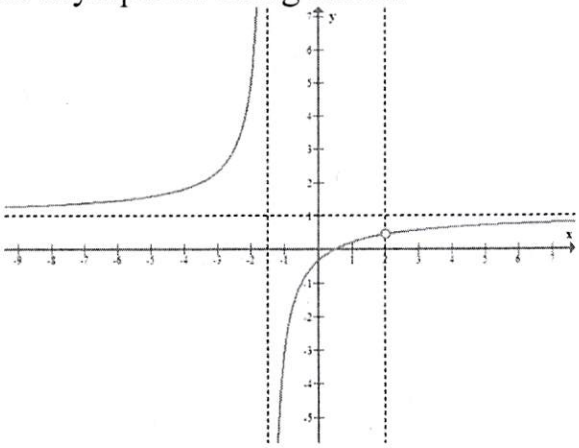
Name: Solutions

Precalc 12

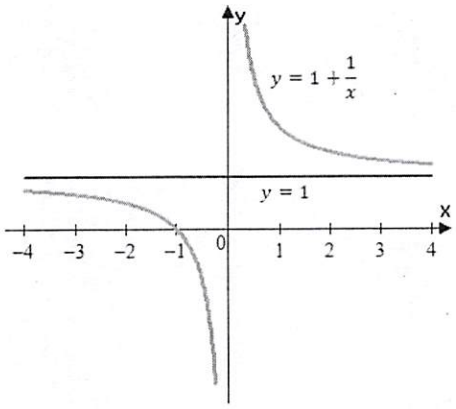
Chapter 9 – Extra Practice

Express the asymptotes using limits:



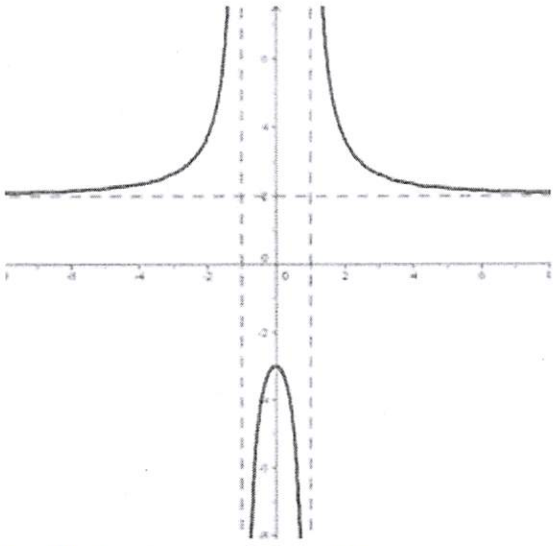
$\lim_{x \to +\infty} f(x) = 1$ $\lim_{x \to 2} f(x) = 0.4$

$\lim_{x \to -\infty} f(x) = 1$ $\lim_{x \to -1.5^+} f(x) = -\infty$ $\lim_{x \to -1.5^-} f(x) = +\infty$



