turn p 142

a) $g(x) = (x-2)^2(x+1)$

b) $f(x) = -x^3 + 13x + 12$

) verif w your calc.

<u>Hwk</u>: p 147 # 1 – 10, 12 – 16.