Name: Solutions

$\overline{}$ 27

Precalc 12

Chapter 7&8 TEST – Part II
Part I – CALCULATOR allowed

Multiple Choice

D 1. To the nearest year, how long would an investment need to be left in the bank at 5%, compounded annually, for the investment to triple?

A 15 years
B 26 years
C 28 years
D 23 years

$3 = 1.05^t$

$\log_{1.05}(3) = t$

B 2. Solve $10^{2x-5} = 7^{x+4}$. Round your answer to two decimal places.

A. 3.06
B. 7.26
C. 2.95
D. −1.40

2/

Problem

3. Jeff buys a new vehicle for \$35 000. It is known that the vehicle will depreciate by 20% of its current value every year.

a) Write an equation to relate the value, V, of the vehicle to the age, t, in years, of the vehicle.

$V = 35000 \times 0.8^t$

1/

b) Use the equation to determine the value of the vehicle 2 years after Jeff buys it.

$V = 35\,000 \times 0.8^2$

$V \simeq 22400\$$

1/

c) Approximately how long will it take the vehicle to depreciate to \$3000?

$3000 = 35000 \times 0.8^t$

$\frac{3}{35} = 0.8^t$

$\log\left(\frac{3}{35}\right) = t\log(0.8)$

$t = \frac{\log(3/35)}{\log(0.8)}$

$\boxed{t \simeq 11 \text{ years}}$

2/

Name: ______________________________ Precalc 12

4. Cobalt-60, which has a half-life of 5.3 years, is used in medical radiology. A sample of 60 mg of the materia is present today.
a) Write an equation to relate the amount of cobalt-60 remaining and ~~the number of half-life periods~~ the number of years t.

$$A = 60 \times \left(\frac{1}{2}\right)^{t/5.3}$$

1

b) What amount will be present in 10.6 years?

$$A = 60 \times \left(\frac{1}{2}\right)^{10.6/5.3}$$

$$= 60 \times \left(\frac{1}{2}\right)^2 = 60 \times \frac{1}{4} = \boxed{15 \text{ mg}}$$

1

c) How many years will it take for the amount of cobalt-60 to decay to 12.5% of its initial amount?

$$12.5 = 100 \times \left(\frac{1}{2}\right)^{t/5.3}$$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^{t/5.3}$$

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{t/5.3}$$

$$t = 5.3 \times 3$$

$$\boxed{t = 15.9 \text{ years}}$$

2

5. A \$21 500 investment earns 5.25% interest, compounded quarterly.
a) Determine the value of the investment in 5 years.

$$5.25 \div 4 = 1.3125$$

$$V = 21500 \times 1.013125^{4\times5}$$

$$\boxed{V \simeq \$27906.10}$$

2

b) How long will it take the original investment to double in value?

$$2 = 1.013125^{4t}$$

$$\log 2 = 4t \log(1.013125)$$

$$t = \frac{\log 2}{4 \log 1.013125}$$

$$\boxed{t \simeq 13 \text{ years}}$$

$+ 3\frac{1}{2}$ months

2

Name: ______________________________ Precalc 12

6. The magnitude of an earthquake is defined as $M = \log\left(\frac{A}{A_0}\right)$, where A is the amplitude of the ground motion and A_0 is the amplitude corrected for the distance from the actual earthquake that would be expected for a "standard earthquake." On March 2, 2012, an earthquake with a magnitude of 5.3 was recorded in Norman Wells, Northwest Territories. A few hours later, there was an aftershock that was 120 times weaker than the original earthquake.
What was the aftershock's magnitude on the Richter scale?

$$M_{NW} = 5.3 \Rightarrow 5.3 = \log\left(\frac{A_{NW}}{A_0}\right)$$

$$A_{NW} = A_0 \times 10^{5.3}$$

$$A_A = \frac{A_{NW}}{120} \Rightarrow A_A = \frac{1}{120} A_0 \times 10^{5.3}$$

2.5

$$M_A = \log\left(\frac{\frac{1}{120} A_0 \times 10^{5.3}}{A_0}\right) = \log\left(\frac{10^{5.3}}{120}\right)$$

$$\boxed{M_A \simeq 3.2}$$

7. A 200-g sample of a radioactive substance is placed in a chamber to be tested. After 3 h, 140 g of the sample remains.Determine the half-life of this substance, to the nearest hundredth of an hour.

$$140 = 200 \times \left(\frac{1}{2}\right)^{3/n}$$

$$\frac{7}{10} = \left(\frac{1}{2}\right)^{3/n}$$

$$\log\left(\frac{7}{10}\right) = \frac{3}{n} \log\left(\frac{1}{2}\right)$$

2.5

$$n = \frac{3\log\left(\frac{1}{2}\right)}{\log\left(\frac{7}{10}\right)}$$

$$\boxed{n \simeq 5.83 \text{ h}}$$

Name: ______________________________ Precalc 12

Chapter 7 TEST
Part II – NO CALCULATOR

Multiple Choice

Identify the choice that best completes the statement or answers the question.

D 8. A colony of ants has an initial population of 750 and triples every day. Which function can be used to model the ant population, p, after t days?

A $p(t) = 3(750)^t$

B $p(t) = \frac{1}{3}(750)^t$

C $p(t) = 750\left(\frac{1}{3}\right)^t$

D $p(t) = 750(3)^t$

1

Short Answer

9. Solve for x: $16^{x-1} = \left(\frac{1}{4}\right)^{4x-1}$

$4^{2(x-1)} = 4^{-(4x-1)}$

$2(x-1) = -(4x-1)$

$2x - 2 = -4x + 1$

$6x = 3$

$\boxed{x = \frac{1}{2}}$

2

10. Solve the equation $6^{3x+1} = 2^{2x-3}$.

$\log\left(6^{3x+1}\right) = \log\left(2^{2x-3}\right)$

$(3x+1)\log 6 = (2x-3)\log 2$

$3x\log 6 + \log 6 = 2x\log 2 - 3\log 2$

$3x\log 6 - 2x\log 2 = -\log 6 - 3\log 2$

$x(3\log 6 - 2\log 2) = -\log 6 - 3\log 2$

$\boxed{x = \dfrac{-\log 6 - 3\log 2}{3\log 6 - 2\log 2}}$

2

Name: ______________________________ **Precalc 12**

11. Solve for x.
$2\log_4(x+4) - \log_4(x+12) = 1$

• Restrictions: $\cdot\ x+4>0$ $\quad \cdot\ x+12>0$ $\quad D=(-4,+\infty)$
$x>-4$ $\quad x>-12$

• Resolution: $2\log_4(x+4) = 1 + \log_4(x+12)$

$\log_4(x+4)^2 = \log_4 4 + \log_4(x+12)$

$(x+4)^2 = 4(x+12)$

$x^2+8x+16 = 4x+48$

$x^2+4x-32=0$

$(x+8)(x-4)=0$

$\cancel{x=-8}$ Restr x or $\boxed{x=4}$

3