$\lim_{x \to 0^+} f(x) = +\infty$ $\lim_{x \to +\infty} f(x) = 1$

$\lim_{x \to 0^-} f(x) = -\infty$ $\lim_{x \to -\infty} f(x) = 1$

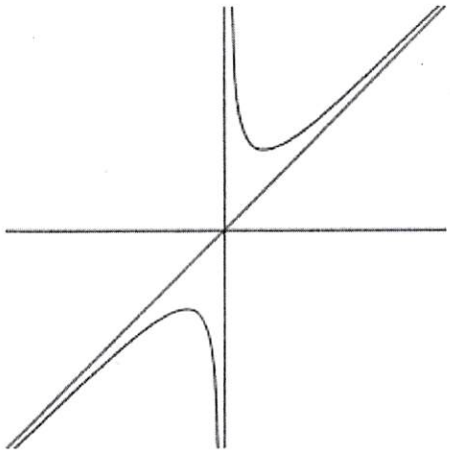


$\lim_{x \to +\infty} f(x) = 2$ $\lim_{x \to 1^+} f(x) = +\infty$

$\lim_{x \to -\infty} f(x) = 2$ $\lim_{x \to 1^-} f(x) = -\infty$

$\lim_{x \to -1^+} f(x) = -\infty$

$\lim_{x \to -1^-} f(x) = +\infty$



$\lim_{x \to 0^+} f(x) = +\infty$ $\lim_{x \to +\infty} f(x) = +\infty$

$\lim_{x \to 0^-} f(x) = -\infty$ $\lim_{x \to -\infty} f(x) = -\infty$

① $D = \mathbb{R} \setminus \{0\}$ $\quad 4 = x^2$

$\boxed{x = \pm 2}$

② $D = \mathbb{R} \setminus \{4\}$

$2x = 8 + x - 4$

$x = 4$ ~~$x \neq 4$~~ no solution.

③ $D = \mathbb{R} \setminus \{1\}$ $\quad (x-1)^2 = 4$

$x^2 - 2x + 1 = 4$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\boxed{x = 3 \text{ ou } x = -1}$

④ $D = \mathbb{R} \setminus \{-1; 5\}$

$x(x+1) - 3(x-5) = 30$

$x^2 + x - 3x + 15 = 30$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

~~$x = 5$~~ or $\boxed{x = -3}$

⑤ $D = \mathbb{R} \setminus \{-5; 1\}$

$(x+5)(x-1) - 2(x-1) = (x+3)(x+5)$

$x^2 - x + 5x - 5 - 2x + 2 = x^2 + 5x + 3x + 15$

$-6x = 18$

$\boxed{x = -3}$

⑥ $D = \mathbb{R} \setminus \{-3; \frac{1}{2}\}$

$4(x+3) = 2(2x-1)$

$4x + 12 = 4x - 2$

$12 = -2$

no solution!

⑦ $D = \mathbb{R} \setminus \{-1; 2\}$

$(x+1)^2 - (x-3)(x-2) = 3x$

$x^2 + 2x + 1 - (x^2 - 2x - 3x + 6) = 3x$

$7x - 5 = 3x$

$4x = 5$

$\boxed{x = \frac{5}{4}}$

⑧ $D = \mathbb{R} \setminus \{0; 7\}$

$10x - (x-7) = 9x + 7$

$10x - x + 7 = 9x + 7$

$7 = 7$

Solution: All real numbers except for 0 and 7.

⑨ $D = \mathbb{R} \setminus \{2\}$ $\quad x = 2 + 2(x-2)$

~~$2 = x$~~ no sol.

⑩ $D = \mathbb{R} \setminus \{-3\}$ $\quad x - 2(x+3) = -3$

~~$-3 = x$~~ no sol

⑪ $D = \mathbb{R} \setminus \{+2; \frac{1}{2}\}$ $\quad \frac{5}{2(2x-1)} + \frac{1}{2x-1} = \frac{7}{3(x-2)}$

$15(x-2) + 6(x-2) = 14(2x-1)$

$21x - 42 = 28x - 14$

$-28 = 7x$

$\boxed{x = -4}$

⑫ $D = \mathbb{R} \setminus \{\pm 5\}$

$2(x-5) + 20 = 3(x+5)$

$2x + 10 = 3x + 15$

~~$-5 = x$~~ no solut°

(see page 4)

Name: _____ **Precalc 12**

Determine the following rational functions:

a)

- vertical asymptote $x = -1$
- horizontal asymptote $y = 3$
- point of discontinuity @ 4
- no zero

$$y = \frac{3(x-4)^2}{(x+1)(x-4)}$$

b)

- vertical asymptote $x = -2$
- horizontal asymptote $y = 0$
- zero @ 4
- point of discontinuity @ $(1; -\frac{1}{3})$

