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Precalc 12

Chapter 6 TEST

Multiple Choice

Identify the choice that best completes the statement or answers the question.

A 1. What does the expression $\cot\theta \sin\theta \csc\theta$ simplify to?

A. $\cot\theta$
B. $\tan\theta$
C. $\cos\theta$
D. $\sin\theta$

C 2. Which expression is equivalent to $\dfrac{\cos\theta}{1+\sin\theta}$?

A. $\dfrac{\cos\theta(1+\sin\theta)}{1+\sin^2\theta}$
B. $\dfrac{\cos\theta}{1-\sin\theta}$
C. $\dfrac{1-\sin\theta}{\cos\theta}$
D. $\dfrac{1+\sin\theta}{\cos\theta}$

$$\frac{\cos\theta}{1+\sin\theta} \times \frac{(1-\sin\theta)}{(1-\sin\theta)} = \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta} = \frac{\cancel{\cos\theta}(1-\sin\theta)}{\cos^{\cancel{2}}\theta}$$

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Short Answer

3. Prove that $\dfrac{\sin 3\theta+\sin\theta}{\cos 3\theta+\cos\theta} = \tan 2\theta$ for $\theta = \frac{\pi}{6}$

$$\text{L.S}: \frac{\sin\frac{3\pi}{6}+\sin\frac{\pi}{6}}{\cos\frac{3\pi}{6}+\cos\frac{\pi}{6}} = \frac{\sin\pi/2+\sin\pi/6}{\cos\pi/2+\cos\pi/6} = \frac{1+\frac{1}{2}}{0+\frac{\sqrt{3}}{2}} = \frac{\frac{3}{2}}{\frac{\sqrt{3}}{2}} = \sqrt{3}$$

$$\text{R.S}: \tan\frac{2\pi}{6} = \tan\frac{\pi}{3} = \sqrt{3} \quad \text{L.S} \checkmark$$

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4. Solve $sin2x + 2cosx = 0$, for $-\pi \le x \le 3\pi$.

- no restrictions
- $2\sin x \cdot \cos x + 2\cos x = 0$

$2\cos x(\sin x + 1) = 0$

$\cos x = 0$ or $\sin x = -1$


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Solution: $\left\{-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}\right\}$

5. What is the general solution, in radians, to the equation $(4\cos^2 2\theta + 1)\sin\frac{1}{3}\theta = 0$?

- no restriction
- $4\cos^2 2\theta + 1 = 0$ or $\sin\frac{1}{3}\theta = 0$

$\cos^2(2\theta) = -\frac{1}{4}$

impossible.

$p = 6\pi$

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$\frac{1}{3}\theta = 0$ or $\frac{1}{3}\theta = \pi$

$\theta = 0 + 6n\pi$ $\theta = 3\pi + 6n\pi$

solution : $\{0 + 3n\pi, n \in I\}$

6. Solve $2cos^2x + 3sinx - 3 = 0$ over the different domains. NO RESTRICTION.

a) $0 \le x < 2\pi$

$2(1 - \sin^2 x) + 3\sin x - 3 = 0$

$-2\sin^2 x + 3\sin x - 1 = 0$

$\Delta = 9 - 4\times(-2)\times(-1) = 1$

$\sin x = \frac{-3 \pm 1}{-4}$

$\sin x = 1$ or $\sin x = \frac{1}{2}$

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solution : $\{\pi/6, \pi/2, 5\pi/6\}$

b) General solution.

$x = \pi/6 + 2n\pi$ or $\frac{\pi}{2} + 2n\pi$ or $\frac{5\pi}{6} + 2n\pi$, $n \in I$.

7. Consider: $cscx + cotx = \frac{sinx}{1-cosx}$.

a) Determine the restrictions.

$\sin x \neq 0 \qquad \cos x \neq 1$

 

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Restrictions : $x \neq 0 + n\pi \, , \, n \in I$.

b) Prove that it's an identity.

$$\underline{L.S}: \frac{1}{\sin x} + \frac{\cos x}{\sin x} = \frac{1+\cos x}{\sin x} \times \frac{(1-\cos x)}{(1-\cos x)}$$

$$= \frac{1-\cos^2 x}{\sin x(1-\cos x)}$$

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$$= \frac{\sin^2 x}{\sin x(1-\cos x)}$$

$$= \frac{\sin x}{1-\cos x} \quad R.S \checkmark$$

8. Determine the **exact** value of $\cos 15°$ without a calculator (show your work)

$$\cos 15° = \cos(45° - 30°)$$

$$= \cos 45° \times \cos 30° + \sin 45° \times \sin 30°$$

$$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$$

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$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

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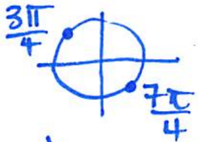
9. Consider $\dfrac{1+\tan\theta}{1+\cot\theta} = \dfrac{1-\tan\theta}{\cot\theta - 1}$.

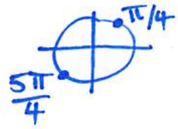
a) Determine the restrictions.

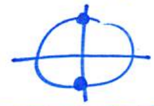
$1+\cot\theta \neq 0$ $\qquad$ $\cot\theta - 1 \neq 0$ $\qquad$ $\cos\theta \neq 0$ $\qquad$ $\sin\theta \neq 0$

$\cot\theta \neq -1$ $\qquad$ $\cot\theta \neq 1$

$\tan\theta \neq -1$ $\qquad$ $\tan\theta \neq 1$







$\boxed{\theta \neq 0 + n\pi,\ n\in I}$

$\boxed{\theta \neq \frac{\pi}{2} + n\pi,\ n\in I}$

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$\boxed{\theta \neq \frac{\pi}{4} + n\frac{\pi}{2},\ n\in I}$

Restrictions

$x \neq 0 + n\frac{\pi}{4},\ n\in I$

b) Prove that it's an identity.

L.S: $\dfrac{1+\frac{\sin\theta}{\cos\theta}}{1+\frac{\cos\theta}{\sin\theta}} = \dfrac{\frac{\cos\theta+\sin\theta}{\cos\theta}}{\frac{\cos\theta+\sin\theta}{\sin\theta}}$

$= \dfrac{\cancel{\cos\theta+\sin\theta}}{\cos\theta} \times \dfrac{\sin\theta}{\cancel{\cos\theta+\sin\theta}}$

$= \dfrac{\sin\theta}{\cos\theta}$

R.S: $\dfrac{1-\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}-1} = \dfrac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\cos\theta-\sin\theta}{\sin\theta}}$

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$= \dfrac{\cancel{\cos\theta-\sin\theta}}{\cos\theta} \times \dfrac{\sin\theta}{\cancel{\cos\theta-\sin\theta}}$

$= \dfrac{\sin\theta}{\cos\theta}$ L.S ✓