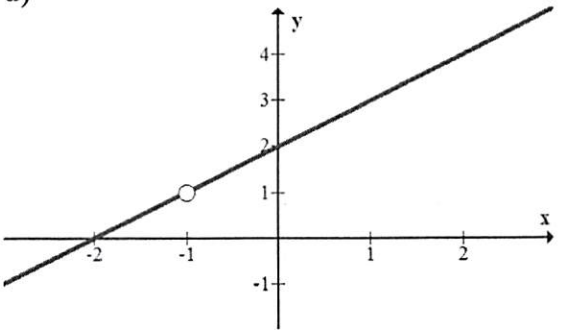
$$y = \frac{(x-4)(x-1)}{(x+2)^2(x-1)}$$

c)

- vertical asymptote $x = -2$
- horizontal asymptote $y = 0$
- zero @ 4
- point of discontinuity @ $(1; -1)$

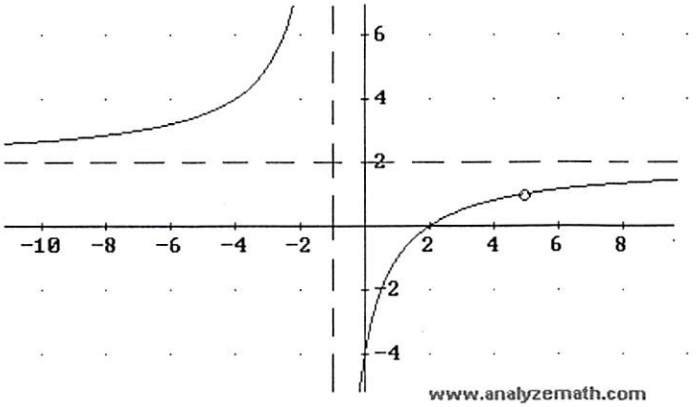
$$y = \frac{3(x-4)(x-1)}{(x+2)^2(x-1)}$$

d)



$$y = \frac{(x+1)(x+2)}{x+1}$$

e)



$$y = \frac{2(x-2)(x-5)}{(x+1)(x-5)}$$

Name: _____ **Precalc 12**

?

Fill the table and graph:

	$f(x) = \frac{3x^2 + 14x - 5}{x^2 - 25}$	$g(x) = \frac{x^3 - x}{x^2 - x - 2}$	$h(x) = \frac{2x^2 + 12x + 18}{x^3 + x^2 - 6x}$
Factored form	$f(x) = \frac{(3x-1)(x+5)}{(x+5)(x-5)}$	$g(x) = \frac{x(x+1)(x-1)}{(x-2)(x+1)}$	$h(x) = \frac{2(x+3)^2}{x(x+3)(x-2)}$
Equations of vertical asymptotes	$x = 5$	$x = 2$	$x = 0$ and $x = 2$
Equations of oblique or horizontal asymptotes	horiz. $y = 3$	oblique $y = x + 1$	horiz. $y = 0$
y-intercept	$\frac{1}{5}$	0	$\emptyset$
x-intercepts	$\frac{1}{3}$	0 and 1	$\emptyset$
Points of discontinuity	$(-5; \frac{8}{5})$	$(-1; -\frac{2}{3})$	$(-3; 0)$
Table of 4 values In order to graph more precisely	x: 1.5, 3, 7, 8.5 / y: -1, -4, 10, 7	x: 0.5, 3, 4, -2 / y: 0.2, 6, 6, -1.5	x: 1, 1.5, 0.9, -2, 3 / y: -8, -12, -9, 0.25, 4

(13) $D = \mathbb{R} \setminus \{-2; -1; 1\}$

$$\frac{3}{(x+2)(x-1)} - \frac{1}{(x+1)(x-1)} = \frac{7}{2(x+2)(x+1)}$$

$$6(x+1) - 2(x+2) = 7(x-1)$$

$$4x + 2 = 7x - 7$$

$$9 = 3x$$

$$\boxed{x = 3}$$